Electric dipole transitions between Gamow-Teller and spin-dipole states

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We study electric dipole transitions between Gamow-Teller (GT) and spin-dipole (SD) states. SD and GT excitations are calculated within the Hartree-Fock+Tamm-Dancoff approximation for ^{48}Sc and ^{90}Nb. The electric dipole transitions are found to be rather selective, and strong $E1$ transitions occur to some specific spin-dipole states. Calculated $E1$ transition strengths between GT and SD states are compared with the analytic sum rules within one-particle–one-hole ($1p$-$1h$) configuration space and within both $1p$-$1h$ and $2p$-$2h$ model space. Possible implications for charge-exchange reactions may help to understand the quenching problem of spin excitations. [S0556-2813(98)03401-3]

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Spin excitation modes in nuclei are interesting and stimulating subjects for investigation. Quenching and spreading of Gamow-Teller (GT) strength in nuclei are under theoretical [1,2] and experimental [3] investigations with refined accuracy stimulated by the recent development of experimental facilities [4]. The problem of a quenching mechanism in a quantitative level, that is, how much of it comes from the Δ-hole excitations or the two-particle–two-hole ($2p$-$2h$) excitations, still remains an unsettled issue.

Spin-dipole (SD) excitations, i.e., $\Delta L=1$ spin-flip excitations, have also been studied experimentally [5]. Theoretical investigations of the SD mode [6,2] arise also as an interesting problem in relation to nuclear structure and also astrophysical issues. In a recent work [7], we studied a possible enhancement of the magnetic dipole ($M1$) transitions between GT states and isobaric analog states (IAS's). Here we extend our study and investigate electric dipole ($E1$) transitions between GT and SD states in the daughter nucleus. We use the Tamm-Dancoff model for simplicity. The more elaborate random phase approximation (RPA) should not alter essentially our main conclusions.

We first obtain GT and SD states in Hartree-Fock (HF)+Tamm-Dancoff approximation (TDA) with Skyrme forces, and then we calculate $E1$ transitions between them. Analytic formulas for the sum rules of $E1$ transitions within $1p$-$1h$ TDA model space are derived and compared with numerical HF+TDA results. We also derive a sum rule including $2p$-$2h$ configuration space beyond the $1p$-$1h$ TDA model space and discuss the relations with $E1$ transitions in the parent nucleus. This point is particularly interesting for checking the validity of Brink's hypothesis on giant resonances built on top of excited states [8], since the SD states can also be considered as giant dipole states on top of an excited state (the GT state).

Let us consider $E1$ transitions between GT and SD states in ^{48}Sc and ^{90}Nb within the $1p$-$1h$ TDA configuration space. This configuration space does not have any effect of the ground state correlations which might be small in the charge-exchange τ_- excitations.

In the TDA framework, we can derive analytic formulas for the total transition rate. We first define the operators

$$\hat{G}=\sum_{im}\tau_-^i\sigma_m^i,$$

$$\hat{S}=\sum_{im\mu}\tau_-^i\sigma_m^i r_i Y_1^\mu(\hat{r}_i),$$

$$\hat{D}=\sum_{i\mu}\frac{1}{2}\tau_3^i r_i Y_1^\mu(\hat{r}_i), \tag{1}$$

where $\tau_\pm=\frac{1}{2}(\tau_x\pm i\tau_y)$ are the isospin-raising and -lowering operators. The doorway state for the GT excitation can be defined as

$$|\mathrm{GT}\rangle=\frac{1}{\sqrt{N_{\mathrm{GT}}}}\hat{G}|\hat{0}\rangle, \tag{2}$$

where $|\hat{0}\rangle$ is the parent state and $N_{\mathrm{GT}}=\langle\hat{0}|\hat{G}^\dagger\hat{G}|\hat{0}\rangle$ is the normalization factor. Similarly, the SD state will be given by

$$|\mathrm{SD}\rangle=\frac{1}{\sqrt{N_{\mathrm{SD}}}}\hat{S}|\hat{0}\rangle, \tag{3}$$

with the corresponding normalization factor $N_{\mathrm{SD}}=\langle\hat{0}|\hat{S}^\dagger\hat{S}|\hat{0}\rangle$. We will first discuss the commutation relations between the above operators. The $E1$ transition matrix ele-

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ment between the GT state and the SD state can be related with the following matrix elements containing commutators:

$$\langle 0|\hat{S}^\dagger \hat{D}\hat{G}|0\rangle = \langle 0|\hat{S}^\dagger[\hat{D},\hat{G}] + [\hat{S}^\dagger,\hat{G}]\hat{D} + \hat{G}\hat{S}^\dagger\hat{D}|0\rangle$$
$$= \langle 0|\hat{S}^\dagger[\hat{D},\hat{G}] + [\hat{S}^\dagger,\hat{G}]\hat{D}|0\rangle. \quad (4)$$

Here the parent state is taken to be a closed core, and $\hat{G}^\dagger|0\rangle = 0$ and $\langle 0|\hat{G} = 0$ are obtained within the TDA model. Note that $\hat{S}^\dagger|0\rangle \neq 0$ even in the TDA model. One can show that

$$[\hat{D},\hat{G}] = -\hat{S} \quad (5)$$

by using $[\tau_3^i, \tau_\pm^j] = \pm 2\delta_{ij}\tau_\mp^i$. It can also be shown by using $[\tau_+^i, \tau_-^j] = \delta_{ij}\tau_3^i$ and $\Sigma_{mm'}$ $[\sigma_m^{i\dagger}, \sigma_{m'}^j] = 4\delta_{ij}(\sigma_1^i - \sigma_{-1}^i)$ that

$$\langle 0|[\hat{S}^\dagger,\hat{G}]\hat{D}|0\rangle = \sum_n \langle 0|[\hat{S}^\dagger,\hat{G}]|n\rangle\langle n|\hat{D}|0\rangle$$
$$= \sum_n \langle 0|6\hat{D}^\dagger|n\rangle\langle n|\hat{D}|0\rangle$$
$$= 6\langle 0|\hat{D}^\dagger\hat{D}|0\rangle, \quad (6)$$

where the states $|n\rangle$ are $1p$-$1h$ states with non-spin-flip excitations, so that $\langle 0|\sigma_1^i - \sigma_{-1}^i|n\rangle = 0$. We thus obtain

$$\langle 0|\hat{S}^\dagger\hat{D}\hat{G}|0\rangle = \langle 0|-\hat{S}^\dagger\hat{S} + 6\hat{D}^\dagger\hat{D}|0\rangle$$
$$= 6\sum_n |\langle n|\hat{D}|0\rangle|^2 - \sum_n |\langle n|\hat{S}|0\rangle|^2. \quad (7)$$

The following equality is also derived:

$$\langle 0|[\hat{S}^\dagger,\hat{S}]|0\rangle = \frac{9}{4\pi}\left\langle 0\left|\sum_i \tau_3^i r_i^2\right|0\right\rangle,$$
$$\langle 0|\hat{S}^\dagger\hat{S}|0\rangle - \langle 0|\hat{S}\hat{S}^\dagger|0\rangle = \frac{9}{4\pi}(N\langle r^2\rangle_n - Z\langle r^2\rangle_p). \quad (8)$$

The normalization factor $N_{\rm GT}$ is evaluated to be

$$\langle 0|\hat{G}^\dagger\hat{G}|0\rangle = \left\langle 0\left|\sum_{ij}[\tau_+^i,\tau_-^j]\sum_{mm'}\sigma_m^{i\dagger}\sigma_{m'}^j + \sum_{ij}\tau_+^i\tau_-^j\sum_{mm'}[\sigma_m^{i\dagger},\sigma_{m'}^j]\right|0\right\rangle = \left\langle 0\left|\sum_i 3\tau_3^i\right|0\right\rangle = 3(N-Z). \quad (9)$$

Then, the total $E1$ transition rate within the $1p$-$1h$ configuration space is given as

$$S_{1p\text{-}ih} = |\langle 0|\hat{S}^\dagger\hat{D}\hat{G}|0\rangle|^2/[\langle 0|\hat{G}^\dagger\hat{G}|0\rangle\langle 0|\hat{S}^\dagger\hat{S}|0\rangle]$$
$$= [6\langle 0|\hat{D}^\dagger\hat{D}|0\rangle - \langle 0|\hat{S}^\dagger\hat{S}|\rangle]^2/[3(N-Z)\langle 0|\hat{S}^\dagger\hat{S}|0\rangle]$$
$$= \left[6\langle 0|\hat{D}^\dagger\hat{D}|0\rangle - \langle 0|\hat{S}\hat{S}^\dagger|0\rangle - \frac{9}{4\pi}(N\langle r^2\rangle_n - Z\langle r^2\rangle_p)\right]^2 \Big/ [3(N-Z)\langle 0|\hat{S}^\dagger\hat{S}|0\rangle], \quad (10)$$

where Eq. (8) is used in the last line. Thus the total $E1$ transition rate between the GT and SD states in the daughter nucleus is related to the rates of $E1$ and SD transitions in the parent nucleus.

Calculated HF+TDA GT and SD transition strengths with the use of the SGII interaction [9] are shown in Figs. 1(a) and 1(b) for ^{48}Sc and ^{90}Nb, respectively. The results are also tabulated in Table I for ^{48}Sc and in Table II for ^{90}Nb. Calculated GT states for ^{48}Sc and ^{90}Nb are located at energies of 11.90 and 16.82 MeV with respect to their parents, and 73.9% and 72.5% of the total strengths are concentrated in these states, respectively. The SD states have $J^\pi = 0^-$, 1^-, and 2^-. For 0^- states, more than 50% of the total strengths are concentrated in one state in both ^{48}Sc and ^{90}Nb nuclei. The SD strengths of 1^- and 2^- states are more fragmented than that of 0^- states. Actually, most of the strengths for 1^{-1} states in ^{90}Nb are found in two states while those in ^{48}Sc are fragmented among several states. The strengths for 2^- states are distributed in much wider energy ranges in both ^{48}Sc and ^{90}Nb. Some 2^- states are below the GT state. Let us next discuss the $E1$ transitions between GT and SD states.

According to Eq. (10), the total $E1$ transition rate can be evaluated from the total $E1$ and SD transition rates in the parent nucleus. In the TDA, one gets $\langle 0|\hat{D}^\dagger\hat{D}|0\rangle = 15.90$ fm^2 and $\langle 0|\hat{S}^\dagger\hat{S}|0\rangle = 139.37$ fm^2 for ^{48}Ca and $\langle 0|\hat{D}^\dagger\hat{D}|0\rangle = 35.14$ fm^2 and $\langle 0|\hat{S}^\dagger\hat{S}|0\rangle = 282.44$ fm^2 for ^{90}Zr. Then the sum rule becomes $S_{1p\text{-}1h} = 0.577$ and 0.606 fm^2 for ^{48}Sc and ^{90}Nb, respectively, using Eq. (10).

The transition rate from the GT state

$$|{\rm GT},1^+m\rangle = \sum_{ph} X_G^{ph}|ph^{-1};1^+m\rangle, \quad (11)$$

to the SD state with spin J,

$$|{\rm SD},J^-m'\rangle = \sum_{p_sh_s} X_S^{p_sh_s}|p_sh_s^{-1};J^-m'\rangle, \quad (12)$$

is given by

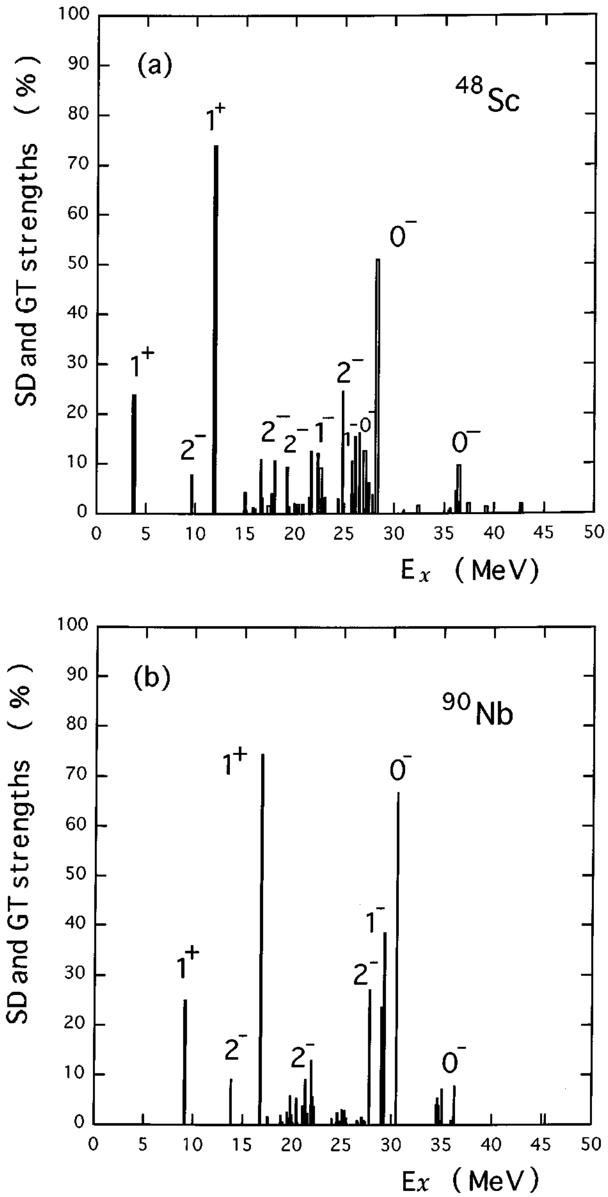

FIG. 1. SD and GT transition strengths from the parent ground state to SD and GT states, respectively, in (a) ^{48}Sc and (b) ^{90}Nb.

$$B(E1)(1^{+}\rightarrow J^{-})=\frac{1}{2J+1}\,|\langle J^{-}\|e\hat{D}\|1^{+}\rangle|^{2}$$

$$=\frac{1}{3}\,(2J+1)$$

$$\times\left|\sum_{ph}\sum_{p_s h_s}\{\delta_{hh_s}(-1)^{p_s-p+J+1}\right.$$

$$\times W(Jp_s1p;h1)\langle p_s\|e\hat{D}\|p\rangle$$

$$+\delta_{pp_s}(-1)^{h_s-h}W(Jh_s1h;p1)$$

$$\left.\times\langle h\|e\hat{D}\|h_s\rangle\}X_S^{p_sh_s}X_G^{ph}\right|^{2}. \qquad (13)$$

The reduced matrix element of the operator $e\hat{D}$ is defined by

TABLE I. SD states and transition strength in ^{48}Sc as well as $E1$ transition strength. The values for SD transitions are given by percentages of SD transition strength for each state referred to the sum of the corresponding J^- states. The values for $E1$ transitions from the GT state at $E_x=11.90$ MeV are percentages of the transitions for each SD state referred to the total sum of all multipoles, $S_{1p\text{-}1h}$ given by Eq. (10).

J^-	E_x (MeV)	SD (%)	$E1$ (%)
0^-	17.41	1.50	0.006
	20.33	1.84	12.27
	22.58	9.23	0.23
	25.88	3.81	0.046
	27.04	12.66	0.115
	28.25	50.96	0.26
	32.43	1.52	0.006
	36.48	9.67	0.044
	37.46	1.95	0.004
	39.23	1.36	0.0007
	42.69	1.91	0.011
	51.54	1.93	0.051
1^-	16.64	2.64	0.0002
	17.70	3.94	0.011
	19.47	1.34	0.80
	20.26	1.25	17.12
	21.51	3.11	0.60
	22.32	12.06	0.47
	23.04	3.16	0.87
	24.37	2.84	0.04
	25.74	10.55	0.11
	26.13	15.40	0.10
	26.50	16.24	0.02
	27.35	5.12	0.003
	27.44	6.12	0.000
	27.83	3.79	0.05
	35.64	1.00	0.10
	36.19	4.51	0.10
2^-	9.62	7.79	0.009
	15.03	4.22	0.22
	16.61	10.85	0.03
	16.78	3.08	3.04
	17.72	3.09	8.53
	18.02	10.66	0.32
	19.25	9.26	0.0002
	20.15	0.04	6.19
	20.86	1.77	0.025
	21.67	12.50	1.11
	22.71	2.97	0.58
	24.82	24.58	0.87
	36.58	2.29	0.027

$$\langle p_s\|e\hat{D}\|p\rangle=(-1)^{p_s+l_p-3/2}\hat{p}_s\hat{p}\,W\left(p_sl_{p_s}pl_p;\frac{1}{2}1\right)$$

$$\times\sqrt{\frac{3}{4\pi}}\,\hat{l}_p(l_p010|l_{p_s}0)\langle p_s|r|p\rangle e_{E1}, \quad (14)$$

where the effective charge e_{E1} takes the value $(N/A)e$ for

TABLE II. SD states and transition strength in ^{90}Nb as well as $E1$ transition strength. The values for SD transitions are given by percentages of SD transition strength for each state referred to the sum of the corresponding J^- states. The values for $E1$ transitions from the GT state at $E_x = 16.82$ MeV are percentages of the transitions for each SD state referred to the total sum of all multipoles, S_{TDA} given by Eq. (10).

J^-	E_x (MeV)	SD (%)	$E1$ (%)
0^-	21.86	3.81	0.95
	24.46	0.48	2.12
	25.31	1.30	3.61
	25.39	0.51	1.50
	27.29	0.59	0.22
	30.44	66.60	12.09
	34.72	3.85	0.39
	35.01	7.15	0.73
	36.28	7.78	0.68
	45.40	1.99	0.10
1^-	20.45	5.26	1.18
	21.52	1.72	0.30
	22.02	5.61	1.52
	24.01	1.02	1.01
	24.36	0.12	4.51
	24.58	2.45	0.57
	25.17	0.05	4.32
	25.27	2.75	5.61
	25.47	1.38	0.18
	28.97	23.64	6.08
	29.21	38.50	11.10
	34.60	5.34	0.83
	36.21	1.58	0.13
2^-	13.83	9.13	0.03
	19.53	2.56	0.06
	19.82	5.78	0.24
	21.10	3.79	0.0006
	21.35	9.10	0.47
	21.59	2.24	0.78
	21.90	12.94	1.35
	22.17	3.65	5.37
	24.37	0.001	3.05
	25.02	2.92	5.28
	27.77	27.07	3.96
	34.52	4.03	0.25

protons and $(-Z/A)e$ for neutrons. Use of the effective charge e_{E1} instead of the bare isovector charge $\pm\frac{1}{2}e$ leads to only a slight difference.

In Tables I and II are shown, for the various J^- SD states, the calculated values of $B(E1)(1^+\rightarrow J^-)$ expressed in percentage of $S_{1p\text{-}1h}$ and their corresponding SD transition strengths from the parent ground state. Tables I and II are for ^{48}Sc and ^{90}Nb, respectively. The sums of $E1$ transition strengths from the GT state to 0^-, 1^-, and 2^- states are 13.1%, 20.8%, and 21.5%, respectively; i.e., their total amounts to 55.4% of $S_{1p\text{-}1h}$ in ^{48}Sc. Note that the GT state ($E_x = 11.90$ MeV) exhausts 73.9% of the total GT sum rule strength. Another part of $E1$ transition strength comes from the 1^+ state at $E_x = 3.749$ MeV which exhausts 23.8% of the total GT strength. The sum of the $E1$ transition strength from the 1^+ state at $E_x = 3.749$ MeV amounts to 26.6% of $S_{1p\text{-}1h}$ for ^{48}Sc. The sum of percentages of the $E1$ transition strengths from these two 1^+ states does not necessarily become 100% as the sum is an incoherent one. Strong $E1$ transitions in the daughter nucleus do not necessarily occur to the SD states which are strongly excited from the parent. The $E1$ transitions between the GT and SD states are quite selective compared to the distributions of the SD states.

In the case of ^{90}Nb, the sums of strengths to the 0^-, 1^-, and 2^- states are 23.3%, 40.2%, and 26.1%, respectively, and the net sum of the transitions from the GT state amounts to 89.6% of $S_{1p\text{-}1h}$. The GT state ($E_x = 16.82$ MeV) exhausts 72.5% of the total GT strength. The sum of the $E1$ transition strength from the 1^+ state at $E_x = 9.16$ MeV, which exhausts 25.0% of the total GT sum rule, amounts to 81.5% of $S_{1p\text{-}1h}$ for ^{90}Nb. Strong $E1$ transitions occur here, in contrast to the case of ^{48}Sc, mainly to the SD states, which are rather strongly excited from the parent except for 2^- states.

It was pointed out by Brink [8] that giant resonances can be possibly built on top of not only the ground state, but also of every excited state. One well-known example is the giant dipole state at finite temperatures in deformed nuclei [10]. One can consider that the SD states are also giant dipole states on top of the excited GT states. The systematic energy of giant dipole resonances is given by $E_x = 78/A^{1/3}$ MeV, which is 21.5 MeV for ^{48}Sc and 17.4 MeV for ^{90}Nb. Brink's hypothesis seems to be valid in the case of ^{90}Nb as far as the excited energy is concerned since the major $E1$ strength in Fig. 2(b) appears at around $E_x = 30$ MeV, which is about 13 MeV above the GT state. On the other hand, the main dipole transitions in ^{48}Sc occur at $E_x = 20$ MeV, i.e., $E_{\mathrm{SD}} - E_{\mathrm{GT}} \simeq 8$ MeV, which is about one-half smaller than the energy expected from Brink's hypothesis and the systematic energy of giant dipole states. However, in Brink's picture the SD states would be $2p$-$2h$ states with respect to the parent ground state, whereas in the present TDA model they are just $1p$-$1h$ states. It would be interesting to study whether the Brink's hypothesis is recovered or not for ^{48}Sc when the model space is extended to include the $2p$-$2h$ states. The study with the extension of the space is left to future investigation.

We note that most of the SD states are energetically higher than the two main components of GT excitations, and therefore one must expect inverse transitions from SD to GT states. The strongest calculated transitions from GT to specific 0^-, 1^-, and 2^- SD states range from 0.04 to 0.10 e^2 fm^2 in ^{48}Sc. Experimentally, inverse transitions from SD to GT states are observed. The transition strengths range from 0.02 to 0.21 e^2 fm^2, i.e., 2–25 % of the Weisskopf unit,

$$B_W(E1) = \frac{(1.2)^2}{4\pi}\left(\frac{3}{4}\right)^2 A^{2/3}\ e^2\ \mathrm{fm}^2 = 0.851\ e^2\ \mathrm{fm}^2.$$

For ^{90}Nb, the strongest calculated $E1$ transitions from specific SD states to the GT state range from 0.02 to 0.22 e^2 fm^2, i.e., 1.5–17 % of the Weisskopf unit (1.295 e^2 fm^2) for this nucleus. Since this order of magnitude of the transition strength could be easily accessed experimentally, it would be quite interesting to observe SD

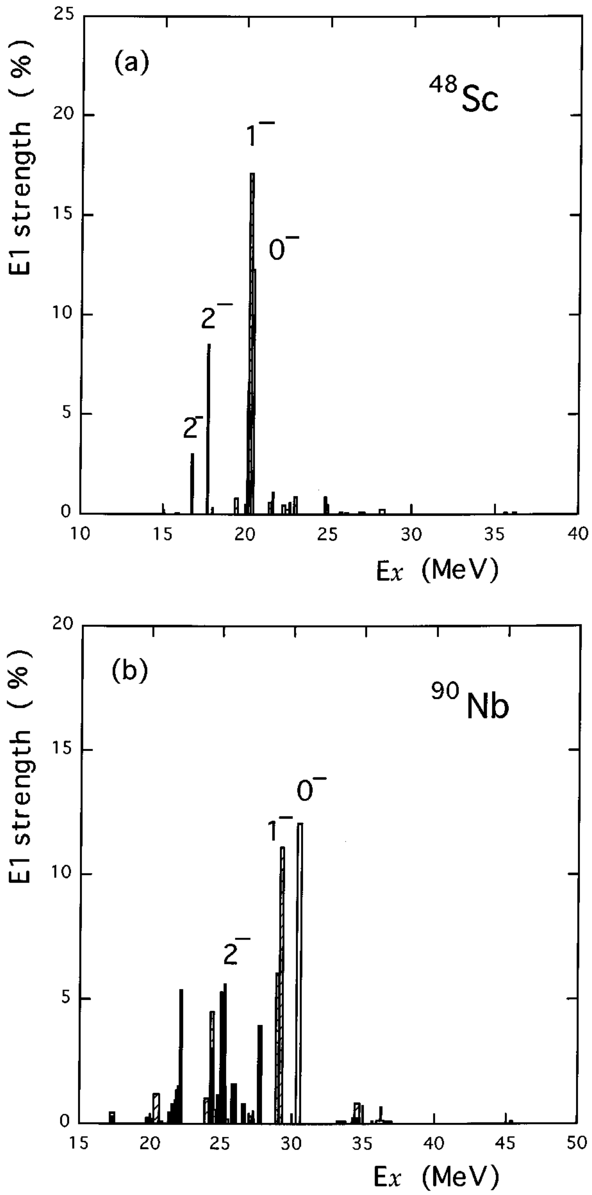

FIG. 2. Electric dipole transition strengths from the main GT state to SD states in (a) ^{48}Sc and (b) ^{90}Nb.

states and measure $E1$ transitions from the SD to GT states. One can excite SD states, for example, by (p,n) or $(^3\mathrm{He},t)$ reactions and measure γ rays emitted when the $E1$ transitions to the GT state occur.

We finally comment on a sum rule obtained without restriction on the configuration space. We define the sum of all possible $E1$ transitions from the GT state:

$$S_A=\langle 0|\hat{G}^{\dagger}\hat{D}^{\dagger}\hat{D}\hat{G}|0\rangle/\langle 0|\hat{G}^{\dagger}\hat{G}|0\rangle, \tag{15}$$

where $\langle 0|\hat{G}^{\dagger}\hat{G}|0\rangle=3(N-Z)$. Since the GT state is constructed on the parent, which is taken to be a closed core, the $E1$ transitions from the GT state lead not only to the SD states with $1p$-$1h$ configurations, but also to $2p$-$2h$ configurations with $J^{\pi}=0^-$, 1^-, and 2^-. Under the same assumptions (5) and (6) as before, $\hat{G}^{\dagger}|0\rangle=\langle 0|\hat{G}=0$, we can derive

$$S_A=\left(1-\frac{4}{N-Z}\right)\langle 0|\hat{D}^{\dagger}\hat{D}|0\rangle+\frac{1}{3(N-Z)}\langle 0|\hat{S}^{\dagger}\hat{S}+\hat{S}\hat{S}^{\dagger}|0\rangle. \tag{16}$$

Making use of the equality

$$\langle 0|\hat{S}^{\dagger}\hat{S}+\hat{S}\hat{S}^{\dagger}|0\rangle=12\langle 0|\hat{D}^{\dagger}\hat{D}|0\rangle, \tag{17}$$

we obtain

$$S_A=\langle 0|\hat{D}^{\dagger}\hat{D}|0\rangle. \tag{18}$$

It is interesting to notice that the sum of the $E1$ transition strength from the GT state is equal to that from the ground state in the parent nucleus. This sum rule suggests that the SD states are typical examples of giant resonances built on top of an excited state suggested by Brink [8]. The total sum rule S_A is much larger than $S_{1p\text{-}1h}$ in Eq. (10). The values of S_A are evaluated to be 15.90 and 35.14 fm^2 in ^{48}Sc and ^{90}Nb, respectively, while $S_{1p\text{-}1h}$ is only 0.577 and 0.606 fm^2 in ^{48}Sc and ^{90}Nb, respectively. Thus the $S_{1p\text{-}1h}$ exhausts only a few percent of the total sum S_A.

In general, the charge-exchange reactions excite dominantly the $1p$-$1h$ states as the direct process. Thus the observed states with $J^{\pi}=0^-$, 1^-, and 2^- will provide information about the excitation energies of $1p$-$1h$ SD states. These SD states will couple within their survival time to more complicated many-p–many-h configurations with $J^{\pi}=0^-$, 1^-, and 2^-, which make larger energy spreadings than those of the calculated $1p$-$1h$ SD states. Moreover, the summed $E1$ strength between SD and GT states might be significantly enhanced compared with the $S_{1p\text{-}1h}$ value as is expected from the large difference between $S_{1p\text{-}1h}$ and S_A. Thus these experimental data will be quite useful to obtain quantitative information about the many-p–many-h states with $J^{\pi}=0^-$, 1^-, and 2^- at the corresponding energies of $1p$-$1h$ SD states.

The charge-exchange reactions of light projectiles, like (p,n) or $(^3\mathrm{He},t)$ reactions, have been used experimentally to excite both the GT and SD states. It is known that the transition form factor to the GT state is the volume type, while that to SD states might be dominated by the surface region. There might be some advantage in using the charge-exchange reactions with heavier projectiles like $(^{12}\mathrm{C}, ^{12}\mathrm{N})$ for the study of SD states since the reactions occur mainly at the nuclear surface.

In summary, we have studied the $E1$ transitions between GT and SD states in ^{48}Sc and ^{90}Nb by using both HF+TDA calculations and the analytic sum rule approach. It is shown that the $E1$ transitions are rather selective between those states, having the order of the Weisskopf unit for several SD states. These results are obtained in a $1p$-$1h$ space, and our evaluation of sum rules in a more extended $2p$-$2h$ space indicates that dipole transitions could be actually somewhat stronger. Thus, experimental observation of such transitions should be feasible. Brink's hypothesis remains valid in ^{90}Nb as far as the transition energies of dipole decays are concerned. On the other hand, the transition energies are about twice lower than that of Brink's hypothesis in ^{48}Sc within the $1p$-$1h$ model space calculations. We have evaluated two different analytic $E1$ sum rules between GT and SD states: the one $S_{1p\text{-}1h}$ is within $1p\text{-}1h$ configuration space and the other one S_A includes both the $1p\text{-}1h$ and $2p\text{-}2h$ states with $J^\pi = 0^-$, 1^-, and 2^-. It is found that the $E1$ sum rule $S_{1p\text{-}1h}$ is much smaller than the total sum S_A. Studies of transitions from such $2p\text{-}2h$ configurations will help to understand quantitatively the important problem of the coupling of spin excited states with these configurations.

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