

## Level density parameter in a refined Thomas-Fermi theory

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We present calculations for the nuclear level density parameter over a wide range of nuclear mass and temperature. The hot nucleus is modeled in a refined Thomas-Fermi approximation that allows the confinement of the hot evaporating metastable system through an external pressure. The position and temperature dependence of the momentum and frequency dependent effective masses are taken into account. The agreement with experimental data, particularly its temperature dependence, is reasonably reproduced. [S0556-2813(98)05503-4]

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### I. INTRODUCTION

Nuclei with large excitations and high temperatures can now be created in the laboratory from intermediate energy or high energy nucleus-nucleus collisions [1]. Properties of such highly excited nuclear systems are studied by linking the deexcitation of the hot nuclei to the residual systems that are usually observed. This involves statistical analysis of the evaporation spectra where the nuclear level density  $\rho(E^*)$  plays a central role. In Bethe's [2] independent particle model, the expression for the level density is given by

$$\rho(E^*) \approx \exp(2\sqrt{aE^*})/E^{*2}, \quad (1)$$

where  $E^*$  is the excitation energy of the nucleus and  $a$  the level density parameter. It is related to the mass  $A$  of the nucleus as

$$a = A/K \quad (2)$$

and from experimental analysis of nuclear resonances and evaporation spectra, it is found that for cold nuclei  $K \approx 8$  MeV. From homogeneous Fermi-gas model, however, it is predicted to be  $K \approx 4\varepsilon_F/\pi^2$  ( $\varepsilon_F$  is the Fermi energy)  $\approx 16$  MeV at zero temperature [3].

Understanding this well-known discrepancy has been attempted by many authors by taking into account effects not included in the Fermi-gas model, namely surface and curvature effects [4–6], pairing and shell effects [3,7], deformation and angular momentum dependence [3,8], effects from both momentum and frequency dependent effective mass [5,6,9,10] and effects of continuum [6,11,12]. Recently there has been a renewed interest in the understanding of the level density parameter, particularly its temperature dependence as considerable amount of experimental data has poured in from energetic nucleus-nucleus collisions where nuclei could be created at very high temperatures. It was found that for high temperatures like  $T \approx 5$  MeV,  $K$  is around 13 MeV [13–16]. Giant dipole resonance (GDR) gamma-ray as well as particle emission in coincidence with the evaporation residues are expected to be dominated by emission from equilibrium nuclear configurations and analyses of these emis-

sion spectra indicate a temperature dependence of the level density parameter [15,16]. Temperature dependent Hartree-Fock (HF) [17] or semiclassical calculations [18,19] with different types of effective interactions fail to reproduce the trend of the data. It has however been recognized [6,9,10] that the observed  $T$  dependence of the level density parameter can be accounted for by taking into account the effects of correlations, i.e., the  $T$  dependence of the frequency dependent effective mass  $m_\omega$ . In a recent simplified but sufficiently realistic calculation in the Thomas-Fermi framework, the level density parameter  $a$  was calculated [6,20] at different temperatures for different nuclei including the finite size effects, the momentum and frequency dependence of the effective mass, the effects of continuum and the shell effects. The agreement with experimental data is reasonable.

Before extracting the level density parameter of a hot nucleus, it is necessary to have a consistent description of the excited nuclear system. The hot nucleus is thermodynamically unstable as it deexcites by emission of nucleons and light particles; the modeling of such a hot system therefore poses some problems. The main difficulty arises in taking proper account of the continuum states which are occupied at finite temperature as a result of which the particle density does not vanish at large distances. The extracted observables then depend on the size of the box in which the calculations are performed. This problem was addressed in a so-called subtraction procedure [17,18] where the nucleus was assumed to be a thermalized system in equilibrium with a surrounding gas. It was studied by means of a thermodynamic potential calculated as the difference between the thermodynamic potentials for the nucleus in equilibrium with the gas and that for the gas alone. In a recent refined Thomas-Fermi (TF) description [19], however, the tendency of the nucleons to leave the hot nucleus from the continuum states was compensated by imposing a suitable external pressure to maintain thermodynamic equilibrium. The nucleon densities of a nucleus at different temperatures are then independent of the size of the box in which calculations are done. The level density parameter calculated with these refined TF densities is then found to be nearly temperature independent in disagreement with the experimental data.

The purpose of the present paper is to see whether by suitably exploiting the refined TF model of the hot nucleus we get a better understanding of the level density parameter. The main difference between the self-consistent TF prescription pursued here with the ones done earlier [17,18] is the choice of the boundary conditions. In both cases, the thermodynamic equilibrium is maintained by external gas pressure; but whereas in Refs. [17,18], the gas contribution is subtracted in the evaluation of the TF (or HF) density, in our calculation, it is incorporated through a modified occupation probability as shown later. The effect of the temperature dependence of the frequency dependent effective mass was taken into account following the prescription of Refs. [6,20]. We also estimate the magnitude of the  $\hbar^2$  corrections to the Thomas-Fermi approximation. However, compared to the model calculations of Refs. [6,20], the present calculations provides a self-consistent treatment of the temperature dependence of the momentum dependent effective mass and an assessment of the limiting temperature [1]. Moreover, the calculations of Refs. [6,20] were carried out for a specific definition of the level density parameter (relating the excitation energy to temperature). In this work we also consider other definitions of the level density parameter and compared with the corresponding experimental data (see also Ref. [21]).

The paper is organized as follows. In Sec. II, we briefly review the refined TF theory and present equations relevant for the calculations of the level density parameter  $a$ . Results from the calculations are presented in Sec. III and the concluding remarks are given in Sec. IV.

## II. THEORETICAL FRAMEWORK

### A. Modeling the hot nucleus

The hot unstable nucleus is constrained in thermodynamic equilibrium by imposing a suitable external pressure on it. This constraint appears somewhat artificial for an isolated nucleus, but may be relevant in the astrophysical context where a nucleus embedded in a hot nucleon gas is a possible scenario. This gas supplies the necessary external pressure. Stability implies that the internal pressure exerted by the nucleus is balanced by the external gas pressure and that the chemical potentials of the neutrons and protons of the nucleus match exactly the corresponding chemical potentials of the gas environment. By minimization of the thermodynamic potential [22]

$$G = E - TS - \sum_{\tau} \mu_{\tau} N_{\tau} + P\Omega, \quad (3)$$

one is then led to the occupation probability [19]

$$n_{\tau}(\mathbf{r}, \mathbf{p}) = \left[ 1 + \exp \left[ p^2/2m_{\tau} + V_{\tau}^0(r) + p^2 V_{\tau}^1(r) + V_{\tau}^2(r) + \delta_{\tau p} V_C(r) - \mu_{\tau} + \frac{P}{\rho(r)} \right] / T \right]^{-1}. \quad (4)$$

In Eq. (3),  $E$  and  $S$  are the total energy and entropy of the nucleus,  $N_{\tau}$  refer to the number of neutrons or protons ( $\tau$  is the isospin index),  $\mu_{\tau}$  their chemical potentials,  $\Omega$  the vol-

ume and  $P$  the external pressure. The contribution of the  $P\Omega$  term in the minimization of  $G$  for a homogeneous bulk system with uniform density  $\rho$  is  $P/\rho$  to the single particle energy. For the finite system with diffuse density distribution, we take recourse to the local density approximation and then the  $P/\rho(r)$  term appears in the occupancy function given by Eq. (4). We mention that we have used a Seyler-Blanchard-type momentum and density dependent effective interaction [19]. Here  $m_{\tau}$  is the nucleon mass and  $V_{\tau}^2(r)$  is the rearrangement potential term that appears because of the density dependence in the interaction. The nuclear single-particle (SP) potential is given by  $V_{\tau}^0 + p^2 V_{\tau}^1$  where the momentum dependent part determines the effective nucleon  $k$ -mass  $m_{\tau,k}$  as

$$\frac{p^2}{2m_{\tau,k}} = \frac{p^2}{2m_{\tau}} + p^2 V_{\tau}^1. \quad (5)$$

The Coulomb potential  $V_C(r)$  contains both the direct and exchange terms. The occupation probability  $n_{\tau}(\mathbf{r}, \mathbf{p})$  given by Eq. (4) can also be expressed as the SP occupancy in the energy space as

$$f(\varepsilon_{\tau}) = [1 + \exp\{(\varepsilon_{\tau} - \mu_{\tau})/T\}]^{-1}. \quad (6)$$

The expression for the SP energy  $\varepsilon_{\tau}$  can easily be obtained by comparing Eqs. (4) and (6) and can be written as

$$\varepsilon_{\tau} = \frac{p^2}{2m_{\tau,k}} + \mathcal{V}_{\tau}(r). \quad (7)$$

Here  $\mathcal{V}_{\tau}$  is the effective single particle potential given by

$$\mathcal{V}_{\tau}(r) = V_{\tau}^0(r) + V_{\tau}^2(r) + \delta_{\tau p} V_C(r) + \frac{P}{\rho(r)}. \quad (8)$$

The density is obtained from (we drop the suffix  $\tau$  from now on)

$$\rho(r) = \frac{2}{h^3} \int n(\mathbf{r}, \mathbf{p}) d\mathbf{p}, \quad (9)$$

$$= A_T^* J_{1/2}(\eta(r)), \quad (10)$$

where

$$A_T^* = \frac{4\pi}{h^3} [2m_k(r)T]^{3/2}, \quad (11)$$

$J_K(\eta)$  is the Fermi integral

$$J_K(\eta) = \int_0^{\infty} \frac{x^K}{1 + \exp(x - \eta)} dx, \quad (12)$$

and

$$\eta(r) = \left[ \mu - V^0(r) - V^2(r) - \delta_{\tau,p} V_C(r) - \frac{P}{\rho(r)} \right] / T. \quad (13)$$

The zeroth order free energy and entropy density are given as

$$F(r) = \mu\rho - \frac{2}{3}TA_T^*J_{3/2}(\eta), \quad (14)$$

$$\sigma(r) = -\eta\rho + \frac{5}{3}A_T^*J_{3/2}(\eta). \quad (15)$$

The density profiles of neutrons and protons for a particular temperature are solved self-consistently with an external pressure  $P$  so chosen that the chemical potential of the neutrons and protons in the nucleus match exactly those in the gas phase that exerts the same pressure  $P$ . At zero temperature, the pressure is zero as there is no gas outside. Full expressions for the single particle potential with the chosen effective interaction as well as the methodology for solving the density self-consistently are given in [19].

Further second order ( $\hbar^2$ ) corrections to the density, free energy and entropy density are given as [23],

$$\rho_2(r) = \frac{1}{24} \frac{\hbar^2}{2m} A_T^* \left[ \frac{b_2(r)}{T} J_{-1/2}(\eta) + \frac{d_2(r)}{T^2} J_{-3/2}(\eta) + \frac{3}{4T^3} (\nabla\mathcal{V})^2 J_{-5/2}(\eta) \right], \quad (16)$$

$$F_2(r) = \mu\rho_2(r) + \frac{1}{12} \frac{\hbar^2}{2m} A_T^* \left[ -b_2(r) J_{1/2}(\eta) + \frac{d_2(r)}{T} J_{-1/2}(\eta) + \frac{1}{4T^2} (\nabla\mathcal{V})^2 J_{-3/2}(\eta) \right], \quad (17)$$

$$\sigma_2(r) = -\eta\rho_2(r) + \frac{1}{24} \frac{\hbar^2}{2m} A_T^* \left[ 3 \frac{b_2(r)}{T} J_{1/2}(\eta) - \frac{d_2(r)}{T^2} J_{-1/2}(\eta) + \frac{1}{4T^3} (\nabla\mathcal{V})^2 J_{-3/2}(\eta) \right]. \quad (18)$$

In Eqs. (16)–(18),

$$b_2(r) = \frac{7}{4} (\nabla q)^2 / q - 5 \Delta q, \quad (19)$$

and

$$d_2(r) = q \Delta \mathcal{V} - \frac{1}{2} \nabla q \cdot \nabla \mathcal{V}, \quad (20)$$

where  $q(r) = m/m_k(r)$ .

### B. The level density parameter

In this section, we present the basic expressions relevant for the calculation of the single particle level density  $g(\varepsilon)$  and the level density parameter. We will omit in the following the isospin index  $\tau$ .

The level density is given by

$$g(\varepsilon) = \text{Tr}(\varepsilon - \hat{H}) = \sum_i \delta(\varepsilon - \varepsilon_i), \quad (21)$$

where  $\hat{H}$  is the corresponding single particle Hamiltonian in which the single particle potential is given by the self-consistent mean field generated in the temperature dependent TF theory as discussed earlier.

The chemical potential is determined from number conservation

$$N = \int g(\varepsilon) f(\varepsilon, \mu, T) d\varepsilon. \quad (22)$$

Since

$$\begin{aligned} \rho(r) &= \frac{2}{h^3} \int n(\mathbf{r}, \mathbf{p}) d\mathbf{p} \\ &= \frac{1}{4\pi^2} \int_{\mathcal{V}(r)}^{\infty} (2m_k(r)/\hbar^2)^{3/2} \sqrt{\varepsilon - \mathcal{V}(r)} f(\varepsilon, T) d\varepsilon \end{aligned} \quad (23)$$

and  $N = \int \rho(\mathbf{r}) d\mathbf{r}$ , we have

$$g(\varepsilon) = \frac{2\sqrt{2}}{\pi\hbar^3} \int [m_k(r)]^{3/2} \sqrt{\varepsilon - \mathcal{V}(r)} r^2 dr. \quad (24)$$

In arriving at Eq. (23), use has been made of Eq. (4) and  $\varepsilon$  and  $\mathcal{V}(r)$  are defined as in Eqs. (7) and (8). Here the condition that  $(\varepsilon - \mathcal{V}(r)) > 0$  effectively limits the integration range. At large  $r$ ,  $P/\rho(r) \Rightarrow \infty$  and hence  $\mathcal{V}(r) \Rightarrow \infty$ . From Eq. (23), one sees that at  $T = 0$

$$\rho(r) = \frac{1}{6\pi^2} (2m_k(r)/\hbar^2)^{3/2} (\varepsilon_F - \mathcal{V}(r))^{3/2}. \quad (25)$$

In the independent particle model, the level density parameter is defined as [3]

$$a = \frac{\pi^2}{6} g(\varepsilon = \varepsilon_F), \quad (26)$$

where  $g(\varepsilon_F)$  is obtained from Eq. (24). At finite temperature, to keep the definition analogous to Eq. (26), we use Eq. (25) in the expression for  $g(\varepsilon_F)$  (the Thomas-Fermi approximation) and obtain

$$a_{\text{TF}} = \frac{1}{24} \int (2m_k(r)/\hbar^2)^{3/2} (\varepsilon_F - \mathcal{V}(r))^{1/2} d\mathbf{r}. \quad (27)$$

For the purpose of comparison, we will also calculate (for  $T > 0$ )

$$a_\mu = \frac{\pi^2}{6} g(\varepsilon = \mu). \quad (28)$$

Once the occupation probability  $f$  is known, the entropy  $S$  and the internal energy  $E$  can be determined from

$$S(T) = - \int g(\varepsilon) [f \ln f + (1-f) \ln(1-f)] d\varepsilon, \quad (29)$$

$$E(T) = \int \varepsilon g(\varepsilon) f(\varepsilon, \mu, T) d\varepsilon, \quad (30)$$

and the excitation energy is given by

$$E^*(T) = E(T) - E(0). \quad (31)$$

In the zero temperature limit, one has  $S = 2aT$ ,  $E^* = aT^2$ . Then one may define the level density parameter for a given temperature using any of the following relations:

$$a_E(T) = E^*/T^2, \quad a_S(T) = S/2T, \quad a_{SE}(T) = S^2/4E^*, \quad (32)$$

which coincide in the limit  $T \rightarrow 0$ . Any significant difference that may arise between them at low temperatures ( $T \leq 2$  MeV) is due to shell effects. In the present calculation, we ignore shell effects as we are more interested in the high temperature behavior of the level density parameter.

The effective mass has both the  $k$ -mass and the  $\omega$ -mass and is defined as

$$m^* = m(m_k/m)(m_\omega/m). \quad (33)$$

The temperature dependent  $k$ -mass [defined as  $m_k(r)$  earlier] is given by the momentum dependence of the single-particle (SP) potential, but the frequency-dependent effective mass  $m_\omega$  is due to the coupling of the single particle motion with the collective degrees of freedom. This effective mass  $m_\omega$  varies with position and temperature and is known to increase the level density substantially [9,10]. The origin and the magnitude of the effects due to  $m_\omega$  have been discussed and calculated earlier; see Refs. [6,9,10]. To simulate these effects in our calculation, we make an *ad hoc* inclusion of  $m_\omega$ , for which we use the phenomenological form [5,6]

$$\frac{m_\omega}{m} = 1 - 0.4A^{1/3} \exp[-(T/(21A^{1/3}))^2] \frac{d\rho(r)}{dr} \bigg/ \rho_m, \quad (34)$$

where  $\rho_m$  is the central density of the density distribution and the temperature  $T$  is measured in MeV.

A self-consistent calculation including the  $T$  dependence of  $m_\omega$  is very involved and not within the scope of the present work. We have therefore adopted the following approach which is a realistic extension to the approach used in Ref. [6] when the complexities are omitted. For each fixed temperature  $T$ , we assume that the nucleus is well described by the mean field and the effective mass given by Eqs. (5) and (34). We note that the self-energy of a particle in the medium is usually approximated by a local field  $(m^*/m)U(r)$ , with the effective mass of Eq. (5) and  $U(r)$  is a local (Woods-Saxon) potential well, see Ref. [5]. In our case  $m_k$  is already included in our  $\mathcal{V}(r)$ . Therefore, due to the frequency dependence in the effective mass,  $g(\varepsilon)$  is modified from Eq. (24) as

$$g(\varepsilon) = \frac{2\sqrt{2}}{\pi\hbar^3} \int (m^*(r))^{3/2} \left[ \sqrt{\varepsilon - \frac{m}{m_\omega}} \mathcal{V}(r) - \Theta(\varepsilon - V_B) \sqrt{\varepsilon - V_B} \right] \Theta\left(\varepsilon - \frac{m}{m_\omega} \mathcal{V}(r)\right) r^2 dr. \quad (35)$$

The term  $\Theta(\varepsilon - V_B) \sqrt{\varepsilon - V_B}$  in Eq. (35) is included in order to correct for the effects of continuum [6], where  $V_B$  is the Coulomb barrier, being zero for neutrons. The effect of the continuum on the single-particle level density  $g(\varepsilon)$  was investigated in detail in Refs. [12,24,25], it decreases with  $\varepsilon$  for  $\varepsilon > 0$ . It was found in Refs. [6,20] that for temperatures above 2 MeV, this has a significant effect on the level density parameter  $a$  (it decreases). The resulting  $g(\varepsilon)$  is now assumed to be independent of  $T$  and used to determine the corresponding ground state Fermi energy  $\varepsilon_F$ , the cold energy  $E(0)$ , the chemical potential  $\mu(T)$ , the excitation energy  $E^*(T)$  and the entropy  $S(T)$ . The level density parameters  $a_{TF}$ ,  $a_\mu$ ,  $a_E$ ,  $a_S$  and  $a_{SE}$  for neutrons and protons are obtained from Eqs. (26), (27), (28) and (32). Using  $a = a_n + a_p$ , the corresponding inverse level density parameters  $K_{TF}$ ,  $K_\mu$ ,  $K_E$ ,  $K_S$  and  $K_{SE}$  are then deduced from Eq. (2).

### III. RESULTS AND DISCUSSIONS

The explicit form of the effective interaction used for the calculation of the Thomas-Fermi interaction energy density is given by

$$\mathcal{V}_{\text{eff}}(\mathbf{r}_1, \mathbf{r}_2, p, \rho) = -C_{l,u} \left[ 1 - \frac{p^2}{b^2} - d^2 \{ \rho(\mathbf{r}_1) + \rho(\mathbf{r}_2) \}^n \right] \times \frac{\exp(-|\mathbf{r}_1 - \mathbf{r}_2|/a)}{|\mathbf{r}_1 - \mathbf{r}_2|/a}. \quad (36)$$

Here  $\mathbf{r}$  and  $\mathbf{p}$  are the relative separation of the interacting nucleons in configuration and momentum space,  $\rho(\mathbf{r}_1)$  and  $\rho(\mathbf{r}_2)$  are the densities at the sites of the two nucleons and  $C_l$  and  $C_u$  are the strengths for like pair ( $n$ - $n$ ,  $p$ - $p$ ) and unlike pair ( $n$ - $p$ ) interactions. The six parameters  $C_l$ ,  $C_u$ ,  $a$ ,  $b$ ,  $d$  and  $n$  are determined by reproducing the volume energy per particle of symmetric nuclear matter, its saturation density, volume asymmetry energy, surface energy, the energy dependence of the real part of the nucleon-nucleus optical potential and energies of giant monopole resonances (GMR). The values of the parameters can be found in Ref. [19]. The incompressibility of symmetric nuclear matter obtained from this interaction comes out to be  $K_\infty = 238$  MeV, in close agreement with that obtained from an extended analysis by Blaizot [26].

To study the mass dependence of the level density parameter, we have considered four nuclei, namely  $^{40}\text{Ca}$ ,  $^{90}\text{Zr}$ ,  $^{150}\text{Sm}$  and  $^{208}\text{Pb}$  which covers a wide mass spectrum. In studying the temperature dependence of the level density parameter, we note that isolated hot nuclei cannot remain in stable equilibrium and therefore in modeling stable hot nuclei, we assume that they are embedded in an environment of nucleonic gas at the same temperature whose composition (neutron-proton ratio) and density can be regulated so that

the nucleus can remain in complete thermodynamic equilibrium. However, the equilibrium conditions can be realized only up to a certain maximum temperature called the limiting temperature  $T_{\text{lim}}$  above which the nucleus ceases to exist as a bound system. Level density parameter is thus defined only up to the limiting temperature. It is found that the limiting temperature decreases with mass number, mainly because of the Coulomb effects. A comprehensive discussion on finding out the equilibrium conditions and the limiting temperature for a two-component hot nucleus in the Thomas-Fermi model is given in Ref. [19].

The basic ingredients in the evaluation of the level density parameter are the density profile, the effective mass, effective single particle potential and the chemical potential. The density profile calculated with inclusion of second order corrections is found to become more diffuse with increase in temperature and can be well approximated by a Woods-Saxon form

$$\rho(r, T) = \rho_0(T) / \left[ 1 + \exp\left\{ \frac{r - R(T)}{a(T)} \right\} \right], \quad (37)$$

where

$$R(T) \approx r_0 A^{1/3} (1 + \alpha T^2), \quad (38)$$

$$a(T) \approx a_0 (1 + \beta T^2) \quad (39)$$

and

$$\rho_0(T) \approx \rho_{00} (1 - \gamma T^2). \quad (40)$$

The values of  $r_0$ ,  $a_0$  and  $\rho_{00}$  are weakly dependent on the mass number. For neutrons in  $^{150}\text{Sm}$ , we get  $r_0 = 1.154$  fm,  $a_0 = 0.458$  fm and  $\rho_{00} = 0.0863$  fm $^{-3}$ . The coefficients of the quadratic temperature dependence  $\alpha, \beta$  and  $\gamma$  are 0.00052, 0.006 and 0.0018 MeV $^{-2}$ , respectively.

The nuclear part  $V^0(r) + V^2(r)$  of the effective single-particle potential  $\mathcal{V}(r)$  can be parametrized in a Woods-Saxon form

$$V(r, T) = V_0(T) / [1 + \exp\{r - R_V(T)\}/a_V(T)]. \quad (41)$$

The temperature dependence of the potential parameters can also be approximated by a quadratic temperature dependence as in Eqs. (38)–(40). The values of  $r_V(0), a_V(0)$  and  $V_0(0)$  for the neutron single particle potential in  $^{150}\text{Sm}$  are found out to be 1.34 fm, 0.594 fm and  $-63.69$  MeV and the corresponding temperature coefficients are 0.00065, 0.0026 and 0.0012 MeV $^{-2}$ , respectively. The radius and diffuseness parameters are greater than those for the density as expected. It may be pointed out that the real part of the nucleon-nucleus optical potential is given by  $V_0(r) + p^2 V^1(r)$  which is different from the potential as given by Eq. (41). The effective  $k$ -mass of the nucleus can also be parametrized in a similar form [5,6]

$$1 - \frac{m_k(T)}{m} = \left[ 1 - \frac{m_k(0)}{m} \right] f_W(r) (1 - \delta T^2), \quad (42)$$

where  $f_W(r)$  is the Woods-Saxon form factor. For neutrons in  $^{150}\text{Sm}$ ,  $m_k(0)/m = 0.664$ ; the radius and the diffuseness parameters for the effective  $k$ -mass profile at zero tempera-

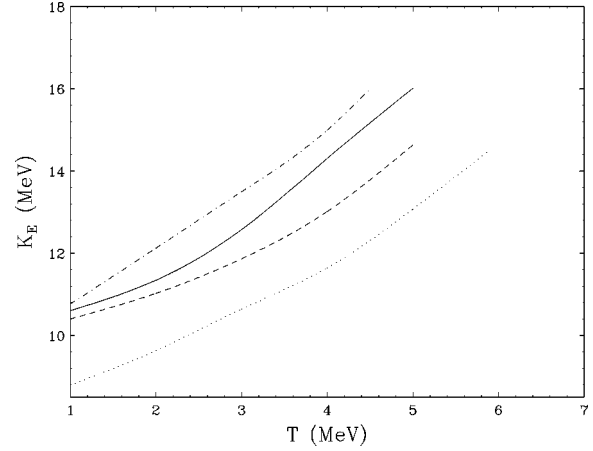


FIG. 1. The inverse level density parameter  $K_E$  plotted as a function of temperature  $T$  for the systems Ca (dotted line), Zr (dashed line), Sm (solid line), and Pb (dash-dotted line).

ture are  $r_m(0) = 1.33$  fm and  $a_m(0) = 0.572$  fm, respectively. The temperature coefficients for the parameters  $r_m(T)$  and  $a_m(T)$  are found out to be 0.0006 and 0.0025 MeV $^{-2}$ . The value of  $\delta$  comes out to be 0.0014 MeV $^{-2}$ . Similar parameters for the density profile, SP potential and effective  $k$ -mass for protons have also been determined. The temperature dependence of the effective  $\omega$ -mass is given by Eq. (34).

In Fig. 1, we display the temperature dependence of the inverse level density parameter  $K_E(T)$  for the nuclei  $^{40}\text{Ca}$ ,  $^{90}\text{Zr}$ ,  $^{150}\text{Sm}$  and  $^{208}\text{Pb}$ , using the Eqs. (32) and (35). Second order corrections as well as effects due to the  $\omega$ -mass have been incorporated in these calculations. The inverse level density parameter  $K_E$  increases both with temperature and with mass number of the nucleus, in qualitative agreement with that observed in Ref. [20]. The differences between the present results and those of Ref. [20] can be attributed to the differences in the adopted mean fields (effective mass, etc.) The shell effects that are important at low temperatures and dissolves at  $T \sim 1.5$  to 2 MeV are not included in the present calculations and therefore the results are displayed from  $T = 1$  MeV. Though the SP potential, density profile and the effective  $k$ -mass are temperature dependent, in previous calculations [19,27] it is found that the level density parameter or its inverse are very weakly dependent on temperature. In the present calculation, the marked temperature dependence is attributed to the  $\omega$ -mass as given by Eq. (34). The second order corrections decrease the inverse level density parameter only by a few percent.

As discussed in Sec. II, we note that there are a few definitions for the level density parameter. The specific definition to be used is determined by the appropriate experimental data and analysis. In certain experiments one determines the excitation energy and the temperature independently. In this case,  $K_E$  is the relevant parameter. In standard statistical model calculation (such as Cascade), one adopts a Bethe-like expression for the level density and thus  $K_{SE}$  is the relevant parameter (see also Ref. [21]). In Fig. 2, we present calculations for the inverse level density parameters  $K_{TF}$ ,  $K_\mu$ ,  $K_S$ ,  $K_E$  and  $K_{SE}$  given from Eqs. (27), (28) and (32) for the system  $^{150}\text{Sm}$ . As expected,  $K_\mu, K_S, K_E$  and  $K_{SE}$  are nearly the same at lower temperatures; they differ somewhat at higher temperatures. The value of  $K_{TF}$  is significantly lower

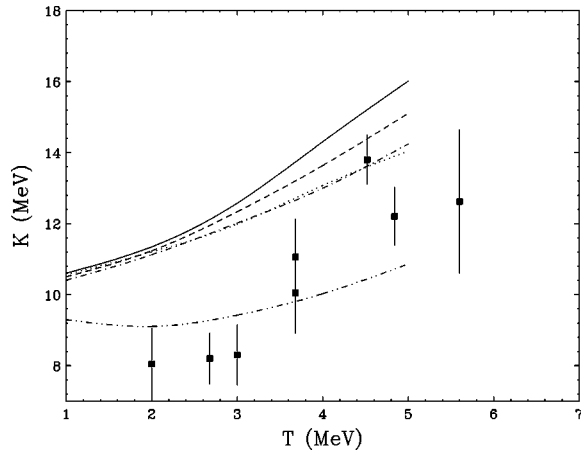


FIG. 2. The calculated inverse level density parameter for the system  $^{150}\text{Sm}$  is shown as a function of temperature along with the experimental data. The dash-dot-dotted, dotted, dashed, full, and dash-dotted lines correspond to  $K_{TF}$ ,  $K_{\mu}$ ,  $K_S$ ,  $K_E$ , and  $K_{ES}$ , respectively.

compared to the other ones and its temperature dependence is relatively weak. We have verified numerically that at very low temperature ( $T=0.1$  MeV) the different definition of  $K$  yield the same result. Note that at higher temperature  $K_{SE}$  is smaller than  $K_E$ , as pointed out in Ref. [21]. As seen from Fig. 2,  $K_{SE}$  can be well approximated by  $K_{\mu}$ . The experimental data [13,28,29] for  $A \approx 160$  at different temperatures are also shown in the figure. Since these experimental values are obtained by extracting the temperature from the slope of the evaporation spectra of light particles and the excitation energies from the linear momentum transfer, they pertain to the definition  $K_E$ .

#### IV. SUMMARY AND CONCLUDING REMARKS

We have calculated the level density parameter of several nuclei covering a wide mass range at different temperatures in a Thomas-Fermi theory including the second order corrections. The stability of the hot nucleus is ensured by placing it in an appropriate nucleon gas environment so that thermodynamic equilibrium is maintained. In our treatment, the temperature-dependent density profile is evaluated in a self-consistent way as a natural consequence of which the temperature dependence of the SP potential and effective  $k$ -mass profiles are obtained. These are basic ingredients in the calculation of level density parameter and we have found that with these alone, the level density parameter is highly underestimated and is practically temperature independent [19]. With the inclusion of a phenomenological temperature dependent effective  $\omega$ -mass, however, the calculated level density parameter compares reasonably with the experimental data, particularly its temperature variation is well reproduced. The second order corrections to the Thomas-Fermi theory do not change the level density parameter much, they increase the level density parameter typically by a few percent.

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- [1] D. Guerreau, in *Towards a Unified Picture of Nuclear Dynamics*, International Symposium, edited by Y. Abe, F. Sakata, and S. M. Lee, AIP Conf. Proc. No. 250 (AIP, New York, 1992), p. 347 and references therein.
  - [2] H. A. Bethe, *Rev. Mod. Phys.* **93**, 5 (1937).
  - [3] A. Bohr and B. R. Mottelson, *Nuclear Structure, Vols. I and II* (Benjamin, Reading, MA, 1969 and 1975).
  - [4] J. Toke and W. J. Swiatecki, *Nucl. Phys.* **A372**, 141 (1981).
  - [5] M. Prakash, J. Wambach, and Z. Y. Ma, *Phys. Lett.* **128B**, 141 (1983).
  - [6] S. Shlomo and J. B. Natowitz, *Phys. Lett. B* **252**, 187 (1990).
  - [7] S. K. Kataria, V. S. Ramamurthy, and S. S. Kapoor, *Phys. Rev. C* **18**, 549 (1978).
  - [8] F. Garcias, M. Barranco, J. Nemeth, and C. Ngo, *Phys. Lett. B* **206**, 177 (1988).
  - [9] R. Hasse, and P. Schuck, *Phys. Lett. B* **179**, 313 (1986).
  - [10] P. F. Bortignon and C. H. Dasso, *Phys. Lett. B* **189**, 381 (1987).
  - [11] D. L. Tubbs and S. E. Koonin, *Astrophys. J.* **232**, L59 (1979).
  - [12] D. R. Dean and U. Mosel, *Z. Phys. A* **322**, 647 (1985).
  - [13] K. Hagel *et al.*, *Nucl. Phys.* **A486**, 429 (1988).
  - [14] M. Gonin *et al.*, *Phys. Lett. B* **217**, 406 (1989).
  - [15] B. J. Fineman, K. T. Brinkmann, A. L. Caraley, N. Gam, R. L. Mcgrath, and J. Velkovska, *Phys. Rev. C* **50**, 1991 (1994).
  - [16] D. Fabris *et al.*, *Phys. Rev. C* **50**, R1261 (1994).
  - [17] P. Bonche, S. Levit, and D. Vautherin, *Nucl. Phys.* **A436**, 265 (1985).
  - [18] E. Suraud, *Nucl. Phys.* **A462**, 109 (1987).
  - [19] J. N. De, N. Rudra, S. Pal, and S. K. Samaddar, *Phys. Rev. C* **53**, 780 (1996).
  - [20] S. Shlomo and J. B. Natowitz, *Phys. Rev. C* **44**, 2878 (1991).
  - [21] J. P. Lestone, *Phys. Rev. C* **52**, 1118 (1995).
  - [22] S. K. Ma, *Statistical Mechanics* (World Scientific, Singapore, 1993) (first reprint) p. 266.
  - [23] J. Bartel *et al.*, *Nucl. Phys.* **A445**, 263 (1985).
  - [24] S. Shlomo, *Nucl. Phys.* **A539**, 17 (1992).
  - [25] S. Shlomo, V. M. Kolomietz, and H. Dejbakhsh, *Phys. Rev. C* **55**, 1972 (1997).
  - [26] J. P. Blaizot, *Phys. Rep.* **64**, 171 (1980).
  - [27] E. Suraud, P. Schuck, and R. W. Hasse, *Phys. Lett.* **164B**, 212 (1985).
  - [28] E. Chulick, M. N. Namboodiri, and J. B. Natowitz (unpublished).
  - [29] M. Gonin *et al.*, *Phys. Rev. C* **42**, 2125 (1991).