Δ excitation in K^+-nucleus collisions

J. A. Oller,[1] E. Oset,[1] A. De Pace,[2] and P. Fernández de Córdoba[3]
[1]*Departament de Física Teòrica, and IFIC Centro Mixto Universidad de Valencia-CSIC, 46100 Burjassot (Valencia), Spain*
[2]*Istituto Nazionale di Fisica Nucleare, Sezione di Torino, via P. Giuria 1, I-10125 Torino, Italy*
[3]*Departamento de Matemática Aplicada, Universidad Politécnica de Valencia, Spain*
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We present calculations for Δ excitation in the (K^+,K^+) reaction in nuclei. The background from quasielastic K^+ scattering in the Δ region is also evaluated and shown to be quite small in some kinematical regions, so as to allow for a clean identification of the Δ excitation strength. Nuclear effects tied to the Δ renormalization in the nucleus are considered and the reaction is shown to provide new elements to enrich our knowledge of the Δ properties in a nuclear medium. [S0556-2813(98)05703-3]

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I. INTRODUCTION

Delta excitation in nuclei has been a topic of permanent interest and it has been studied in connection with pion nucleus collisions [1], photonuclear reactions [2–5], electron scattering on nuclei [6–9], nuclear reactions induced by protons or light nuclei [10–13], neutrino induced reactions [14,15], etc.

In all these reactions the Δ excitation proceeds in a different way: sometimes it is excited by a spin-isospin longitudinal source (pions), other times by a transverse source (photons), and in other cases by a mixture of both. Also, the range of energy and momentum used to excite the Δ varies from one case to another. Differences also appear in the regime of nuclear densities explored. In some reactions the Δ is more neatly excited than in others where background terms are important and, often, distortions of the strongly interacting particles involved in the reaction lead to Δ shapes that differ appreciably from each other. All these differences, however, serve to enrich our knowledge of the Δ properties in a nuclear medium and of its coupling to the nuclear components. Delta excitation in K^+ nuclear reactions has not yet been explored and clearly deserves some attention in view of its complementarity with respect to other reactions mentioned.

The K^+ is a meson belonging, like the pion, to the octet of pseudoscalar mesons. However it has peculiar features. In a sense, the small K^+N cross sections allow the kaons to explore inner regions of the nucleus, while pion nuclear reactions are usually more peripheral. Another big difference is the fact that the pion can be absorbed by one nucleon to give the Δ, while this is not possible with the K^+ due to its strangeness. One can also not excite strange baryons (of negative strangeness) with the K^+. Hence the K^+ in this case can only release some momentum and energy and keep traveling as a K^+ (or K^0). In this sense the Δ excitation induced by K^+ is similar to the proton induced one in (p,p') or (p,n) reactions, with the difference that in the K^+ case the Δ is excited only with a transverse source as we shall see.

The modifications of the Δ properties in a nuclear medium have been the object of much theoretical attention [16–21]. Also early empirical studies of pion-nucleus scattering lead to parametrizations of the Δ spreading potential [22], although caution has to be exerted to compare to theoretical models because the empirical spreading potential incorporates elements which in some theoretical models are part of the Δh interaction [1]. Experiments on K^+-nucleus scattering in the Δ excitation region would bring additional information by means of which to test our present understanding of the Δ properties in nuclei and enrich it.

Experiments on inclusive K^+-nucleus scattering are already available [23,24], but they restrict themselves to the quasielastic excitation region. These data have proved useful in order to learn about the strength of the residual nuclear forces [25], since they have offered new information with respect to the one obtained from electron scattering at low momentum transfers [26]. The extension of this work to the Δ excitation region, passing through the dip region should be most useful. We should recall that the dip region has been a permanent theoretical problem in inclusive electron scattering and only the recent thorough many body calculation of Ref. [9] has been able to provide a fair description so far. Given the different dynamics in K^+-nucleus scattering with respect to electrons, we anticipate that this region should pose a challenge to theory.

The elementary $K^+N\to K\Delta$ reaction has not been much studied but there are data for $K^+p\to KN\pi$ in several charge channels which clearly indicate the contribution from Δ excitation [27,28]. A recent study of this reaction using the terms from chiral Lagrangians plus Δ excitation, has been performed in [29] and this provides us with the elementary information needed to tackle the nuclear problem. The other important ingredient is the Δ self-energy in the nuclear medium, which we take from Ref. [18]. This self-energy has been tested in elastic pion nucleus scattering [30] and in quasielastic, single charge exchange, double charge exchange, and absorption of pions in nuclei [31]. It has also been tested in photonuclear reactions [5] and electronuclear reactions [9], and in all cases a good description of the data around the resonance region was found. With these ingredients at hand we tackle now the K^+ nucleus inclusive scattering around the Δ region.

II. THE MODEL

Following the developments in photonuclear and electronuclear reactions [5,9] we evaluate the self-energy of a K^+

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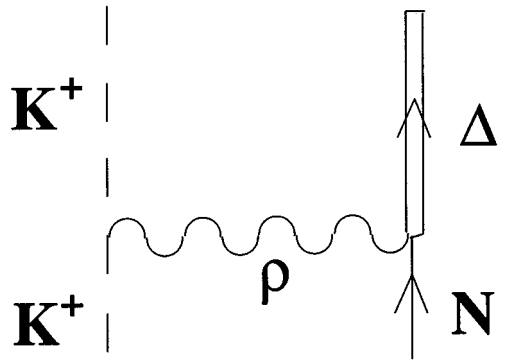

FIG. 1. Δ excitation term mediated by ρ exchange in the $K^+N\to K^+\Delta$ reaction.

in nuclear matter and from there the cross section in nuclei via the local density approximation.

The elementary model of [29] for $K^+N\to K^+\Delta$ is depicted in Fig. 1. The model consists of ρ exchange between the kaon and the baryonic components. The two necessary ingredients are the $K^+K^+\rho$ coupling and the $\rho N\Delta$ coupling, which we take from [29] where a fit to the data was performed. We have for $\rho^0\to K^+K^-$

$$-i\,\delta H_{\rho K^+K^-}=-i\,\tilde{f}_\rho\epsilon_\rho^\mu[p_{K^+}-p_{K^-}]_\mu \tag{1}$$

and for $\rho^0 N\to\Delta$ the vertex function ($\vec{q}\equiv\rho$ momentum)

$$-i\,\delta\tilde{H}_{\rho^0N\Delta}=\sqrt{\frac{2}{3}}\,\frac{f^*}{m_\pi}\sqrt{C_\rho}(\vec{S}^\dagger\times\vec{q})\cdot\vec{\epsilon}_\rho, \tag{2}$$

where ϵ_ρ^μ is the polarization vector of the ρ and $S^\dagger$ the spin transition operator from spin 1/2 to 3/2. The coefficient $\sqrt{2/3}$ is an isospin coefficient. In addition we use a monopole form factor for the $\rho N\Delta$ vertex of the type

$$F_\rho(q)=\frac{\Lambda^2-m_\rho^2}{\Lambda^2-q^2}, \tag{3}$$

with $\Lambda=2$ GeV. By fixing $C_\rho=2$ and using the standard value $f^{*2}/4\pi=0.36$, the fit to the data in [29] gave a value $\tilde{f}_\rho=4.2$, 30% higher than the expected SU(3) value $\tilde{f}_\rho=f_\rho/2=3.1$ [32,33]. This value, however, is imposed by our choice of the $\rho N\Delta$ coupling, where we rely again on SU(6) symmetry to relate it to the empirical ρNN coupling used in [5].

The next step is to evaluate the K^+ self-energy in nuclear matter where the intermediate state is K^+ and a Δh excitation. This self-energy diagram is depicted in Fig. 2. By using the sum over Δ spins,

$$\sum_{M_s}S_i|M_s\rangle\langle M_s|S_j^\dagger=\frac{2}{3}\delta_{ij}-\frac{i}{3}\epsilon_{ijk}\sigma_k, \tag{4}$$

and taking into account that the three momenta of the $\rho N\Delta$ coupling must be taken in the Δ c.m. frame, we can write in terms of the K^+A lab frame momenta the kaon self-energy as

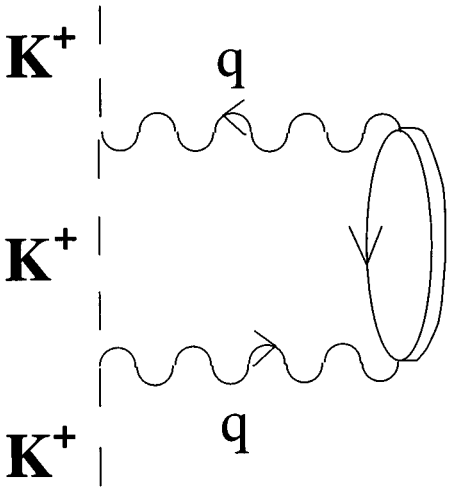

FIG. 2. Self-energy of K^+ associated with an intermediate Δh excitation.

$$\Pi(k)=i\int\frac{d^4q}{(2\pi)^4}D_\rho^2(q)\left(\frac{f^*}{m_\pi}\right)^2C_\rho\tilde{f}_\rho^2\tilde{U}_\Delta(q)\times\frac{16}{9}\left(\frac{M}{M_I}\right)^2(\vec{k}\times\vec{k}')^2D_{K^+}(k-q)F_\rho^2(q), \tag{5}$$

where M is the nucleon mass, M_I the invariant mass of the Δ, $M_I^2=p_\Delta^{02}-\vec{p}_\Delta^{\,2}$, D_{K^+} and D_ρ are the K^+ and ρ propagators respectively, and $\tilde{U}_\Delta(q)$ is the Δh Lindhard function with the normalization

$$\tilde{U}_\Delta(q)=\rho\frac{1}{\sqrt{s}-M_\Delta+i\Gamma(s)/2}, \tag{6}$$

with ρ the nuclear density.

The step from $\Pi(k)$ to a nuclear cross section is readily done by recalling that the reaction probability per unit time is ($2\omega V_{\rm opt}\equiv\Pi$)

$$\Gamma=-2\,\mathrm{Im}\;V_{\rm opt}=-\frac{1}{\omega}\mathrm{Im}\Pi(k), \tag{7}$$

with ω the kaon energy. The probability of reaction per unit length is then $-\mathrm{Im}\Pi/k$ and hence the contribution of an element of volume to the cross section is

$$d\sigma=-\frac{1}{k}\mathrm{Im}\Pi(k)d^3r. \tag{8}$$

The local density approximation comes now into action since $\Pi(k)$ is a function of ρ, the nuclear density, and then the cross section in a finite nucleus becomes

$$\sigma=-\frac{1}{k}\int d^3r\,\mathrm{Im}\Pi(k,\rho(\vec{r})). \tag{9}$$

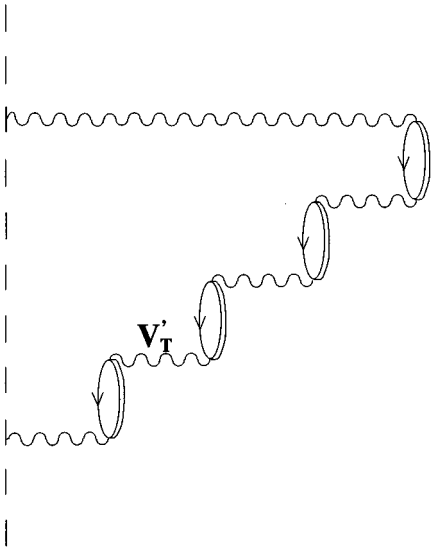

FIG. 3. Contribution to the K^+ self-energy from Tamm-Dancoff propagation of Δh states.

One must now evaluate $\mathrm{Im}\Pi$ from Eq. (5), which is readily done using Cutkosky rules, placing on shell the intermediate states of the self-energy diagram. Technically one has

$$\Pi(k)\to 2i\,\mathrm{Im}\Pi, \tag{10}$$

$$\widetilde{U}_\Delta(q)\to 2i\,\theta(q^0)\,\mathrm{Im}\,\widetilde{U}_\Delta(q), \tag{11}$$

$$D_{K^+}(k-q)\to 2i\,\theta(k^0-q^0)\,\mathrm{Im}\,D_{K^+}(k-q)$$
$$=2i\,\frac{1}{2\omega(\vec{k}-\vec{q})}(-\pi)\,\delta(k^0-q^0-\omega(\vec{k}-\vec{q})).$$

This allows us to write the K^+ differential cross section as

$$\frac{d\sigma}{d\Omega' d\omega'}=\int\frac{d^3r}{(2\pi)^3}\,\frac{k'}{k}\,\frac{8}{9}\left(\frac{f^*}{m_\pi}\right)^2 C_\rho\,\widetilde{f}_\rho^{\,2}(-)\,\mathrm{Im}\widetilde{U}_\Delta(k-k')$$
$$\times\left(\frac{M}{M_I}\right)^2(\vec{k}\times\vec{k}')^2 D_\rho(q)^2 F_\rho^2(k-k), \tag{12}$$

with $D_\rho(q)=(q^2-m_\rho^2)^{-1}$.

So far we have not introduced Δ self-energies into the scheme. There is also another physical effect that must be taken into account which is the distortion of the K^+ waves.

The Δ self-energy is readily introduced adding Σ_Δ from Ref. [18] to the Δ mass in $\widetilde{U}_\Delta(q)$, including Pauli corrections to the Δ width. At the same time one can introduce corrections from the RPA propagation of Δh in the medium to account for the diagrams of the type depicted in Fig. 3, where backward going Δh excitations are omitted since they are negligible in the Δ region. This is also accomplished technically in a very easy way [5] by substituting Σ_Δ by

$$\Sigma_\Delta\to\Sigma'_\Delta=\Sigma_\Delta+\frac{4}{9}\left(\frac{f^*}{m_\pi}\right)^2 V'_T\rho, \tag{13}$$

where V'_T is the transverse part of the spin-isospin interaction,

$$V'_T=\frac{\vec{q}^{\,2}}{q^2-m_\rho^2}C_\rho F_\rho^2(q)+g', \tag{14}$$

and g', the Landau-Migdal parameter, is taken as $g'=0.6$.

The next correction is the distortion of the kaons. This requires some thought because the K^+ is distorted only by quasielastic collisions or conversion into K^0. In the latter case the K^+ disappears after one collision (although it can be generated again in a second collision), but in the quasielastic collisions the K^+ remains, although changing direction and energy. The conventional use of a K^+-nucleus optical potential removes from the K^+ flux all events where there is a quasielastic collision or K^0 conversion. However, for small angles of the emerging K^+ this procedure is numerically accurate since the contribution of two step processes, one quasielastic and the other one the $N\Delta$ transition, is negligible at small angles. This has been found as a general rule in hadronic collisions [34], in the Δ excitation with the $(^3\mathrm{He},t)$ reaction [35,36], and in K^+ quasielastic scattering [25], much closer to the problem we are dealing with.

Since we are going to deal with small K^+ angles, we shall then use distorted waves for the K^+ and the same assumption of small angles allows us to use the eikonal approximation. In this case we must multiply the cross section of Eq. (12) by the distortion factor $D(k,k',\vec{r})$ given by

$$D(k,k',\vec{r})=\exp\left(\int_{-\infty}^{z}\sigma_{KN}^{(1)}\rho(\vec{b},z')dz'\right.$$
$$\left.+\int_z^{\infty}\sigma_{KN}^{(2)}\rho(\vec{b},z')dz'\right), \tag{15}$$

where $\vec{b}$ is the impact parameter corresponding to the point $\vec{r}$ and $\sigma_{KN}^{(1)},\sigma_{KN}^{(2)}$ are the K^+N cross sections of the incoming and outgoing K^+ respectively, which we take from [37].

Summarizing, our final formula for the cross section is given by Eq. (12) multiplying the expression by the distortion factor of Eq. (15) and substituting M_Δ by $M_\Delta+\Sigma'_\Delta$ in $\widetilde{U}_\Delta(q)$ of Eq. (6), with Σ'_Δ given by Eq. (13).

We present results in the next section.

III. RESULTS AND DISCUSSION

In Fig. 4 we show differential cross sections for Δ production for $k=1$ GeV/c and three different angles for ^{12}C. At the same time we calculate the background from quasielastic K^+ collisions in the same region, coming from one and two steps, as discussed in [25]. Since our aim is to single out kinematical regions where this background can be expected to be negligible, these latter calculations have been performed with some simplifications, that is the use of har-

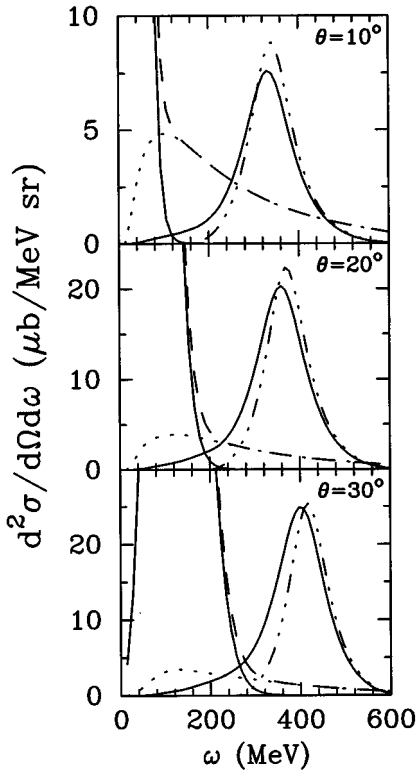

FIG. 4. Double differential cross sections for K^+ scattering on ^{12}C at 1 GeV/c and for three different angles. In the Δ region results for the free Δ (dot-dot-dot-dashed) and for the medium-modified Δ (solid) are displayed. The dashed line represents the total quasielastic background due to one-step (solid) and two-step (dotted) collisions.

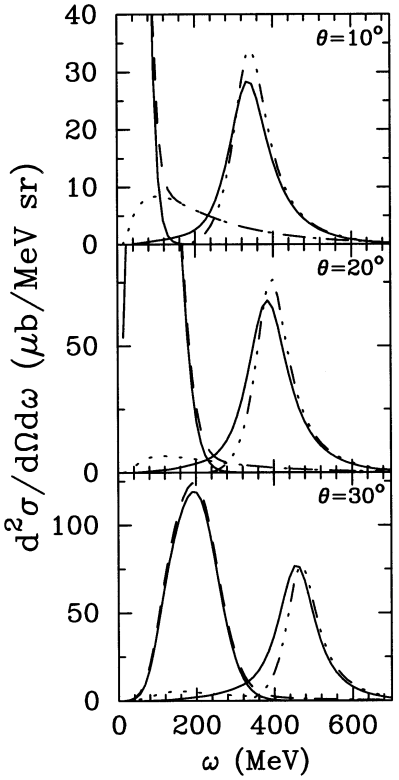

FIG. 5. As in Fig. 4, but for K^+ at 1.25 GeV/c.

monic oscillator states (instead of Woods-Saxon), the omission of the RPA corrections (which were found relevant only on the left-hand side of the quasielastic peak) and the neglect of the width of the ph states. We can see that at $\theta = 10°$ there is a substantial background below the Δ peak coming from two-step quasielastic collisions. The figure also shows the effect of the Δ self-energy and the Δh interaction in the transverse channel (addition of Σ'_Δ to M_Δ). There is a small shift of the peak to smaller excitation energies, a moderate decrease of the strength at the Δ peak and some increased strength at lower excitation energies, which comes as a consequence of the Δ coupling to ph components, i.e., the decay mode of the Δ in the nucleus, $\Delta N \rightarrow NN$. We can see this strength more visibly at bigger angles $\theta = 20°, 30°$. For these latter angles the quasielastic background is relatively smaller, which makes it easier to identify the Δ excitation strength.

In Fig. 5 we show the same results for $k = 1.25$ GeV. The qualitative features here are similar to those in Fig. 4, only the relative strength of the Δ excitation with respect to the quasielastic one is bigger.

In Fig. 6 we show the results for $k = 1.5$ GeV. Once again the features are similar to those in the former figures and the strength of the Δ excitation with respect to the quasielastic one is even higher. At the angle $\theta = 30°$ the Δ strength is bigger than the quasielastic one, but the quasielastic contri-

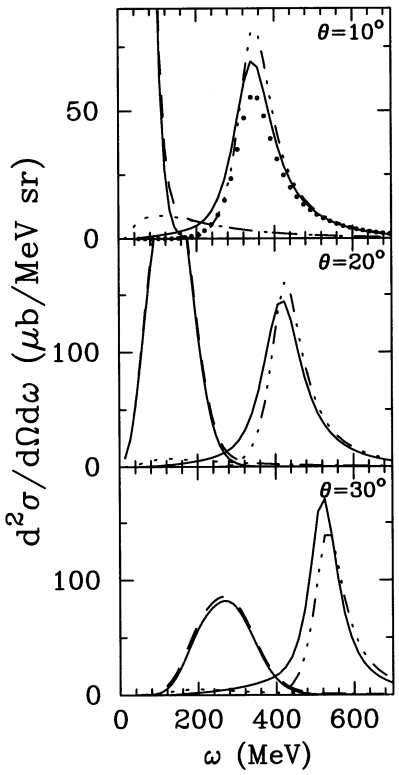

FIG. 6. As in Fig. 4, but for K^+ at 1.5 GeV/c. In the top panel, the amount of Δ strength in the medium due to pionic decay is also shown (heavy dots).

bution has a wide bump that induces an appreciable background below the Δ peak.

The effects of the Δ self-energy in the medium might look moderate by comparing the solid and dash-dotted lines in Figs. 4–6. However, the medium effects are far more relevant than these two lines might indicate. Indeed, in the case of a free Δ, the width is fully associated to the pionic decay of the Δ while in the nuclear medium the width is associated to pion emission and ph excitations and only part of the Δ strength of the figure goes into pion emission. This can be made quantitative by recalling the form for the Δ self-energy from [18]. We have

$$\widetilde{U}_{R,\Delta}(q)=\rho\frac{1}{\sqrt{s}-M_\Delta+i\widetilde{\Gamma}/2-\mathrm{Re}\Sigma_\Delta-\frac{4}{9}\left(\frac{f^*}{m_\pi}\right)^2V_T'\rho+iC_Q\left(\frac{\rho}{\rho_0}\right)^\alpha+iC_{A2}\left(\frac{\rho}{\rho_0}\right)^\beta+iC_{A3}\left(\frac{\rho}{\rho_0}\right)^\gamma}, \tag{16}$$

where $\widetilde{U}_{R,\Delta}$ is the Δh Lindhard function incorporating the self-energy corrections. In Eq. (16) ρ_0 is the normal nuclear matter density, $\widetilde{\Gamma}$ is the Pauli blocked width, and C_Q, C_{A2}, C_{A3} are coefficients parametrized in [18] such that their corresponding terms are associated to Δ pionic decay (C_Q), $2p1h$ decay (C_{A2}), and $3p2h$ decay (C_{A3}).

The strength of the Δ decaying into pions is associated to $\widetilde{\Gamma}$ and the C_Q term and we can write

$$\mathrm{Im}\widetilde{U}_{R,\Delta}(q)=-\rho\frac{\widetilde{\Gamma}/2+C_Q\left(\frac{\rho}{\rho_0}\right)^\alpha+C_{A2}\left(\frac{\rho}{\rho_0}\right)^\beta+C_{A3}\left(\frac{\rho}{\rho_0}\right)^\gamma}{\left(\sqrt{s}-M_\Delta-\mathrm{Re}\Sigma_\Delta-\frac{4}{9}\left(\frac{f^*}{m_\pi}\right)^2V_T'\rho\right)^2+\left(\frac{\widetilde{\Gamma}}{2}+C_Q\left(\frac{\rho}{\rho_0}\right)^\alpha+C_{A2}\left(\frac{\rho}{\rho_0}\right)^\beta+C_{A3}\left(\frac{\rho}{\rho_0}\right)^\gamma\right)^2}. \tag{17}$$

With this separation and bearing in mind the meaning of Cutkosky rules, if we take the first two terms in the numerator of Eq. (17), the resulting strength will go into primary pion emission, while the one coming from the last two terms will go into nucleon emission.

We have thus isolated the pionic decay content of the Δ strength and show it in Fig. 6 at $\theta=10°$. This strength is only about 70% of the corresponding one for a free Δ and the reduction is not due to the Pauli blocked width but to the competition of the other Δ decay channels. Indeed, in the absence of ph Δ decay channels, $\mathrm{Im}\widetilde{U}\sim\widetilde{\Gamma}^{-1}$, and with a reduced $\widetilde{\Gamma}$ width, the Δ peak would increase rather that the opposite, while at the same time the resonance shape would become narrower.

We should also point out that this pionic content refers to the first step of the reaction, before there is any final state interaction. Recall that in our local density formula we are producing the pions in an element of volume d^3r. In their way out, part of these pions will be reabsorbed and will show up as particle emission. In a nucleus like ^{12}C, about 30% of these pions are reabsorbed [15,38], so that finally only about 1/2 of the original strength assuming a free Δ goes into pion emission. It would be interesting to perform some coincidence measurements where pions would be detected together with the K^+.

We should also recall that the present reaction has other added advantages over the (^{3}He,t) reaction which has been thoroughly studied. Indeed, the Δ information on that reaction is essentially limited to 0°, since the cross section falls by about two orders of magnitude when going to about 5° and the shape of the Δ resonance is essentially lost [39]. Here, on the contrary, the cross section remains sizable up to angles of about 30° and more. This offers a wider spectrum of excitation energies and momenta by means of which to study the Δ excitation.

IV. CONCLUSIONS

We have evaluated the cross section for inclusive (K^+,K^+) scattering in nuclei around the Δ resonance region. These are the first evaluations for a reaction on which there are no data yet, but they could be obtained as a continuation of the recent experimental program in the quasielastic region [23,24].

The cross sections obtained are sizable, and the mixture with the quasielastic tail is sufficiently small in some regions to allow for a clean separation of the Δ excitation and the nuclear effects associated with it. The present study should stimulate such measurements that surely will contribute to enrich our knowledge of resonance renormalization in nuclei, which is a subject of continuous debate.

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