Two-nucleon emission in the longitudinal response

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The contribution of the two-nucleon emission in the longitudinal response for inclusive electron scattering reactions is studied. The model adopted to perform the calculations is based upon correlated basis function theory but it considers only first order terms in the correlation function. The proper normalization of the wave function is ensured by considering, in addition to the usually evaluated two-point diagrams, the three-point diagrams. Results for the ^{12}C nucleus in the quasielastic region are presented. [S0556-2813(98)03801-1]

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Electromagnetically induced two-nucleon knockout reactions are considered to be well suited to study short-range correlations (SRC) in nuclei [1]. The basic idea is that the real or virtual photon interacts with a correlated pair of nucleons which are emitted from the nucleus. Though the study of this process was proposed long ago [2], only recently, with the advent of high-intensity monochromatic photon beams and 100% duty cycle electron beams, have the technical difficulties in performing this kind of experiment with adequate statistics been overcome.

The simple picture presented above involves one-body electromagnetic operators and short-range correlations only; however, other mechanisms contribute to the two-nucleon emission, for example, the meson exchange current (MEC) and final state interactions, and this complicates the analysis of the experimental data. It is therefore necessary to deal with experimental situations where the alternative emission mechanisms can be disentangled, or to find kinematical conditions where the emission via SRC becomes the dominant one.

For these reasons it is important to avoid those energy regions dominated by collective excitations of the nucleus, such as the giant resonance region, because in these regions the multinucleon emission is mainly induced by the residual interaction via the excitation of many-particle–many-hole configurations [3]. There is also another reason, a more pragmatical one, to avoid the kinematical regions with relatively low excitation energy. In these regions the excitation energy is just above the two-nucleon emission threshold, the phase space available for the two nucleons emitted is quite small, and, as a consequence, the cross sections are rather small.

It is therefore mandatory to work, at least, at the excitation energies where the quasielastic peak shows up. In this region, however, the emission mechanism we want to study competes with the two-nucleon emission produced by MECs [4]. Since MECs are active predominantly in the transverse response, there is the hope that the two-nucleon emission in the longitudinal response would be dominated by SRC effects.

In this paper we present the results of a calculation of the two-nucleon emission contribution to the inclusive (e,e') longitudinal response. This calculation has been done for the ^{12}C nucleus.

The model we have developed to describe the process is based upon the correlated basis function (CBF) theory [5], but it considers only the terms up to a single correlation line. In CBF theory the many-body Schrödinger equation is solved by means of the variational principle within a subspace of wave functions of the type

$$|\Psi\rangle = F|\Phi\rangle, \tag{1}$$

where $|\Phi\rangle$ is a Slater determinant built up with a set of single particle wave functions properly chosen, and F is the correlation function. The variational method with the ansatz of Eq. (1) has been successfully used to describe few-body systems [6], light nuclei [7], and infinite systems [8], nevertheless, its application to medium and heavy nuclei is still at the beginning stages. Recently, promising attempts to extend CBF theory to these last nuclear systems have been carried out with the help of Fermi hypernetted chain (FHNC) technology [9].

In addition to the known difficulties related to FHNC calculations in finite nuclear systems, ours presents a further complication due to the fact that it is necessary to extend the FHNC theory to the description of the nuclear excited states, in the same spirit of what was done in Refs. [10,11] for nuclear matter.

For these reasons we have developed a model which is an extension of those models used some time ago to calculate ground state density and momentum distributions [12]. These models consider only those terms of the cluster expansion containing a single correlation line. A test of the validity of these models have been recently done comparing their results with those obtained using the same input in a full FHNC calculation [13,14]. The good agreement obtained in this comparison gives us the hope that a truncation of the cluster expansion to the terms with a single correlation line could work also for the description of nuclear transitions, at least for those induced by the charge operator.

The basic hypothesis of our work lies in the ansatz of Eq. (1). The correlation function F is extremely complicated and it has the same operator structure of the nucleon-nucleon interaction. Realistic CBF calculations [6–8] show that the scalar term of the correlation function greatly dominates the other ones. This does not mean, however, that the so-called

state-dependent terms of the correlation can be neglected because their effect is small. For example, the tensor correlations, are extremely important in the calculation of the binding energy [15], and they probably play a crucial role in setting the magnitude of the MEC [11,16] in the quasielastic peak. On the other hand, in this work we consider the charge operator, which has only an isospin dependence, and we believe that for this operator the effects of the state-dependent terms of the correlation should be small. For this reason, and to simplify the calculations, we have considered a purely scalar correlation function of the form

$$F = \prod_{i<j}^{A} f(r_{ij}), \qquad (2)$$

where r_{ij} is the distance between the particles i and j.

The response produced by a generic operator $U(\mathbf{q})$ is

$$R(\mathbf{q}, \omega) = \sum_{f} \frac{\langle \Psi_i | U^{\dagger}(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | U(\mathbf{q}) | \Psi_i \rangle}{\langle \Psi_i | \Psi_i \rangle \langle \Psi_f | \Psi_f \rangle}$$

$$\times \delta(E_f - E_i - \omega). \qquad (3)$$

Assuming the same correlations for both ground and excited states, the above equation can be rewritten in terms of the amplitude:

$$\xi_{if}(\mathbf{q}) = \frac{\langle \Phi_f | F^{\dagger} U(\mathbf{q}) F | \Phi_i \rangle}{\langle \Phi_i | F^{\dagger} F | \Phi_i \rangle} \left[\frac{\langle \Phi_i | F^{\dagger} F | \Phi_i \rangle}{\langle \Phi_f | F^{\dagger} F | \Phi_f \rangle} \right]^{1/2}. \qquad (4)$$

This is the basic quantity to be studied and it corresponds to the ground state expectation value of the operator $U(\mathbf{q})$ in the case where the state $|\Phi_f\rangle$ becomes the ground state $|\Phi_i\rangle$. For the charge operator, which is the one we consider in our calculations, the quantity $\xi_{if}(\mathbf{q})$ satisfies the property

$$\lim_{q \to 0} \lim_{i \to f} \xi_{if}(\mathbf{q}) = \frac{Z}{A}. \qquad (5)$$

To evaluate $\xi_{if}(\mathbf{q})$, instead of performing the full cluster expansion as has been done for infinite nuclear systems [10], we consider only terms of the cluster expansion containing a single correlation line h defined as

$$h(r_{ij}) = f^2(r_{ij}) - 1. \qquad (6)$$

Cutting an infinite series is always a delicate operation because a wrong choice of the terms retained can produce equations which do not conserve the properties of the system under investigation, such as the number of particles. In constructing our model we have been guided by the rule that the terms considered should provide an approximate amplitude ξ_{fi}^{1} satisfying the limit of Eq. (5).

This model can be used to calculate transitions leading to final states with one or two particles in the continuum. In this work we are interested in two-nucleon emission, and for this process our model produces 4 two-point diagrams and 12 three-point diagrams.

The three-point diagrams describe the situation where three particle are correlated, in spite of the fact that only one two-point (dynamical) correlation is present. In these diagrams, in addition to the dynamical correlation, a statistical
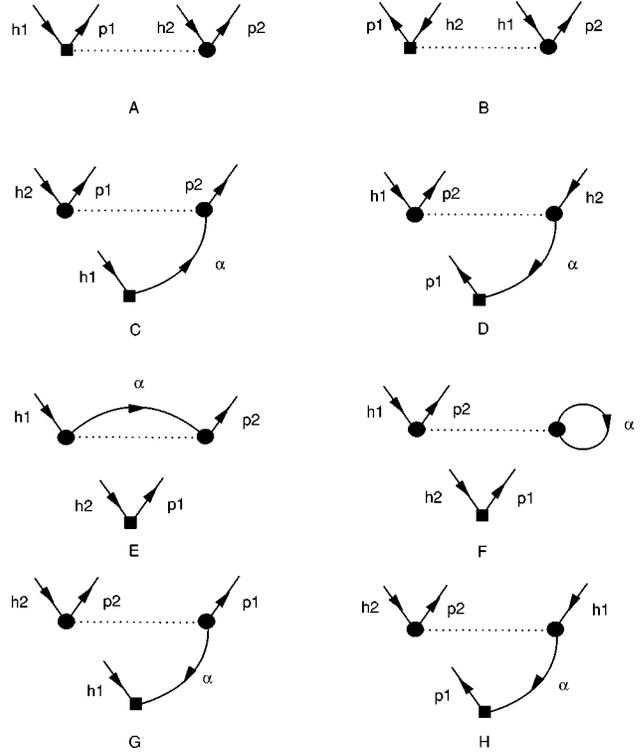

FIG. 1. Diagrams considered in our model. The dotted lines represent the correlation function h. The full oriented lines represent particle and hole wave functions. We have indicated with $p1$, $p2$, $h1$, and $h2$ the wave functions of the two particles and two hole states surviving asymptotically, and with α a generic hole wave function different from $h1$ or $h2$. The black circle indicates an integration point while the black square indicate the integration point where the electromagnetic operator $U(q)$, the charge operator in our case, is acting. In addition to these diagrams we consider also those obtained by exchanging the pairs $(p1,h1)$ with the pairs $(p2,h2)$, as well as those three-points diagrams where the two points linked by the correlation function are exchanged.

correlation, generated by the antisymmetrization of the many-body wave function under the exchange of two particles is also acting.

Considering the symmetry properties of the correlation function $h(r_{ij}) = h(r_{ji})$ and the fact that some of these diagrams are obtained exchanging particle and hole lines, these 16 diagrams reduce to the 8 topologically distinguished diagrams shown in Fig. 1. A more thorough description of the model will be provided in a forthcoming publication.

The calculations we discuss in the following have been done for the ^{12}C nucleus. This nucleus is relatively light, it has only four hole single particle states, and therefore calculations are less time consuming than for heavier nuclei. In addition we have thoroughly studied the quasielastic response of this nucleus [17,4] and this experience gives us some insight in the details of the configuration space to be used. In this respect, all our calculations have been done with the set of single particle wave functions generated by a Woods-Saxon potential used in our previous quasielastic peak calculations. It is worth noticing that the same set of single particle wave functions has been used in the FHNC calculations of Ref. [9].

In Ref. [4] we have calculated the two-nucleon emission in the transverse response induced by the MEC. In the
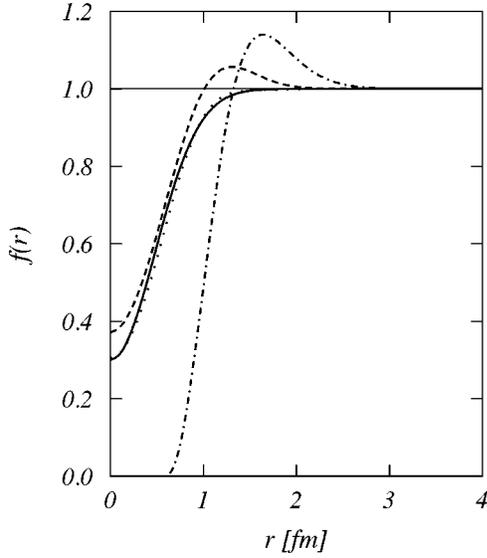
FIG. 2. Correlation functions used in our calculations. The full and dashed lines are the Gaussian and the ACA Euler correlations of Ref. [9]. The dotted line is the scalar part of the nuclear matter correlation function of Ref. [8]. The dashed-dotted line represent the OMY correlation function [18].

present calculation we have used the same angular coupling scheme, this time applied to the longitudinal response and for the charge transition operator modified with the corresponding correlation term.

In Fig. 2 we present the various types of scalar correlation functions used to test the sensitivity of our results to the details of the correlation. The full and dashed lines represent the Gaussian and ACA Euler correlations used in the FHNC calculations of Ref. [9]. These correlations have been fixed by minimizing the binding energy of ^{12}C for the Afnan-Tang S3 semirealistic nucleon-nucleon interaction and for the same set of single particle wave functions adopted in the present work.

The third correlation we have used (dotted line) corresponds to the scalar part of the nuclear matter correlation determined in the FHNC calculations of Ref. [8]. In addition we have also considered the OMY correlation [18], represented by the dashed dotted line, because it has been widely used in the literature.

As far as we know, the $(e,e'2N)$ calculations performed up to now [19,20] consider only two-point diagrams (the A and B diagrams of Fig. 1). In these calculations the normalization of the wave function is not conserved because the limit of Eq. (5) is not satisfied. The first aspect to be investigated with our model is then the importance of the three-point diagrams necessary to fulfill Eq. (5) at the first order in the correlation line.

In Fig. 3 we show the results we have obtained for the contribution of the two-nucleon emission to the inclusive longitudinal responses for three values of the momentum transfer. These calculations have been performed with the Gaussian correlation. The full lines show the results found with all the diagrams, while the dashed lines have been obtained considering only the two-point diagrams.

This figure shows that the contributions of the two- and three-point diagrams sum up to each other. We have obtained analogous results for all the correlations functions
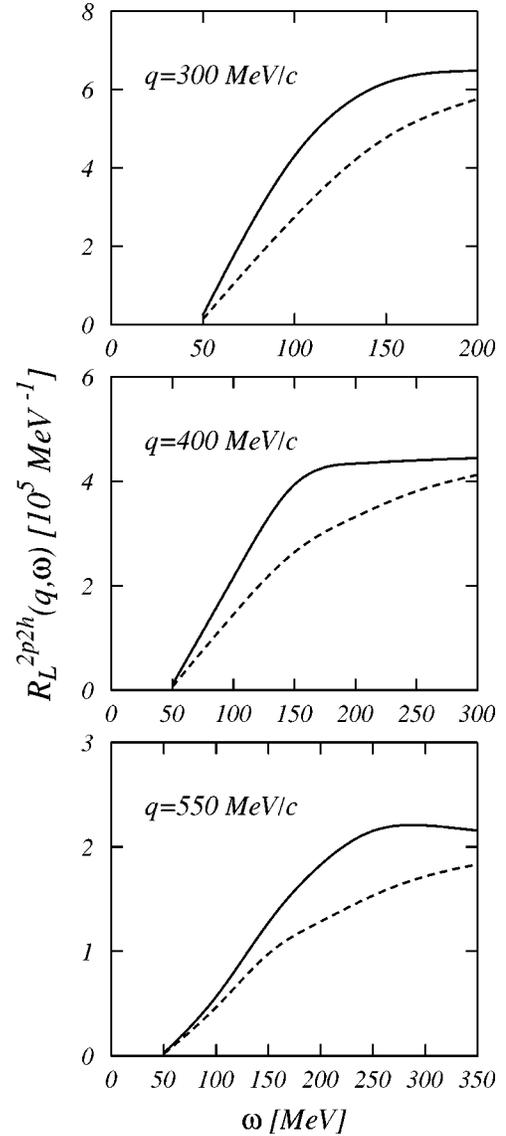

FIG. 3. Inclusive longitudinal response functions for the emission of two nucleons calculated for three different values of the momentum transfer. The calculation was performed with the Gaussian correlation function. The full lines show the result obtained considering all the diagrams presented in Fig. 1, while the dashed lines have been obtained only with the two-point diagrams (diagrams A and B in Fig. 1).

considered. This result is, in principle, surprising, since, from our previous experience in the calculation of density and momentum distributions [13,14], we expected big cancelations between two- and three-point contributions.

In reality the correlations play different roles in the two cases. In the ground state the correlations effects remodel the shape of the mean-field charge distribution without changing the total charge. In this case, every two-point diagram is coupled to a three-point diagram of opposite sign, which, in the limit of Eq. (5), cancels exactly the contribution of the two-point diagram (the uncorrelated charge distribution is already correctly normalized). In the response the three-point diagrams offer an additional mechanism of emitting two nucleons, enlarging the available phase space, and therefore their contribution to the response adds up to that of the two-point diagrams.
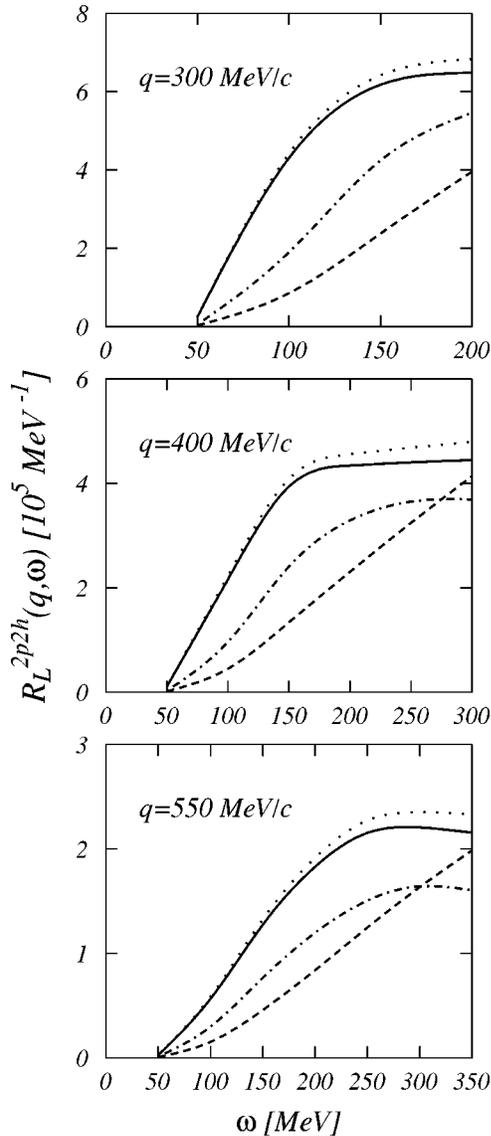
FIG. 4. Inclusive longitudinal response functions for the emission of two nucleons calculated for three different values of the momentum transfer. The various lines represent the results obtained with the correlations of Fig. 1. The full lines have been obtained with the Gaussian correlation, the dashed ones with the Euler ACA, the dotted ones with the nuclear matter correlation, and the dashed-dotted ones with the OMY correlation. Note that the OMY dashed-dotted curves have been multiplied by a 0.1 factor.

A second aspect we want to investigate is the sensitivity of the results to the correlation chosen. In Fig. 4 we show the full responses (that is including two- plus three-point diagrams) obtained with the various correlation functions. The same convention as in Fig. 2 has been used for the different curves. One should notice that, in the figure, the responses obtained with the OMY correlation (dashed-dotted lines)

have been divided by a factor 10. All the other responses are of the same order of magnitude. The results obtained with the Gaussian correlation are very similar to those obtained with the nuclear matter one, as expected because of the large similitude between the correlations one can observe in Fig. 2.

The results of Fig. 4 show high sensitivity to the details of the correlation function. Our approach does not provide any prescription to choose among the correlations we have used. On the other hand, in CBF theory the correlation functions are chosen together with the single particle wave functions in a way to minimize the ground state energy of the system. The lack of an internal criterion to link single particle wave functions and correlations is a weak point of our approach. We think this problem can be overcome by taking these inputs from a microscopic calculation of the ground state energy.

In the calculations we have presented, this has been done, at least partially, for the Gaussian and ACA Euler correlations which, for this set of single particle wave functions, minimize the binding energy of ^{12}C when the S3 nucleon-nucleon interaction is used. There is no link between single particle wave functions and the OMY correlation, which produces responses one order of magnitude larger than the other ones. In this sense the comparison of Fig. 4 is not fully correct, because we should have compared results obtained with correlations and single particle wave functions modified to minimize the nuclear binding energy.

Let us summarize the main points of this report.

(1) In order to get the proper normalization of the wave function in a model considering only terms up to the first order in the correlation line it is necessary to include both two- and three-point diagrams.

(2) The contribution of the three-point diagrams adds strength to the response, contrary to the case of the ground state expectation values, where a strong cancelation between two- and three-point diagrams is found.

(3) A relation between single particle wave functions and correlation functions is necessary to have physically meaningful results.

Before concluding we would like to make some general remarks about our model. As we have said, we infer the validity of our model from the fact that the results of the ground state density and momentum distribution were quite similar to those obtained with the FHNC calculation [14]. We do not claim that models considering only first order terms in the correlation can be blindly applied to any operator. We believe that the good results obtained for the ground state expectation values of the charge distribution are related to the peculiar characteristics of this operator. Because of this, we feel quite confident of our model devised for the calculation of responses, but we think that a comparison with FHNC responses is a necessary test. Work in this direction is in progress.

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