Nucleon charge symmetry breaking and parity violating electron-proton scattering

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The consequences of the charge symmetry breaking effects of the mass difference between the up and down quarks and electromagnetic effects for searches for strangeness form factors in parity violating electron scattering from the proton are investigated. The formalism necessary to identify and compute the relevant observables is developed by separating the Hamiltonian into charge symmetry conserving and breaking terms. Using a set of SU(6) nonrelativistic quark models, the effects of the charge symmetry breaking Hamiltonian are considered for experimentally relevant values of the momentum transfer and found to be less than about 1%. The charge symmetry breaking corrections to the Bjorken sum rule are also studied and shown to vanish in first-order perturbation theory. [S0556-2813(98)03403-7]

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I. INTRODUCTION

If one neglects the mass difference between the up and down quarks and ignores electromagnetic effects, the QCD Lagrangian that governs hadronic physics would be invariant under the interchange of up and down quarks. This invariance is called charge symmetry, which is more restrictive than isospin symmetry, which involves invariance under any rotation in isospin space. Small, but interesting, violations of charge symmetry have been discovered and are described in the reviews [1–3]. All charge symmetry breaking effects arise from the mass difference between the up and down quarks and from electromagnetic effects.

The second European Muon Collaboration (EMC) effect [4], the discovery that valence quarks carry only a small fraction of the nucleon spin, and the resulting search for strangeness in the nucleon have brought some attention to understanding the role of nucleonic charge symmetry breaking. If this symmetry holds, measurements of a parity violating electron left-right asymmetry in electron-proton scattering can determine new form factors whose origin lies only in the strange and antistrange quarks of the nucleon [5,6]. However, the symmetry does not hold precisely and it is of interest to estimate how small the effects can be. This is especially true now that the first measurement of the proton's neutral weak magnetic form factor finds a value of the strange magnetic form factor that is consistent with zero [7].

Another issue concerns the momentum transfer Q^2 dependence of any charge symmetry breaking effects. In principle, the charge symmetry breaking terms, which act as a perturbing Hamiltonian, can cause the nucleon to mix with states which would otherwise be orthogonal. Such components could cause the form factor to have a Q^2 dependence which could emphasize the effects of charge symmetry breaking. The purpose of this paper is to present arguments that such a possibility cannot occur.

It is worthwhile to discuss briefly how the assumption of charge symmetry simplifies the analysis of parity violating electron scattering [6]. The difference in cross section for right and left handed incident electrons arises from the interference of the photon and Z-boson exchange terms. In particular, the photon-electron coupling is vector and the

Z-electron coupling is axial, while the boson-proton coupling is vector. The matrix element for the vector Z-boson proton coupling, $M_{fi}^\mu(Q^2)$ is given by [8]

$$M_{fi}^\mu(Q^2) = \langle p,f|\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d|p,i\rangle - \tfrac{1}{3}\langle p,f|\bar{s}\gamma^\mu s|p,i\rangle$$
$$- 4\sin^2\theta_W J_{p,fi}^\mu(Q^2). \quad (1.1)$$

Our notation is that the $|p,i\rangle$ denotes a proton in an initial state with momentum and spin denoted by i. The terms $\bar{u}\gamma^\mu u$ and $\bar{d}\gamma^\mu d$ are evaluated at the space-time origin. The electromagnetic matrix element of the proton is denoted as $J_{p,fi}^\mu(Q^2)$, and the nucleonic term $N=p,n$ is defined as

$$J_{N,fi}^\mu(Q^2) \equiv \langle N,f| \tfrac{2}{3}\bar{u}\gamma^\mu u - \tfrac{1}{3}\bar{d}\gamma^\mu d - \tfrac{1}{3}\bar{s}\gamma^\mu s|N,i\rangle. \quad (1.2)$$

The second term of Eq. (1.1) is directly related to the strangeness of the nucleon, and is the new feature of parity-violating electron scattering. The third term of Eq. (1.1) is well measured, but to extract the strange properties it is necessary to determine the first term from independent experiments. We define this term as $X_{fi}^\mu(Q^2)$ with

$$X_{fi}^\mu(Q^2) \equiv \langle p,f|\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d|p,i\rangle. \quad (1.3)$$

If charge symmetry holds, the (u,d) quarks in the proton are in the same wave function as the (d,u) quarks in the neutron, and the strange quark wave functions of the neutron and proton are identical. In that case

$$X_{fi}^\mu(Q^2) = J_{p,fi}^\mu(Q^2) - J_{n,fi}^\mu(Q^2), \quad (1.4)$$

and the right-hand side can be well measured. We aim to study the error involved in asserting that the equality holds exactly.

Here is an outline of this paper. The next section is concerned with displaying the charge symmetry formalism which allows a definition of the terms that cause the charge symmetry breaking correction to $X_{fi}^\mu(Q^2)$. This correction $\delta X_{fi}^\mu(Q^2)$ is obtained as a specific matrix element involving the charge symmetry breaking Hamiltonian. This formalism

is general, but our application involves the nonrelativistic quark model. This model is well enough founded as to allow reasonable estimates of the charge symmetry breaking effects and is simple enough so that some general conclusions, that go beyond the specific calculations, can be drawn. Three different nonrelativistic quark models are defined in Sec. III. Computing the perturbative corrections to the form factors involves summing over all of the unperturbed intermediate states. This sum can be simplified by using an approximation in which the unperturbed Hamiltonian can be treated as a number, an average excited state mass M^*, so that the sum over states can be performed using closure. The mass M^* can be chosen so that the first correction to the closure approximation vanishes, with the result that M^* depends on Q^2 and on the perturbing Hamiltonian. This closure treatment is worked out in Sec. IV. The charge symmetry breaking observables are computed in Sec. V. Section VI discusses the charge symmetry breaking correction to the Bjorken sum rule [9]. Section VII is reserved for a summary and a discussion of the implications of the calculations. In addition, a comparison with other theories of nucleonic charge symmetry breaking [10–12] is presented.

II. CHARGE SYMMETRY FORMALISM

The isospin formalism is elaborated in several reviews [1–3]. Here we apply it to the nucleon and to the calculation of the quantity $X_{fi}^{\mu}(Q^2)$. The starting point is to realize the approximate invariance of the Lagrangian under the interchange of u and d quarks. This makes it worthwhile to define the charge symmetry operator, which is an isospin rotation by 180° about the y axis (taking the z axis to be associated with the charge). This is defined by

$$P_{cs}^{\dagger} u P_{cs} = d, \qquad (2.1)$$

with

$$P_{cs} = \exp(i\pi T_2), \qquad (2.2)$$

and

$$T_2 = \frac{1}{2}\bar{q}\tau_2 q, \qquad (2.3)$$

where q is the light (u,d) quark field operator.

The Hamiltonian consists of a charge symmetry conserving term H_0 and a breaking term H_1 such that

$$H = H_0 + H_1, \qquad (2.4)$$

with

$$[H_0, P_{cs}] = 0, \qquad (2.5)$$

and

$$[H, P_{cs}] = [H_1, P_{cs}]. \qquad (2.6)$$

The unperturbed states are denoted by a subscript 0 and defined by

$$H_0|p,i\rangle_0 = \sqrt{\bar{M}^2 + \vec{p}^2}|p,i\rangle_0 = E_i|p,i\rangle_0, \qquad (2.7)$$

where $\bar{M}$ is the average of the neutron and proton masses. We work to first order in H_1 such that the physical proton is expressed in terms of the unperturbed states by

$$|p,i\rangle = |p,i\rangle_0 + \frac{1}{E_i - H_0}\Lambda_i H_1|p,i\rangle_0. \qquad (2.8)$$

The quantity Λ_i is a projection operator on to states orthogonal to the unperturbed ground state isospin doublet:

$$\Lambda_i = I - |p,i\rangle\langle p,i| - |n,i\rangle\langle n,i|. \qquad (2.9)$$

The measured electromagnetic matrix elements are then obtained using first-order perturbation theory as

$$J_{p,fi}^{\mu}(Q^2) = {}_0\left\langle p,i\left|\left(\frac{2}{3}\bar{u}\gamma^{\mu}u - \frac{1}{3}\bar{d}\gamma^{\mu}d\right)\right.\right. \\ \left.\left.\times\left(1 + \frac{1}{E_i - H_0}2\Lambda_i H_1\right)\right|p,i\right\rangle_0, \qquad (2.10)$$

and

$$J_{n,fi}^{\mu}(Q^2) = {}_0\left\langle n,f\left|\left(\frac{2}{3}\bar{u}\gamma^{\mu}u - \frac{1}{3}\bar{d}\gamma^{\mu}d\right)\right.\right. \\ \left.\left.\times\left(1 + \frac{1}{E_i - H_0}2\Lambda_i H_1\right)\right|n,i\right\rangle_0. \qquad (2.11)$$

We may relate the neutron and proton matrix elements by using charge symmetry which holds for unperturbed states:

$$|n,i\rangle_0 = P_{cs}|p,i\rangle_0, \qquad (2.12)$$

and which also gives

$$P_{cs}^{\dagger}\bar{u}\gamma^{\mu}u P_{cs} = \bar{d}\gamma^{\mu}d. \qquad (2.13)$$

This along with Eq. (2.6) allows one to obtain the relation

$$P_{cs}^{\dagger}H_1 P_{cs} = H_1 + \Delta H, \qquad (2.14)$$

where

$$\Delta H \equiv P_{cs}^{\dagger}H P_{cs} - H. \qquad (2.15)$$

This equation is useful in identifying the charge symmetry breaking parts of the Hamiltonian which are relevant here. In particular, the isospin-vector operators are selected and doubled in taking the difference between the neutron and proton. The evaluation of ΔH will proceed by using the identity

$$P_{cs}^{\dagger}\hat{\tau}_3 P_{cs} = -\hat{\tau}_3, \qquad (2.16)$$

expressed in terms of field operators $\hat{\tau}_3 = \int d^3x[u(x)^{\dagger}u(x) - d(x)^{\dagger}d(x)]$. In first-quantized notation this is

$$P_{cs}^{\dagger}\tau_3(i)P_{cs} = -\tau_3(i). \qquad (2.17)$$

Using Eqs. (2.10) and (2.11) and recalling the definition (1.3) of the relevant quantity $X_{f,i}^\mu(Q^2)$ which involves matrix elements of the physical proton state leads to the desired result

$$X_{f,i}^\mu(Q^2) = J_{p,fi}^\mu(Q^2) - J_{n,fi}^\mu(Q^2) + \delta X_{f,i}^\mu(Q^2), \quad (2.18)$$

where

$$\delta X_{f,i}^\mu(Q^2) \equiv {}_0\!\left\langle p, f \left| \left(\frac{2}{3} \bar{d} \gamma^\mu d - \frac{1}{3} \bar{u} \gamma^\mu u \right) \frac{\Lambda_i}{E_i - H_0} 2\Delta H \right| p, i \right\rangle_0 . \quad (2.19)$$

Obtaining the equation for $\delta X_{f,i}^\mu(Q^2)$ is the main result of the present formalism. This term contributes to the observed parity violating signal in just the same way as the interesting strangeness matrix element. It is therefore necessary to have some understanding about its magnitude and its Q^2 dependence. The quantity $\delta X_{f,i}^\mu(Q^2)$ can be related to charge symmetry breaking modifications of the form factors $G_{E,M}$ or $F_{1,2}$. In particular, the F_i so obtained are the same as $\frac{1}{2}(^{u-d}F_i^{p+n} - {}^{u+d}F_i^{p-n})$ of Dmitrasinovic and Pollock [10].

III. NONRELATIVISTIC QUARK MODELS

The preceding formalism is completely general. Here we adopt the view that it is reasonable to use a set of nonrelativistic quark models to understand the rough size of effects at low Q^2 and to make first estimates of the Q^2 dependence. With these models, the necessary evaluations are not difficult and one gains insight into the physics of charge symmetry breaking.

In nonrelativistic quark models the spin and momentum of the proton are not related so that we may specify our notation by the replacement

$$|p,i\rangle \rightarrow |p,\uparrow\rangle, \quad (3.1)$$

for a spin up proton. The spin index will be treated implicitly so that $|p,\uparrow\rangle \rightarrow |p\rangle$.

The Hamiltonian is specified by a set of terms

$$H = K + V_{\text{con}} + V_{\text{em}} + V_g, \quad (3.2)$$

including the kinetic energy operator K, the confining potential V_{con} which respects charge symmetry, and the residual electromagnetic V_{em} and gluon exchange V_g interactions.

We shall use Eq. (2.15) to identify the charge symmetry breaking Hamiltonian ΔH as a sum or contributions from the different terms of the Hamiltonian. Thus we shall obtain

$$\Delta H = \Delta K + \Delta V_{\text{em}} + \Delta V_g \quad (3.3)$$

in which each term is obtained via the operation indicated in Eq. (2.15), i.e., $\Delta K = P_{\text{cs}}^\dagger K P_{\text{cs}} - K$.

Specifically, the kinetic energy term is given by

$$K = \sum_{i=1}^3 \left(m_i + \frac{p_i^2}{2m_i} \right), \quad (3.4)$$

where m_i depends on whether the ith quark is an up or down quark. We use the notation

$$m_i = \bar{m} + \frac{\Delta m}{2} \tau_3(i), \quad (3.5)$$

in which $\Delta m = m_u - m_d$. Then

$$K = K_0 + \Delta K, \quad (3.6)$$

with

$$K_0 = 3\bar{m} + \sum_i \frac{p_i^2}{2\bar{m}} \quad (3.7)$$

and

$$\Delta K = \Delta m \sum_i \tau_3(i) + \frac{\Delta m}{\bar{m}} \sum_i \frac{p_i^2}{2\bar{m}} \tau_3(i). \quad (3.8)$$

The first term of Eq. (3.8) does not modify the unperturbed wave function and is henceforth ignored.

The electromagnetic interaction contains charge symmetry breaking and more general charge dependent terms. This operator is given by

$$V_{\text{em}} = \alpha \sum_{i<j} q_i q_j \left(\frac{1}{r_{ij}} - \frac{\pi}{2} \delta(\vec{r}_{ij}) \left[\frac{2}{\bar{m}^2} + \frac{4}{3} \frac{\vec{\sigma}(i) \cdot \vec{\sigma}(j)}{\bar{m}^2} \right] \right), \quad (3.9)$$

where $q_i = \frac{1}{6} + \frac{1}{2} \tau_3(i)$ and $\vec{r}_{ij} \equiv \vec{r}_i - \vec{r}_j$. The charge asymmetric part of V_{em} is given according to Eq. (2.15) as

$$\Delta V_{\text{em}} = -\frac{\alpha}{6} \sum_{i<j} [\tau_3(i) + \tau_3(j)] \left(\frac{1}{r_{ij}} - \frac{\pi}{\bar{m}^2} \delta(\vec{r}_{ij}) \right.$$

$$\left. \times \left[1 + \frac{2}{3} \vec{\sigma}(i) \cdot \vec{\sigma}(j) \right] \right). \quad (3.10)$$

We take the gluon exchange operator to be

$$V_g = -\alpha_s \sum_{i<j} \lambda_i \cdot \lambda_j \left[\frac{\pi}{2} \delta(\vec{r}_{ij}) \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{3} \frac{\vec{\sigma}(i) \cdot \vec{\sigma}(j)}{m_i m_j} \right) \right], \quad (3.11)$$

where for three quark baryons $\lambda_i \cdot \lambda_j = -\frac{2}{3}$. The long range $1/r_{ij}$ term respects charge symmetry and is not included here. Such a term is included, in principle, as part of the flavor independent confining interaction. The charge symmetry breaking piece of V_g is given by

$$\Delta V_g = \alpha_s \frac{2\pi}{3} \frac{\Delta m}{\bar{m}^3} \sum_{i<j} [\tau_3(i) + \tau_3(j)]$$

$$\times \left[1 + \frac{2}{3} \vec{\sigma}(i) \cdot \vec{\sigma}(j) \right] \delta(\vec{r}_{ij}). \quad (3.12)$$

We note that the short-range terms of the electromagnetic and gluon exchange operators are rather similar, so that we may rewrite the charge symmetry breaking Hamiltonian as

$$\Delta H = \Delta K + \Delta V_L + \Delta V_S, \quad (3.13)$$

where

$$\Delta V_L = -\frac{\alpha}{6}\sum_{i<j}[\tau_3(i)+\tau_3(j)]\frac{1}{r_{ij}}, \qquad (3.14)$$

and

$$\Delta V_s = \left(-\frac{\alpha}{6}+\frac{2}{3}\alpha_s\frac{\Delta m}{m}\right)\frac{\pi}{\overline{m}^2}\sum_{i<j}[\tau_3(i)+\tau_3(j)]\delta(\vec{r}_{ij})$$

$$\times\left(1+\frac{2}{3}\vec{\sigma}(i)\cdot\vec{\sigma}(j)\right). \qquad (3.15)$$

These expressions are used to simplify the evaluations performed in the next section.

To proceed further we need to specify the confining potential and its ground state wave function. We shall use oscillator confinement for most of the calculations of this paper. Thus we write

$$|p\rangle_0 = |\Psi\rangle\frac{1}{\sqrt{2}}(|\phi_S\rangle|\chi_S\rangle+|\phi_A\rangle|\chi_A\rangle), \qquad (3.16)$$

where

$$\langle\vec{r}_i|\Psi\rangle = \Psi(\rho,\lambda) = Ne^{-(\rho^2+\lambda^2)/2\beta}. \qquad (3.17)$$

Here $\vec{\rho}\equiv 1/\sqrt{2}(\vec{r}_1-\vec{r}_2)$ and $\vec{\lambda}\equiv 1/\sqrt{6}(\vec{r}_1+\vec{r}_2-2\vec{r}_3)$, and the dependence on the position of the center of mass is not made explicit. Standard [13,14] mixed symmetric spin (ϕ_S) and isospin (χ_S) wave functions are used. The mixed antisymmetric ones are denoted by the subscript A. If oscillator confinement is used, the full charge-asymmetric kinetic energy operator can be incorporated exactly into the operator H_0. This is the procedure of Ref. [10]. We keep the first-order perturbative treatment here for two reasons. First, all effects of first order in m_d-m_u can be treated in the same way and second we wish to go beyond the effects of oscillator confinement. However, this difference in procedure does not lead to differences in the results of first order in m_d-m_u.

The above wave function can be used to compute the electric G_E and magnetic (G_M) form factors. In the nonrelativistic quark model these are given by the expressions

$$G_E(Q^2) = \left\langle p\left|\sum_i\frac{1}{2}\left[\frac{1}{3}+\tau_3(i)\right]e^{i\vec{q}\cdot\vec{r}_i}\right|p\right\rangle \qquad (3.18)$$

and

$$G_M(Q^2) = \bar{M}\left\langle p\left|\sum_i\frac{1}{2}\left[\frac{1}{3}+\tau_3(i)\right]\frac{\sigma_3(i)}{m_i}e^{i\vec{q}\cdot\vec{r}_i}\right|p\right\rangle, \qquad (3.19)$$

in which $Q^2=\vec{q}\cdot\vec{q}$. These expressions need to be discussed because the equations that relate $J^\mu_{N,f,i}$ of Eq. (1.2) to the form factors G_E and G_M depend on the nucleon mass and one must therefore specify whether it is the proton or neutron mass or the average that enters. The discussion in [10] shows that in the Breit frame the quantity G_M/M_N is proportional to the matrix element of the quark magnetic moment operator $\sigma_3(i)/m_i$. However, the discussion in Halzen and Mar-

tins [14] shows that the $1/M_N$ factor does not appear in the definition of G_M, but that there is a factor of M_N in the definition of G_E. The difference arises because of different choices of the normalization of nucleon spinors. One cannot tell which is more appropriate without doing a more complete treatment in which the nucleon-spinor representation is derived from the quark model. Since the spin and total momentum degrees of freedom are uncoupled, such derivation is beyond the scope of the nonrelativistic quark model. Thus we simply use the average mass $\bar{M}$ in Eq. (3.19). This introduces a difference between our approach and that of Ref. [10]. In principle, the differences are of order $(M_n-M_p)/\bar{M}\approx 1.3\times10^{-3}$ and ignorable [15]. We remind the reader that it is the quark mass difference, not the neutron proton mass difference, that sets the scale of the charge symmetry breaking effects. The former quantity is larger than the latter because it must compensate for the effects of the electromagnetic interaction and the quark-mass dependence of the gluon exchange interaction which would cause the proton to be more massive than the neutron. The values that m_d-m_u might take in different models are discussed next.

Neutron-proton mass difference and model parameters. The parameters of the nonrelativistic quark model shall be determined from the neutron proton mass difference and a consideration of pionic effects. In first-order perturbation theory the mass difference between the neutron and the proton can be expressed as a matrix element of ΔH:

$$M_n-M_p = {}_0\langle p|\Delta H|p\rangle_0. \qquad (3.20)$$

Evaluating the individual terms of Eq. (3.13) yields the following results:

$${}_0\langle p|\Delta K|p\rangle_0 = -\frac{(m_d-m_u)}{2\beta\overline{m}^2}, \qquad (3.21)$$

$${}_0\langle p|\Delta V_{\text{em}}|p\rangle_0 = -\frac{\alpha}{3}\sqrt{\frac{2}{\pi\beta}}\left(1-\frac{5}{12\overline{m}^2\beta}\right), \qquad (3.22)$$

and

$${}_0\langle p|\Delta V_g|p\rangle_0 = -\frac{\alpha_S(m_d-m_u)}{\overline{m}^3\beta^{3/2}}\sqrt{\frac{2}{\pi}}\frac{5}{9}. \qquad (3.23)$$

Adding the individual terms leads to

$$M_n-M_p = (m_d-m_u)\left[1-\frac{1}{2\beta\overline{m}^2}-\frac{\alpha_s}{\overline{m}^3\beta^{3/2}}\sqrt{\frac{2}{\pi}}\frac{5}{9}\right]$$

$$-\frac{\alpha}{3}\sqrt{\frac{2}{\pi\beta}}\left(1-\frac{5}{12\overline{m}^2\beta}\right). \qquad (3.24)$$

The parameters to be determined are β, α_s, and $\bar{m}$. We shall use $\bar{m}=337$ MeV as this leads to a proton magnetic moment of 2.79 μ_N. The model used does not include pionic effects because these are essentially charge symmetric (as discussed below), but any consideration of the parameters should take implicit account of the pion cloud. We follow the ideas of the cloudy bag model [16] in which a perturbative

TABLE I. Parameters of the nonrelativistic quark models.

Model	1	2	3
$\sqrt{\beta}$ (fm)	0.7	0.6	0.5
$\sqrt{\beta}\bar{m}$	1.20	1.02	0.85
α_s	2.3	1.20	0.35
$m_d - m_u$ (MeV)	5.2	3.8	2.3
γ	0.80	0.67	0.33

treatment of pions as quantum fluctuations converges for bag radii greater than about 0.6 fm. The importance of pionic effects decreases as the bag radius R_B increases. The parameter β is essentially the mean square radius of the nucleon (which corresponds to about 0.6 of R_B^2). We use the calculation of the Δ-nucleon mass difference as a measure of pionic effects. The gluonic contribution is given by

$$(M_\Delta - M_N)_g = \frac{2}{3}\sqrt{\frac{2}{\pi\beta}}\frac{\alpha_s}{\bar{m}^2\beta}. \qquad (3.25)$$

The physical value of this difference is taken here to be 300 MeV, but pionic effects also contribute. So $(M_\Delta - M_N)_g$ is a fraction γ of 300 MeV. Larger values of γ correspond to smaller pionic contributions and larger values of β. Three typical choices of parameters are shown in Table I. We shall investigate the charge symmetry breaking using each of the three models.

IV. CLOSURE APPROXIMATION

We are interested in computing the charge symmetry breaking observables represented by Eq. (2.19). The different values of μ and the different helicities specified by the quantum numbers i,f can be used to specify the contributions to the electric E and magnetic M terms. Separating these terms and using the nonrelativistic wave function allows specifies Eq. (2.19) to

$$\delta G_{E,M}(Q^2) = {}_0\langle p|O_{E,M}(q)\frac{\Lambda}{\bar{M}-\bar{H}_0}2\Delta H|p\rangle_0, \qquad (4.1)$$

where

$$O_E(q) \equiv \sum_i \left(\frac{1}{6} - \frac{\tau_3(i)}{2}\right)e^{i\vec{q}\cdot\vec{r}_i}, \qquad (4.2)$$

and

$$O_M(q) \equiv \frac{\bar{M}}{\bar{m}}\sum_i \left(\frac{1}{6} - \frac{\tau_3(i)}{2}\right)\sigma_3(i)e^{i\vec{q}\cdot\vec{r}_i}. \qquad (4.3)$$

The operator $\bar{H}_0$ removes the center-of-mass kinetic energy operator from H_0:

$$\bar{H}_0 = H_0 - \frac{(\Sigma_i p_i)^2}{2\Sigma_i m_i}. \qquad (4.4)$$

The expression (4.1) depends only on internal coordinates ρ and λ, so that the projection operator Λ does not depend on the initial and final nucleon momentum:

$$\Lambda = I - |p\rangle_0 {}_0\langle p| - |n\rangle_0 {}_0\langle n|. \qquad (4.5)$$

The evaluation of Eq. (4.1) depends on knowing the energies and wave functions of all of the eigenstates of $\bar{H}_0$. We shall replace $\bar{H}_0$ by a number $M^*_{E,M}(Q^2,\Delta H)$ which is expected to depend on the momentum transfer, whether the electric or magnetic term is to be evaluated, and on the operator ΔH. This quantity is determined from the condition that the first correction to the simplification of the energy denominator by treating $\bar{H}_0$ as a number vanishes. This determination is accomplished by adding and subtracting $M^*[Q^2,\Delta H]$ to $\bar{H}_0$:

$$\bar{H}_0 = M^*(Q^2,\Delta H) + [\bar{H}_0 - M^*(Q^2,\Delta H)], \qquad (4.6)$$

and rewriting the energy denominator of Eq. (2.19) as

$$\frac{1}{\bar{M}-\bar{H}_0} \approx \frac{1}{\bar{M}-M^*(Q^2,\Delta H)} + \frac{1}{[\bar{M}-M^*(Q^2,\Delta H)]^2}[\bar{H}_0 - M^*(Q^2,\Delta H)]. \qquad (4.7)$$

The requirement that the (unperturbed) ground state expectation value of the second term of Eq. (4.7) vanishes leads to the result

$$M^*_{E,M}(Q^2,\Delta H) - \bar{M} = \frac{1}{2}\frac{{}_0\langle p|[[O_{E,M}(q),\bar{H}_0],\Delta H]|p\rangle_0}{{}_0\langle p|O_{E,M}(q)\Lambda\Delta H|p\rangle_0}. \qquad (4.8)$$

The use of the double commutator allows a straightforward evaluation of the various average masses of the excited states. Observe that these masses depend on the operator ΔH and will be different for the different contributions to ΔH. It is convenient to define

TABLE II. Relevant integrals.

$J_i(Q^2) \equiv \int d^3\rho\, d^3\lambda\, |\psi(\rho,\lambda)|^2 e^{i\vec{q}\cdot\vec{r}_3}O_i$	
$O_1 = \dfrac{1}{r_{12}}$	$J_1(Q^2) = \sqrt{\dfrac{2}{\pi\beta}}e^{-Q^2\beta/6}$
$O_2 = \delta(\vec{r}_{12})$	$J_2(Q^2) = \dfrac{1}{4}\sqrt{\dfrac{2}{\pi^3\beta^3}}e^{-Q^2\beta/6}$
$O_3 = \dfrac{1}{r_{13}}$	$J_3(Q^2) = \sqrt{\dfrac{2}{\pi\beta}}e^{-Q^2\beta/24}S_1(Q^2\beta/2)$
$O_4 = \delta(\vec{r}_{13})$	$J_4(Q^2) = \dfrac{1}{4}\sqrt{\dfrac{2}{\pi^3\beta^3}}e^{-Q^2\beta/24}$

$$\Delta E_{E,M}(Q^2, \Delta H) \equiv \bar{M} - M^*_{E,M}(Q^2, \Delta H), \qquad (4.9)$$

and also to use corresponding definitions for the individual contributions to ΔH.

The contributions to the electric terms can be obtained in a straightforward manner. One simplification is that ${}_0\langle p|O_E(q)|p\rangle_0 = 0$. Then

$$\delta G_E(Q^2) = \frac{{}_0\langle p|O_E(q)2\Delta K|p\rangle_0}{\Delta E_E(Q^2, \Delta K)} + \frac{{}_0\langle p|O_E(q)2\Delta V_L|p\rangle_0}{\Delta E_E(Q^2, \Delta V_L)}$$

$$+ \frac{{}_0\langle p|O_E(q)2\Delta V_S|p\rangle_0}{\Delta E_E(Q^2, \Delta V_S)}, \qquad (4.10)$$

and

$$\delta G_M(Q^2) = \frac{{}_0\langle p|O_M(q)\Lambda 2\Delta K|p\rangle_0}{\Delta E_M(Q^2, \Delta K)}$$

$$+ \frac{{}_0\langle p|O_M(q)\Lambda 2\Delta V_S|p\rangle_0}{\Delta E_M(Q^2, \Delta V_S)}$$

$$+ \frac{{}_0\langle p|O_M(q)\Lambda 2\Delta V_L|p\rangle_0}{\Delta E_M(Q^2, \Delta V_L)}. \qquad (4.11)$$

The evaluation of the various terms $\Delta E_{E,M}(Q^2, \Delta H)$ is a straightforward but tedious procedure, simplified by the feature that only the K_0 part of H_0 contributed to the commutator $[O_{E,M}, \bar{H}_0]$ [17]. Some of the relevant integrals are given in Table II. Using the ΔK in the double commutator leads to the result

$$\Delta E_{E,M}(Q^2, \Delta K) = -\frac{2}{\bar{m}\beta} = -2\hbar\omega. \qquad (4.12)$$

That the above result must be obtained is an immediate consequence of the oscillator confinement: the p^2 operator acting on the ground state leads either to the ground state or to the $2\hbar\omega$ excited state. Here the procedure of evaluating the double commutator was followed as a check on the algebraic procedure.

The use of the long range $1/r_{ij}$ part of the electromagnetic operator leads to the following result for the related average excitation energy:

$$\Delta E_{E,M}(Q^2, \Delta V_L) = \frac{-1}{3\bar{m}} \frac{(5/6)\, Q^2 e^{-Q^2\beta/24} S_1(Q^2\beta/2) + (e^{-Q^2\beta/24}/\beta)\, S_2(Q^2\beta/2)}{e^{-Q^2\beta/24} S_1(Q^2\beta/2) - e^{-Q^2\beta/6}}, \qquad (4.13)$$

where

$$S_1(x) \equiv \sum_{n=0}^{\infty} (-x)^n \frac{n!}{(2n+1)!}, \qquad (4.14)$$

and

$$S_2(x) = 4x \frac{dS_1}{dx}. \qquad (4.15)$$

Note that the average excitation energy turns out to be the same for magnetic and electric probes. This is a consequence of the simple wave functions employed and is related to the feature that the electric and magnetic form factors have the same Q^2 dependence.

The low momentum transfer limit

$$\lim_{Q^2 \to 0} \Delta E_{E,M}(Q^2, \Delta V_L) = -\frac{4}{\bar{m}\beta} \qquad (4.16)$$

shows that the $1/r_{ij}$ operator excites states of higher energy than does the kinetic energy operator.

Using the delta function contribution to ΔH leads to the following result:

$$\Delta E_{E,M}(Q^2, \Delta V_S) = -\frac{5}{9} \frac{(Q^2/\bar{m})\, e^{-Q^2\beta/24}}{e^{-Q^2\beta/24} - e^{-Q^2\beta/6}}, \qquad (4.17)$$

and the low Q^2 limit is given by

$$\lim_{Q^2 \to 0} \Delta E_{E,M}(Q^2, \Delta V_S) = -\frac{40}{9m\beta}. \qquad (4.18)$$

The latter expression shows that the delta function operator is the most effective (of the ones we consider) at exciting the highest energy states.

The terms $\Delta E_{E,M}$ depend only on the variable $Q^2\beta/2$. If one multiplies $\Delta E_{E,M}$ by $\bar{m}\beta$ the result is a function that is independent of the three models used here. This is shown in Fig. 1. Note that $\bar{m}\beta$ decreases by a factor of about two as one changes from model 1 to model 3. Thus model 1 corresponds to the smallest energy denominators. We shall display results for $Q^2\beta/2 \leq 10$. Thus the maximum value of Q^2 is 1.6, 2.2, and 3.1 GeV$^2/c^2$ for models 1–3. The planned parity violation experiments are planned for values of Q^2 ranging from about 0.1 to 3 GeV$^2/c^2$ [18].

V. CHARGE SYMMETRY BREAKING OBSERVABLES

We are now in a position to evaluate the effects of charge symmetry breaking for any value of Q^2. The charge symmetry breaking interactions ΔV_{em} and ΔV_g include two-body interactions that can be expected to lead to effects that fall off slowly with increasing values of Q^2. We must compare such effects with the form factors G_E and G_M computed in the limit in which charge symmetry holds. This is because the gluon exchange interaction V_g includes a charge symmetric term which will also lead to slowly falling form factors. We will see that this feature of the strong form factors pre-
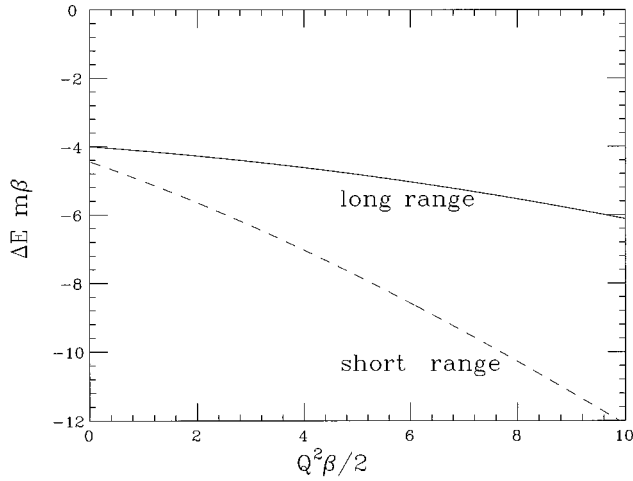
FIG. 1. Energy denominators vs $Q^2\beta/2$. The term m of the figure is $\bar{m}$ of the text. Solid-energy denominator for the long range operator Eq. (4.13). Dashed-energy denominator for the short range operator Eq. (4.17).

cludes a significant enhancement of charge symmetry breaking effects for even the highest values of Q^2 that we consider. Thus the first task is to evaluate $G_{E,M}$ using the wave function $|p\rangle_0$.

A. $G_{E,M}(Q^2)$ with charge symmetry

We shall evaluate $|p\rangle_0$ as arising from the harmonic confining potential including also the first-order effects of V_g. Starting with perturbation theory is reasonable because the first-order contribution of V_g to the nucleon mass is only -60 MeV for model 1 and -25 MeV for model 3. We shall see that for the range of Q^2 between 0 and 3 GeV^2/c^2 relevant here the influence of V_g on the computed form factors can be reasonably large. This is especially true for model 1 for which $\alpha_s = 2.3$ as shown in Table I. We find

$$G_E(Q^2) = \exp(-Q^2\beta/6) + \Delta G_E(Q^2), \tag{5.1}$$

with

$$\Delta G_E(Q^2)$$
$$= -4\alpha_s \frac{\pi}{3\bar{m}^2} \frac{J_2(Q^2) - J_2(0) - 2[J_4(Q^2) - J_4(0)]}{\Delta E_e(Q^2, \Delta V_s)}.$$

The integrals $J_i(Q^2)$ are tabulated in Table II.

Similarly the magnetic form factor is obtained as

$$G_M(Q^2) = \mu_p e^{-Q^2\beta/6} + \Delta G_M(Q^2), \tag{5.2}$$

where $\mu_p = 2.79$ and

$$\Delta G_M(Q^2) = \alpha_s \mu_p \frac{8\pi}{3\bar{m}^2} \frac{\tilde{J}_4(Q^2)}{\Delta E_m(Q^2, \Delta V_S)}, \tag{5.3}$$

with

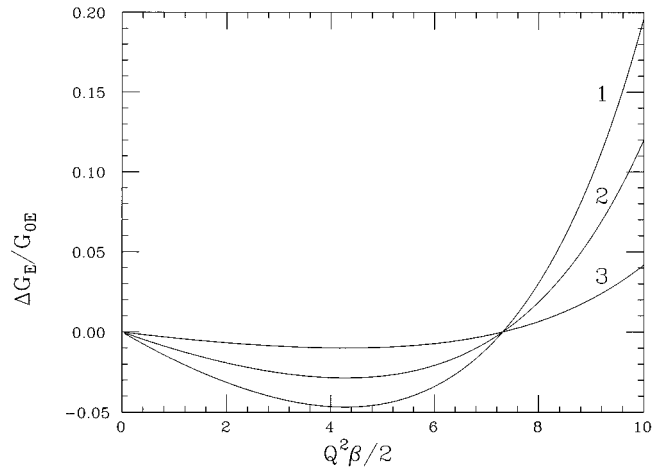$$\tilde{J}_{3,4}(Q^2) \equiv J_{3,4}(Q^2) - J_{3,4}(0) e^{-Q^2\beta/6}. \tag{5.4}$$

FIG. 2. Changes in the electric form factor due to V_g. The ratio of the second to first terms of Eq. (5.1). The numbers refer to models 1–3.

The ratios $\Delta G_{E,M}/G_{0E,M}$, where the form factors in the absence of gluon exchange are given by $G_{0E}(Q^2) = \exp(-Q^2\beta/6)$ and $G_{0M}(Q^2) = \mu_P \exp(-Q^2\beta/6)$ are shown in Figs. 2 and 3. Both ratios vanish at $Q^2 = 0$. Charge conservation mandates that this be so for the electric form factor. However, the change in the magnetic term vanishes also for $Q^2 = 0$ because of the specific simplicities in the model unperturbed wave function—the spatially symmetric wave function multiplies the symmetric spin-isospin wave function. The correction ΔG_E is reasonably small, less than 20% for all of the values of Q^2 that we consider, but the magnetic correction ΔG_M can be very large. If the absolute magnitude ratio $\Delta G_M/G_{,M}$ is larger than about 0.3, we can expect that the perturbative treatment errs by more than about 10%. Hence, the largest values of $Q^2/2\beta$ for which the models can be considered well defined are ≈ 5 and 7 for models 1 and 2. We will display the charge symmetry breaking form factors for values $Q^2/2\beta$ larger than those limits to provide information about the models, but the reader is cautioned against taking those results seriously.
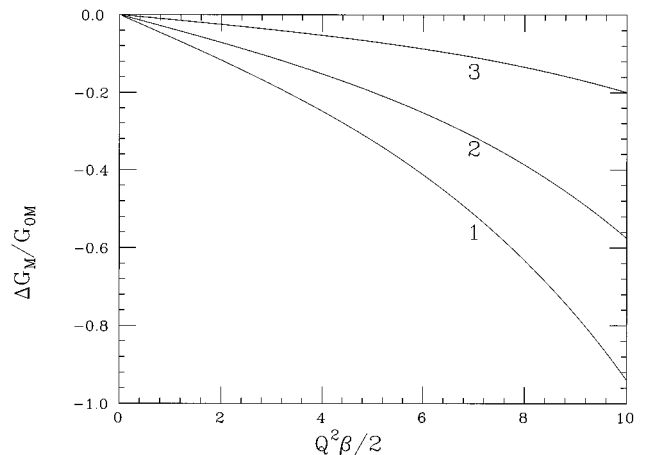

FIG. 3. Changes in the magnetic form factor due to V_g. The ratio of the second to first terms of Eq. (5.2). The numbers refer to models 1–3.

B. Charge symmetry breaking

We are now ready to evaluate the influence of charge symmetry on the measured electric and magnetic form factors. We work to first order in perturbation theory (considering the charge symmetry conserving one gluon exchange interaction as a first order effect). The necessary equations (4.10) and (4.11) are evaluated using the charge symmetry breaking interactions of Eqs. (3.8), (3.10), and (3.12). The average excitation energies are given in Eqs. (4.12), (4.13), and (4.17).

The evaluations are straightforward, so we simply express the results. We consider the influence of each charge symmetry breaking interaction ΔK, ΔV_{em}, and ΔV_g separately. Thus the contribution of ΔK to the electric form factor is given by

$$\delta G_E(Q^2,K) = -\frac{1}{9}\frac{\Delta m}{\Delta E_e(Q^2,K)}\frac{Q^2}{\overline{m}^2}e^{-Q^2\beta/6}, \quad (5.5)$$

while the magnetic form factor has a term

$$\delta G_M(Q^2,K) = -\frac{1}{27}\frac{\Delta m}{\Delta E_m(Q^2,K)}Q^2\beta e^{-Q^2\beta/6}. \quad (5.6)$$

We see that the effects are order $\delta m/\Delta E \ll \delta m/\overline{m}$ times a small coefficient. Furthermore, the Q^2 dependence $\delta G_{E,M}(Q^2,K) \sim Q^2\beta e^{-Q^2\beta/6}$ is different than that of the leading order dominant term $\sim \beta e^{-Q^2\beta/6}$ and this enhances the importance of charge symmetry breaking at the higher values of Q^2 that we consider.

Including the effects of the electromagnetic interaction between quarks leads to the following contributions to the form factors:

$$\delta G_E^{(\text{em})}(Q^2) = -\frac{4\alpha}{9}\left[\frac{[J_1(Q^2)-J_3(Q^2]}{\Delta E_e(Q^2,\Delta V_L)}\right.$$
$$\left. -\frac{\pi}{\overline{m}^2}\frac{5}{3}\frac{[J_2(Q^2)-J_4(Q^2)]}{\Delta E_e(Q^2,\Delta V_s)}\right] \quad (5.7)$$

and

$$\delta G_M^{(\text{em})}(Q^2) = \frac{8}{27}\alpha\mu_P\left[\frac{2\widetilde{J}_3(Q^2)}{\Delta E_m(Q^2,\Delta V_L)}\right.$$
$$\left. -\frac{\pi}{3\overline{m}^2}\frac{7\widetilde{J}_4(Q^2)}{\Delta E_m(Q^2,\Delta V_s)}\right]. \quad (5.8)$$

Here negligible effects are anticipated because of the small value of $\alpha \approx 1/137$ and because of the large energy denominators. These terms include the integrals J_3 and J_4 which fall much more slowly than the leading order term, recall Table II.

Including the effects of the gluon exchange interaction between quarks leads to the following contributions to the form factors:
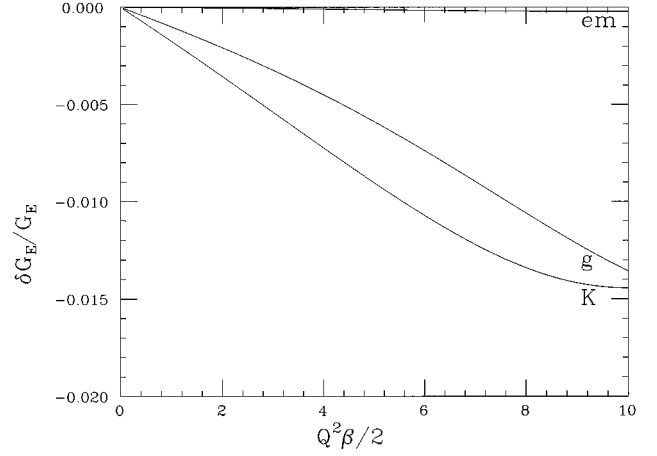

FIG. 4. Charge symmetry breaking electric form factor. The different contributions are shown for model 1.

$$\delta G_E^{(g)}(Q^2) = \frac{\alpha_s}{\Delta E_e(Q^2,\Delta V_s)}\frac{\Delta m}{\beta\overline{m}^3}\frac{20}{27}\sqrt{\frac{2}{\pi\beta}}$$
$$\times(e^{-Q^2\beta/6}-e^{-Q^2\beta/24}) \quad (5.9)$$

and

$$\delta G_M^{(g)}(Q^2) = \frac{-4}{81}\frac{\alpha_s\mu_P}{\Delta E_m(Q^2,V_s)}\sqrt{\frac{2}{\pi}}\frac{\Delta m}{\overline{m}^3\beta^{3/2}}\frac{7}{2}$$
$$\times(e^{-Q^2\beta/24}-e^{-Q^2\beta/6}). \quad (5.10)$$

The explicit formulas show the appearance of the $e^{-Q^2\beta/24}$ term which, at higher values of Q^2 is much bigger than the $e^{-Q^2\beta/6}$ variation of the leading order term. One might expect that this feature would allow the charge symmetry breaking effects to stand out. However, the leading order charge symmetric form factors also have a term, caused by gluon exchange, which also varies as $e^{-Q^2\beta/24}$.

The computed charge symmetry breaking electric form factors are shown in Figs. 4 and 5 which display $\delta G_E/G_E$ as a function of $Q^2/2\beta$ using G_E of Eq. (5.1). Figure 4 shows
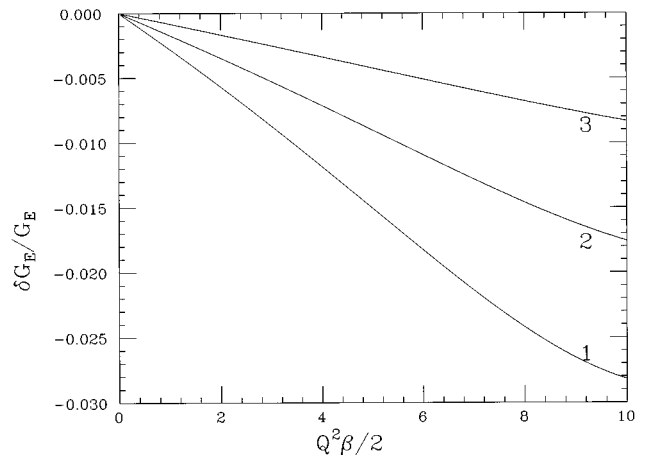

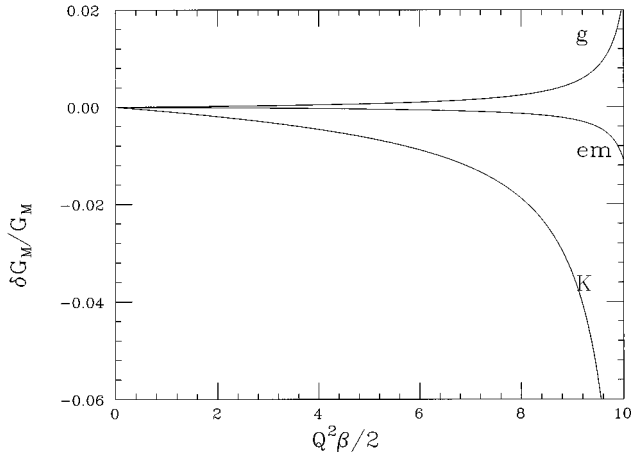FIG. 5. Charge symmetry breaking electric form factors for each of the three models.

FIG. 6. Charge symmetry breaking magnetic form factor. The different contributions are shown for model 1.

FIG. 7. Charge symmetry breaking magnetic form factors for each of the three models.

the three contributions to $\delta G_E/G_E$ arising, in model 1, from the individual charge symmetry breaking terms: kinetic energy (K) electromagnetic interaction (em) and gluon exchange (g). The electromagnetic term gives a negligible contribution, but the other terms can give contributions that are as large as 1%. The sum of the three contributions are shown in Fig. 5 for each of the three models. The effects are largest for model 1 because of its large value of α_s. It is possible that charge symmetry breaking could be as large as 2%. If one wishes to assert that only small values of α_s are allowed [19], then the maximum charge symmetry breaking would be about 1%.

The computed charge symmetry breaking magnetic form factors are shown in Figs. 6 and 7. The ratio $\delta G_M/G_M$ is displayed as a function of $Q^2/2\beta$ where G_M is given in Eq. (5.2). Figure 6 shows the three contributions to $\delta G_M/G_M$ arising, in model 1, from the individual charge symmetry breaking terms. Once again, the electromagnetic term gives a negligible contribution, but the terms g and K can give contributions that are as large as 1%. In this case the gluon exchange and kinetic energy terms tend to cancel, with the sign difference arising from the different combinations of
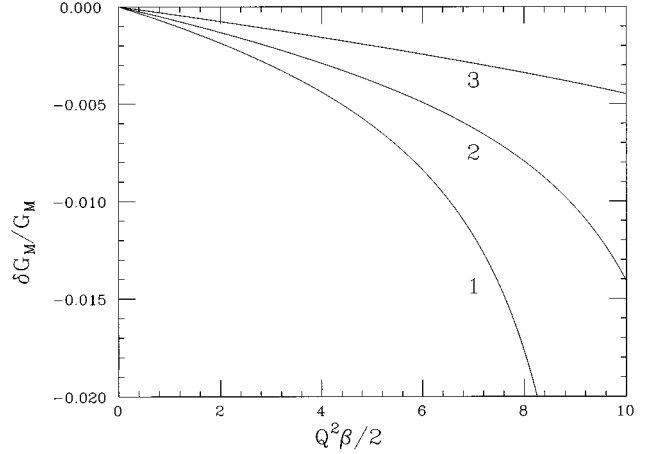
spin matrix elements appearing in the magnetic terms. The net result shown in Fig. 7, for each of the three models, is that the largest effects are less than about 1% for values of $Q^2/2\beta$ for which the models are valid.

It is worthwhile to examine the low Q^2 effects by determining the change in the mean square radii caused by the different terms. The unperturbed form factors each vary as $1-\beta Q^2/6$, and β is the mean square radius. The charge symmetry breaking terms lead to behavior of the form $1-(\beta+\delta\beta)Q^2/6$. We denote the various $\delta\beta$ according to whether related to the electric or magnetic terms and according to the origin of the effects. The results are listed in Table III. The electric terms are much bigger than the magnetic terms, for which the different terms tend to cancel. Thus only $\delta\beta_E$ is changed in a non-negligible manner. For model 1, the sum of the individual contributions gives for model 1 a result $\delta\beta_E/\beta=0.008$, which corresponds to a 1.6% change in the root mean square radius.

C. Dependence on wave function

The previous numerical results have been obtained using the harmonic oscillator wave function. Are the presently ob-

TABLE III. Charge symmetry breaking changes in mean square radii $\Delta m=m_u-m_d$.

Cause	ΔK	ΔV_{em}	δV_g
$\dfrac{\delta\beta_E}{\beta}$	$\dfrac{-1}{3}\dfrac{\Delta m}{\bar m}$	$\dfrac{\alpha}{36\bar m}\sqrt{\dfrac{2\beta}{\pi}}\left(1-\dfrac{9}{8}\dfrac{1}{m^2\beta}\right)$	$\dfrac{-\alpha_s}{8}\dfrac{\Delta m}{\bar m}\dfrac{1}{m\sqrt{\beta}}\sqrt{\dfrac{2}{\pi}}$
model 1	0.0051	-1.14×10^{-5}	0.0029
model 2	0.0038	1.67×10^{-5}	0.0014
model 3	0.0022	4.38×10^{-5}	0.0003
Cause	ΔK	ΔV_{em}	δV_g
$\dfrac{\delta\beta_M}{\beta}$	$\dfrac{-1}{9}\dfrac{\Delta m}{\bar m}\bar m^2\beta$	$\dfrac{-5\alpha}{27\bar m}\sqrt{\dfrac{2\beta}{\pi}}$	$\dfrac{-7\alpha_s}{240}\dfrac{\Delta m}{\bar m}\dfrac{1}{m\sqrt{\beta}}\sqrt{\dfrac{2}{\pi}}$
model 1	0.0024	-0.0013	-0.00069
model 2	0.0013	-0.0011	-0.00031
model 3	0.0005	-0.0004	-0.00007

tained very small values of the charge symmetry breaking effects a simple consequence of this? Another way to ask this question is: Is it possible to find a wave function for which the effects of charge symmetry breaking are enhanced?

The purpose of this section is to address these questions through the use of wave functions other than the harmonic oscillator. Such an investigation is necessarily limited but will allow us to make arguments that are more general.

We start by considering the simple wave function introduced by Henley and Miller (HM) [20]. First the SU(6) nature of the three-quark wave function spin-isospin wave function of Eq. (3.16) is unchanged. Then $\Psi(\rho,\lambda)$ is replaced by a function $\Psi(\rho^2+\lambda^2)$. The generalization is to expand the square of the wave function in terms of harmonic oscillator wave functions of the form given by Eq. (3.17),

$$\Psi_{HM}^2(\rho,\lambda)=\int_0^\infty d\beta g_{HM}(\beta)e^{-(\lambda^2+\rho^2)/\beta}. \quad (5.11)$$

Henley and Miller chose the function $g(\beta)$ so that the resulting electric form factor of Eq. (3.18) is of the usual dipole type $G_E(Q^2)=\Lambda^4/(Q^2+\Lambda^2)^2$. In this case

$$g_{HM}(\beta)(\pi\beta)^3=\frac{1}{36}\Lambda^4\beta\,\exp(-\Lambda^2\beta/6) \quad (5.12)$$

and

$$\Psi_{HM}^2(R)=\frac{\sqrt{6}\Lambda^5}{108\pi^3 R}K_1\left(\sqrt{\frac{2}{3}}\Lambda R\right), \quad (5.13)$$

with $R\equiv\sqrt{\rho^2+\lambda^2}$ and $K_1(x)$ is a Bessel function of an imaginary argument. This wave function was originally used along with a semirelativistic Hamiltonian in which the kinetic plus rest mass energy is given by $\sqrt{p^2+m^2}$ and is not suited for calculations with the nonrelativistic operator of Eq. (3.4). This is because of the nonrelativistic kinetic energy operator has an infinite expectation value in the wave function Ψ_{HM}. This very same wave function was also used in Ref. [21].

We shall proceed here by using a different function $g(\beta)$, one which leads to a finite expectation values of the kinetic energy, but which also leads to a power law falloff of the form factor. Using this wave function will allow us to see if the very small effects of charge symmetry are associated with the rapid Gaussian fall off of form factors obtained from the oscillator model. In particular we take

$$g(\beta)=\frac{\Lambda^8}{\pi^3 6^5}e^{-\Lambda^2\beta/6}, \quad (5.14)$$

which gives

$$\Psi^2(\rho,\lambda)=\int_0^\infty d\beta g(\beta)e^{-(\lambda^2+\rho^2)/\beta}, \quad (5.15)$$

and therefore

$$\Psi^2(R)=\frac{2\sqrt{6}\Lambda^7}{\pi^3 6^5}RK_1\left(\sqrt{\frac{2}{3}}\Lambda R\right). \quad (5.16)$$

The integral form (5.15) is useful for evaluating matrix elements of local operators such as $\exp i\vec{q}\cdot\vec{r}_i$ or $v(r_{ij})$. For such operators the actions of taking the matrix element in a harmonic oscillator wave function and integrating over β commute, i.e., one may integrate the harmonic oscillator matrix element times $g(\beta)(\pi\beta)^3$ over β to obtain the final answer. In particular, the evaluation of Eq. (3.18) now is the integral of $\exp(-Q^2\beta/6)g(\beta)(\pi\beta)^3$ which leads to the result

$$G_E(Q^2)=\left(\frac{\Lambda^2}{Q^2+\Lambda^2}\right)^4. \quad (5.17)$$

To see how this works in studying charge symmetry breaking effects, we evaluate the effects of the magnetic hyperfine interaction. Recalling Eq. (5.10) we now have

$$\delta G_M^{(g)}(Q^2)=-\frac{4}{81}\frac{7}{2}\frac{\alpha_s\mu_P}{\Delta E_m(Q^2,V_s)}\sqrt{\frac{2}{\pi}}\frac{\Delta m}{\bar{m}^3}\int_0^\infty d\beta\frac{\Lambda^8}{6^5}\beta^{3/2}$$
$$\times e^{-\Lambda^2\beta/6}(e^{-Q^2\beta/24}-e^{-Q^2\beta/6}), \quad (5.18)$$

which is evaluated as

$$\delta G_M^{(g)}(Q^2)=-\frac{14}{81}\frac{\alpha_s\mu_P}{\Delta E_m(Q^2,V_s)}\sqrt{2}\frac{3/4}{6^{5/2}}\frac{\Delta m}{\bar{m}^3}$$
$$\times\left[\frac{1}{(\Lambda^2+Q^2/4)^{5/2}}-\frac{1}{(\Lambda^2+Q^2)^{5/2}}\right]. \quad (5.19)$$

For large values of Q^2 this form factor falls roughly as Q^{-5} which is slower than the Q^{-8} behavior of Eq. (5.17). Thus it might seem that at large enough Q^2 the charge symmetry breaking effects would dominate. This, of course, is not true. The strong form factor of Eq. (5.2) has a term $\Delta G_M(Q^2)$ of the same momentum dependence as that of the charge symmetry breaking term of Eq. (4.11). Thus the strong form factor would also have a Q^{-5} behavior and would not be encumbered by the small factor $\Delta m/\bar{m}$.

There is a general lesson that can be drawn from this exercise. Small charge symmetry breaking effects derived from a perturbative term in the strong Hamiltonian cannot lead to form factors of a different asymptotic form than that of the strong form factors.

The only possibility to get new effects is from the charge symmetry breaking in the kinetic energy operator ΔK. One might think that the kinetic energy acting on $|\Psi\rangle$ might generate a state vector with different behavior. To assess the importance of ΔK the relevant expressions of Sec. IV must be re-evaluated using the wave function of Eq. (5.16). The calculations are tedious but straightforward, so the results will be presented after the model parameters are discussed. The size parameter Λ is chosen so that G_E of Eq. (5.17) is consistent with a root mean square radius of 0.83 fm. This gives $\Lambda=5.90$ fm^{-1}. One may compare the relevant size of our present wave function with that of the harmonic oscillator in another manner by computing the contribution of the kinetic energy operator to the neutron-proton mass difference
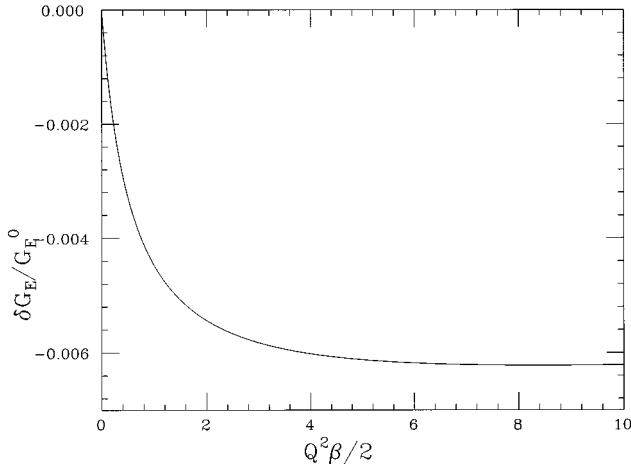
FIG. 8. Use of the wave function of Eq. (5.16). Change in electric form factor.

$$M_n - M_p = -\frac{(m_d - m_u)}{3\bar{m}^2}(4\pi)^2 \int \rho^2 d\rho \int \lambda^2 d\lambda \frac{\lambda^2}{R^2}\left(\frac{\partial\psi}{\partial R}\right)^2,$$
$$(5.20)$$

which may be equated with the harmonic oscillator result of Eq. (3.21). To obtain an equivalent harmonic oscillator parameter β_{eq} in the latter such that $\sqrt{\beta_{eq}} = 0.77$ fm. The present wave function corresponds to a larger size than the oscillators used here. For purposes of estimation, we take $m_d - m_u = 5.2$ MeV, which is the value of model 1. A calculation of all of the relevant charge symmetry breaking terms would probably lead to a value a bit larger than that because a value of α_s larger than the 2.3 of model 1 would be needed to reproduce the Δ-nucleon mass splitting.

The results are shown in Figs. 8 and 9. The computed ratios of charge symmetry breaking effects to charge symmetry conserving ones are displayed as $\delta G_E/G_E$ or $\delta G_M/G_M$ as a function of $Q^2\beta/2$ where $\sqrt{\beta} = 0.77$ fm. Here the electric and magnetic form factors have the functional form of Eq. (5.17). Observe that the computed ratios are once again very small.
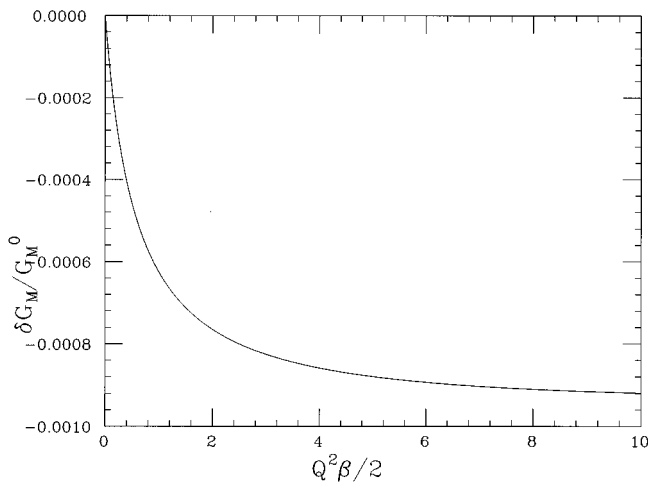

FIG. 9. Use of the wave function of Eq. (5.16). Change in magnetic form factor.

VI. BJORKEN SUM RULE

The structure function $g_1(x,Q^2)$ can be measured in lepton-nucleon deep inelastic scattering (DIS) by using a polarized beam and a polarized target [4]. See the reviews [22]. Here x is the Bjorken variable. The Q^2 dependence of $g_1(x,Q^2)$ arises from perturbative QCD evolution effects and from higher twist and target mass corrections. For the present purpose of evaluating the influence of charge symmetry breaking using nonrelativistic quark models it is sufficient to consider the Bjorken sum rule within the framework of the naive parton model.

The naive parton model interpretation of the spin-dependent DIS data is that the valence quarks contribute very little to the proton's spin. This startling finding motivated the studies of parity violation in electron-proton scattering studies discussed here. The parton model structure functions measure the probability for finding a quark with momentum fraction x in the proton and which is polarized either in the same $\uparrow$ or the opposite $\downarrow$ direction to the proton's polarization $\uparrow$, and the structure functions are described by the four independent parton distributions $(q\pm\bar{q})^\uparrow(x); (q\pm\bar{q})^\downarrow(x)$. The function $g_1(x)$ is given by

$$g_1(x) = \frac{1}{2}\sum_q e_q^2 \Delta q(x), \qquad (6.1)$$

where $\Delta q(x) = (q^\uparrow + \bar{q}^\uparrow)(x) - (q^\downarrow + \bar{q}^\downarrow)(x)$ is the polarized quark distribution and e_q denotes the quark charge.

In the naive parton model the integral

$$\Delta q = \int_0^1 dx \Delta q(x)$$

determines the fraction of the proton's spin which is carried by quarks (and antiquarks) of flavor q. Thus, as reviewed recently [22], one obtains

$$\Gamma_{1,p} \equiv \int_0^1 dx\, g_1(x) = \frac{1}{18}\langle p,\uparrow|(4\Delta u + \Delta d + \Delta s)|p,\uparrow\rangle.$$
$$(6.2)$$

We shall not explicitly write the proton spin $\uparrow$ in the following development. Thus $|p\rangle$ is to be understood as denoting $|p,\uparrow\rangle$. The operators Δq are the axial current operators for the different quark flavors q: $\bar{q}\gamma_\mu\gamma_5 q$. The axial charge g_A measured in beta decays is given by

$$g_A = \langle p|\Delta u - \Delta d|p\rangle. \qquad (6.3)$$

The Bjorken sum rule also involves the neutron matrix element

$$\Gamma_{1n} = \frac{1}{18}\langle n|4\Delta u + \Delta d + \Delta s|n\rangle. \qquad (6.4)$$

The use of the formalism of Sec. II, and the fact that the strange quark field operator is not influenced by rotations in isospin space, allows one to express this quantity as a proton matrix element:

$$\Gamma_{1n} = \frac{1}{18} \langle p | 4\Delta d + \Delta u + \Delta s | p \rangle$$

$$+ \frac{1}{18} {}_{0}\left\langle p \left| (4\Delta u + \Delta d) \frac{\Lambda}{\bar{m} - H_0} 2\Delta H \right| p \right\rangle_0 . \quad (6.5)$$

Taking the difference between Eqs. (6.2) and (6.5) leads to the result

$$\Gamma_{1p} - \Gamma_{1n} = \frac{g_A}{6} + \Delta\Gamma, \quad (6.6)$$

where

$$\Delta\Gamma = -\frac{1}{18} {}_{0}\left\langle p \left| (4\Delta u + \Delta d) \frac{\Lambda}{\bar{m} - H_0} 2\Delta H \right| p \right\rangle_0 . \quad (6.7)$$

The first term of Eq. (6.6) represents the Bjorken sum rule and the second term is the naive parton model correction to it caused by charge symmetry breaking.

We shall use the nonrelativistic quark models [23] to estimate the size of $\Delta\Gamma$. In this case the relevant operators are

$$\Delta u = \sum_i \frac{1 + \tau_3(i)}{2} \sigma_3(i) \quad (6.8)$$

and

$$\Delta d = \sum_i \frac{1 - \tau_3(i)}{2} \sigma_3(i), \quad (6.9)$$

so that

$$4\Delta u + \Delta d = \frac{5}{2} \sum_i \sigma_3(i) + \frac{3}{2} \sum_i \sigma_3(i)\tau_3(i). \quad (6.10)$$

The first term does not excite the nucleon and is irrelevant. The action of the second operator on the nucleon leads to either a nucleon or to a Δ. This simplifies the calculation enormously since only the Δ intermediate state needs to be included in the sum over intermediate states required to evaluate Eq. (6.7).

Then the expression for $\Delta\Gamma$ as obtained by using the relevant operator (6.10) in the matrix element of Eq. (6.7) is simply

$$\Delta\Gamma = \frac{1}{6} {}_{0}\left\langle P \left| \sum_i \sigma_3(i)\tau_3(i) \frac{|\Delta\rangle\langle\Delta|}{M_\Delta - M_N} 2\Delta H \right| p \right\rangle_0 . \quad (6.11)$$

Only the spin dependent pieces of the charge symmetry breaking Hamiltonian ΔH can contribute to this matrix element. Thus

$$\Delta H \rightarrow -\left(\frac{\alpha}{9} - \frac{4\alpha_s}{9} \frac{\Delta m}{\bar{m}} \right) \frac{\pi}{\bar{m}^2} \sum_{i<j} [(\tau_3(i) + \tau_3(j)]$$

$$\times \delta(\vec{r}_{ij})\vec{\sigma}_i \cdot \vec{\sigma}_j . \quad (6.12)$$

However, the spin dot products may each be replaced by unity because for the spin symmetric Δ wave function

$$\vec{\sigma}_i \cdot \vec{\sigma}_j |\Delta\rangle = |\Delta\rangle \quad (6.13)$$

for all pairs i, j of quarks. Furthermore the expectation value of the isospin operators vanishes. Consider $i, j = 1, 2$ and note that

$$\langle \Delta | \tau_3(1) + \tau_3(2) | N \rangle = -\left(\frac{4}{\sqrt{12}} \right) \langle \Delta | \phi_s uud \rangle = 0. \quad (6.14)$$

The spatial symmetry of the Δ and nucleon wave functions insures that this vanishing occurs for all pairs i, j.

The result of this calculation is that the influence of the charge symmetry breaking Hamiltonian on the Bjorken sum rule vanishes. This exact 0 is due to the use of first-order perturbation theory within an SU(6) symmetric wave function.

VII. DISCUSSION

Let us summarize. The charge symmetry breaking observables relevant for parity-violating electron scattering and a general formalism for their evaluation are obtained in Sec. II. This formalism is just a simple way to keep track of the effects of the charge symmetry conserving H_0 and violating H_1 Hamiltonians. The observables are evaluated using a set of three nonrelativistic quark models, each with harmonic oscillator confinement and obeying SU(6) symmetry, that is defined in Sec. III. The models are distinguished by their different size parameters, and are required to reproduce the Δ-nucleon mass difference, or a size-dependent fraction thereof. The charge symmetric breaking effects included are the effects of the mass difference between the up and down quarks in the kinetic energy operator and one-gluon exchange interaction, and the electromagnetic interaction. One obtains a reasonable range of values of $m_d - m_u$ needed to reproduce the observed value of the neutron-proton mass difference.

The charge symmetry breaking effects are small and therefore well treated using first-order perturbation theory. One must sum over an infinite set of intermediate states to carry out the necessary calculations. This summation is aided by the approximation of treating H_0 appearing in the energy denominator as a constant. The relevant constant ΔE is chosen so that the first correction to the approximation vanishes; see Eq. (4.8). This means that ΔE depends on the momentum transfer and the operator that excites the proton. This use of a constant allows one to use closure to perform the sum over intermediate states. This procedure is the subject of Sec. IV.

The evaluations are presented in Sec. V. First the electric and magnetic form factors of the eigenstates of H_0 are obtained. The strong one-gluon exchange operator gives a high momentum tail which dominates the Gaussian term obtained from the oscillator wave function. Then the influence of the three charge symmetry breaking terms in the Hamiltonian are evaluated for the three different models. The effects due to the $u - d$ quark mass difference are larger than that of the electromagnetic interaction, but are themselves very small. The largest of the effects we find are of the order of 1% for the change in G_E caused by charge symmetry breaking effects. Some larger values are shown in the figures, but these are for values of the momentum transfer which are outside of

the regime of applicability of the models we use. These small values arise because of the small sizes of the basic effects: the ratio of the quark mass difference to constituent quark mass is about 1/70 and, the tail caused by the strong one gluon exchange potential makes it impossible to find a region of momentum transfer for which these effects can stand out. This result is not a consequence of the use of oscillator confinement. A different wave function, in which the square of the wave functions is an integral of harmonic oscillator wave functions, is also used, and very small effects of charge symmetry breaking are obtained.

The charge symmetry breaking correction to the Bjorken sum rule is examined in Sec. VI. Here the use of SU(6) symmetric wave functions is shown to lead to a vanishing correction in first-order perturbation theory. Charge symmetry breaking therefore has no impact on current studies of the validity of the Bjorken sum rule.

Next consider other computations of the effects of charge symmetry breaking on the nucleon. The present work is most similar to that of Ref. [10], and our results are consistent with those, except for one detail (see Sec. III) that depends on issues beyond the scope of the nonrelativistic quark model. That previous calculation is extended here by including the effects of the strong and electromagnetic hyperfine interaction, by studying the momentum transfer dependence and by using a nonoscillator proton wave function. One difference is that in Ref. [10] the sum over intermediate excited states is saturated by the $\Delta(1550)$. We use closure to carry out the sum. The charge symmetry breaking operators are isovector which act on $T=1/2$ states so that the intermediate states can have either $T=1/2$ or $T=3/2$. The closure approximation used here allows the $T=1/2$ states to be included.

Let us also discuss the work of Ma [11] who presents his results in the form $-\delta G_M^s \approx 0.006 \rightarrow 0.088$ nucleon magnetons. This is small compared to the current experimental error of about 0.2 nucleon magnetons, but relatively important compared to the rather small strange magnetic moment $G_M^s = -0.066$ nucleon magnetons from the baryon-meson fluctuation model of Ref. [24]. The abstract states that the neutron proton mass difference leads to an excess of $n = \pi^- p$ over $p = \pi^+ n$ fluctuations, but two different effects actually lead to the results.

The first is claimed to arise from the light cone treatment of the noninteracting propagator. If the light cone treatment is used, a term $P^+ P^- - \mathcal{M}^2$ with $\mathcal{M}^2 = \Sigma_{i=1}^2 [(k_{\perp i}^2 + m_i^2)/x_i]$ replaces our nonrelativistic inverse propagator $\bar{M} - \bar{H}_0$. The value of $P^+ P^- = M_N^2$, the square of the nucleon mass. In Ma's treatment this takes on the two values of m_p^2 or

m_n^2. This leads to a numerical result $r_{p/n}^\pi = P(p = \pi^+ n)/P(n = \pi^- p) = 0.986$, which corresponds to an excess of 0.2% of $n = \pi^- p$ fluctuations if $P(p = \pi^+ n) \approx P(n = \pi^- p) \approx 0.15$. However, one should perform a light-cone perturbation theory treatment of the charge symmetry breaking, which involves treating the Hamiltonian operator $P^+ P^-$ as a sum of charge symmetry breaking and charge symmetry conserving terms. In this case the relevant eigenvalue, analogous to $\bar{M}$ used here in Sec. II, must be $\frac{1}{2}(m_p^2 + m_n^2)$. Using this value changes the above result of 0.986 to 0.992; the effect is reduced by a factor of 2.

Actually, the biggest effect used by Ma is caused by the assumption that the radius $R \sim 1$ fm of the $n\pi^+$ component of the proton is 2.5% smaller than that of the $p\pi^-$ component of the neutron. This according to [11] could be caused by Coulomb effects. However, the effects of the Coulomb potential and electromagnetic interactions in loop graphs is of order $\alpha/\pi \ll 0.025$, so that Ma's effect, while physically reasonable, is estimated to have too large a value. A reasonable estimate of the effect could be 0.03 nucleon magnetons [25]. This effect is too small to be relevant to experiments.

Finally, consider the work of Celenza and Shakin [12] who computed the deep inelastic structure functions of the nucleon using a quark model which preserves translational invariance. Effects of charge symmetry breaking enter in their calculation of the ratio of $F_2^n(x)/F_2^p(x)$. They can reproduce the experimental values of this ratio by allowing the neutron confinement radius to be about 10% larger than the corresponding proton radius. Such an effect is well motivated, but the 10% value is much larger than the $<1\%$ effects found here.

The net result is that the effects of charge symmetry breaking on the nucleon wave function can be expected to be very small. Only a limited number of models are discussed in the present paper, but it seems very difficult to construct a reasonable model of the nucleon which incorporates large charge symmetry breaking effects. That any charge symmetry violating effect in the Hamiltonian H_1 has its analog in the symmetry preserving Hamiltonian H_0 is a model independent statement. Thus the present conclusion about the lack of import of charge symmetry breaking effects seems to be true independent of the particular model used.

ACKNOWLEDGMENTS

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