

## Systematics of low-lying electric dipole excitations in the $A \approx 130\text{--}200$ mass region

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The data from numerous high resolution photon scattering experiments allow an extensive survey of the lowest electric dipole excitations in the  $A \approx 130\text{--}200$  mass region. In this mass region one can find spherical as well as transitional and strongly quadrupole deformed nuclei. The measured absolute  $E1$  strengths are typically of the order of several milli Weisskopf units and exhibit in general a smooth variation with mass number. [S0556-2813(98)00601-3]

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### I. INTRODUCTION

Throughout the last decade numerous experiments using electromagnetic probes have provided a vast amount of data on low-lying electric dipole excitations in heavy nuclei [1–3]. In general the dipole excitations in these nuclei can be divided into two structurally different groups: in spherical nuclei near closed shells the lowest  $1^-$  state results from a coupling of dynamic quadrupole ( $2^+$ ) and octupole ( $3^-$ ) vibrations of the nuclear surface [4,5]. This “two phonon state” exhibits a relatively strong  $E1$  transition to the ground state [6]. Recently, the expected strong collective  $E2$  decay to the  $3_1^-$  state (destroying the quadrupole part of the two phonons) has been observed in some cases. These experiments represent unambiguous proofs for the two phonon structure of the lowest electric dipole excitation in spherical nuclei around the neutron shell closure at  $N=82$  [7–9].

In deformed nuclei the octupole vibration couples to the static quadrupole deformation giving rise to four octupole vibrational bands in even-even nuclei [10]. These rotational bands are characterized by their  $K$ -quantum number. The bands with  $K=0$  and  $K=1$  have a  $J^\pi=1^-$  band head accessible in photon scattering experiments. The present paper will extend considerably the energy and strength systematics presented previously for well deformed nuclei in Ref. [2] by including new data on spherical, transitional, and well deformed nuclei.

### II. EXPERIMENTAL RESULTS

The resonant scattering of real photons referred to as the nuclear resonance fluorescence (NRF) technique is a powerful experimental tool for a systematic investigation of dipole strength distributions in stable nuclei [3]. The use of a continuous energy bremsstrahlung photon source to excite the dipole transitions from the ground state in conjunction with high resolution Ge detectors for the measurement of the  $\gamma$  rays from the decay of these states yields the following information about the excited state for even-even nuclei: the excitation energy, the decay pattern (which allows the determination of the  $K$ -quantum number in deformed nuclei), the lifetime and absolute transition strength, the spin of the states

from the angular correlation, and the parity if one uses a polarimeter to detect the scattered photons.

Due to the strength selectivity of the method one measurement gives a *complete* survey of the covered energy range which is typically about 1.2 to 4.1 MeV in the present experiments. This means that all dipole excitations are observed which have an excitation strength above the detection

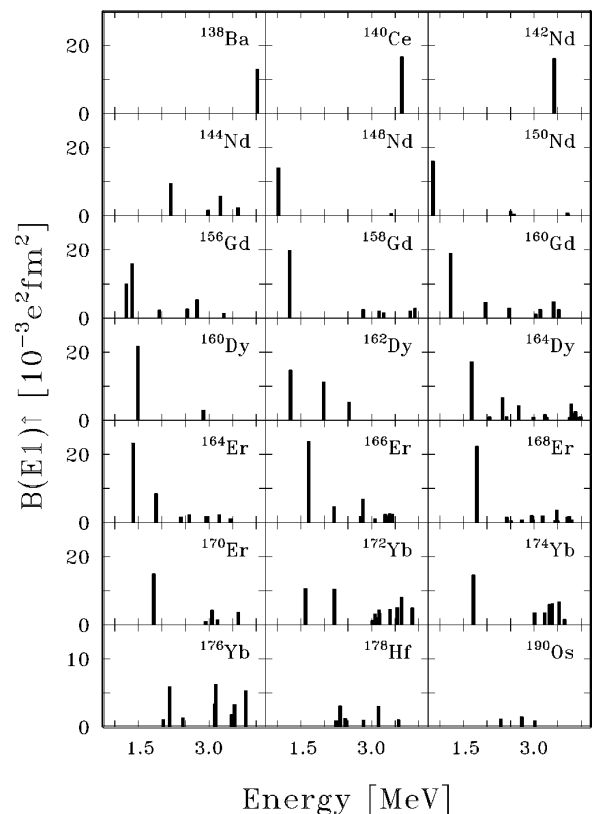


FIG. 1. Distribution of electric dipole strength below about 4 MeV for nuclei ranging from  $^{138}\text{Ba}$  to  $^{190}\text{Os}$  measured in photon scattering experiments. All states have been included for which either the negative parity has been established or for which a  $K$  quantum number  $K=0$  was assigned. Since the  $E1$  strength decreases in the transition for deformed to  $\gamma$ -soft nuclei a different scale was chosen in the plots for  $^{176}\text{Yb}$ ,  $^{178}\text{Hf}$ , and  $^{190}\text{Os}$ .

sensitivity of the actual experiment. A typical sensitivity around 2 MeV in the NRF measurement on  $^{190}\text{Os}$  was, e.g.,  $0.5 \times 10^{-3} e^2 \text{ fm}^2$  [11]. For a more detailed description of the method see [3].

In this paper we discuss the observed  $J=1$  states in even-even nuclei for which either the parity is known to be negative or to which a  $K$  quantum number  $K=0$  has been assigned. For the deformed rare earth nuclei it has been shown empirically that below 4 MeV all strong dipole excitations with  $K=0$  have negative parity [12]. Figure 1 shows the distribution of electric dipole strengths below about 4 MeV in several nuclei ranging from  $^{138}\text{Ba}$  to  $^{192}\text{Os}$ . One can observe a characteristic pattern for different groups of nuclei: The spherical nuclei at the neutron shell closure  $N=82$  exhibit only one strong  $E1$  excitation below 4.1 MeV. In the well deformed nuclei between mass  $A=150$  and  $A=174$  one observes one or two strong excitations around 1.5 MeV and a number of weaker excitations at higher energies. For very heavy nuclei beyond the rare earth ( $Z \geq 72$ ) the distinct, strongly populated low-lying  $1^-$  state vanishes and one observes a number of weaker excitations only. The scale of the  $y$  axis has been enlarged therefore for the three nuclei in the lowest row.

In the following we will focus on the lowest observed electric dipole excitation in the nuclei considered. The measured information about these levels is summarized in Table I. We note that the selection of the lowest state in the nuclei which do not exhibit a distinct pattern is somehow arbitrary. However, most conclusions can be drawn independently of this selection as can be seen from Fig. 1.

Figure 2 shows the energy of the lowest strongly excited  $1^-$  state (or the lowest two  $1^-$  states if they are close in energy and both strongly populated) in nuclei ranging from  $^{134}\text{Ba}$  to  $^{192}\text{Os}$ . This systematics covers nuclei with very different structure: The Ba isotopic chain represents a transition from  $\gamma$ -soft nuclei such  $^{134}\text{Ba}$  [27] with a fluctuating triaxiality to the semimagic, spherical closed shell nucleus  $^{138}\text{Ba}$ . The nuclei near neutron shell closure  $N=82$  are spherical, vibrator nuclei. The stable rare earth nuclei from  $^{150}\text{Nd}$  to  $^{176}\text{Yb}$  are strongly quadrupole deformed and can be described as good rotors. The Hf to Os isotopes represent transitional nuclei from pure rotors to  $\gamma$ -soft nuclei.

The energy of the lowest  $1^-$  state in nuclei near the  $N=82$  shell closure follows closely the sum energy of the  $2_1^+$  and  $3_1^-$  state, which points to the  $2^+ \otimes 3^-$  two phonon quadrupole-octupole structure of this dipole excitation. The lowering of the  $1^-$  energy in the transition from  $N=82$  ( $A \approx 140$ ) to  $N=90$  ( $A \approx 150$ ) nuclei is mainly due to the lowering of the  $2_1^+$  state. In the heavier nuclei where the first strongly populated  $1^-$  level is usually the bandhead of the  $K=0$  octupole vibrational band, the energy rises very smoothly with increasing mass number. This trend for the energy of the  $K=0$  octupole bands has been discussed recently more quantitatively in Ref. [28].

In Fig. 3 we plot the measured  $B(E1)$  values from the ground state to the  $1^-$  states versus the mass number  $A$  [29]. In the mass region  $A=130$  to  $A=200$  one gets a double hump structure with a sharp maximum around  $A=140$  and a smooth plateau around  $A=160$ . The largest  $B(E1)$  values

TABLE I. Excitation energy  $E_x$ , branching ratio  $R = B(E1; 1^- \rightarrow 2_1^+) / B(E1; 1^- \rightarrow 0_1^+)$ , and  $B(E1)\uparrow$  value of the first  $1^-$  state (or the first two  $1^-$  states if they are close in energy and both strongly populated) observed in photon scattering experiments on nuclei in the mass region  $A \approx 130$ –200.

Nucleus	$E_x$ [MeV]	$R$	$B(E1)\uparrow$ [ $10^{-3} e^2 \text{ fm}^2$ ]	Ref.
$^{134}\text{Ba}$	2824	$0.36 \pm 0.21$	$2.30 \pm 0.30$	[13]
$^{136}\text{Ba}$	3436	$\leq 0.06$	$5.01 \pm 0.92$	[14]
$^{138}\text{Ba}$	4026	$\leq 0.19$	$13.1 \pm 2.8$	[15]
$^{140}\text{Ce}$	3643	$\leq 0.23$	$16.65 \pm 0.75$	[6]
$^{142}\text{Ce}$	2187	$2.29 \pm 0.13^a$	$11.7 \pm 3.6$	[15]
$^{142}\text{Nd}$	3425	$0.17 \pm 0.03^b$	$16.3 \pm 2.4$	[16]
$^{144}\text{Nd}$	2185	$1.27 \pm 0.09$	$9.51 \pm 0.64$	[17]
$^{146}\text{Nd}$	1377	$2.29 \pm 0.56$	$5.0 \pm 1.5$	[16]
$^{148}\text{Nd}$	1023	$2.31 \pm 0.86$	$14.2 \pm 6.4$	[16]
$^{150}\text{Nd}$	853	$1.99 \pm 0.30$	$16.1 \pm 6.3$	[16]
$^{144}\text{Sm}$	3226	$0.17 \pm 0.03$	$19.5 \pm 2.6$	[8,18]
$^{148}\text{Sm}$	1465	$2.1 \pm 1.0^a$	$2.71 \pm 0.54$	[18]
$^{150}\text{Sm}$	1166	$2.14 \pm 0.26$	$9.8 \pm 1.1$	[18]
$^{152}\text{Sm}$	963	$1.94 \pm 0.23$	$23.1 \pm 2.2$	[18]
$^{154}\text{Sm}$	921	$1.83 \pm 0.34$	$26.7 \pm 3.9$	[18]
$^{156}\text{Gd}$	1243	$1.51 \pm 0.39$	$10.0 \pm 4.0$	[19]
	1367	$2.48 \pm 0.35$	$16.0 \pm 5.9$	[19]
$^{158}\text{Gd}$	1264	$1.86 \pm 0.22$	$19.9 \pm 4.7$	[19]
$^{160}\text{Gd}$	1224	$1.92 \pm 0.25$	$19.1 \pm 5.3$	[19]
	1966	$2.20 \pm 0.32$	$4.6 \pm 1.1$	[19]
$^{160}\text{Dy}$	1489	$1.79 \pm 0.44$	$21.7 \pm 2.3$	[20]
$^{162}\text{Dy}$	1276	$1.36 \pm 0.33$	$14.7 \pm 2.5$	[2]
	1983	$1.94 \pm 0.25$	$11.2 \pm 2.4$	[2]
$^{164}\text{Dy}$	1675	$1.89 \pm 0.24$	$17.2 \pm 2.1$	[21]
$^{164}\text{Er}$	1387	$2.17 \pm 0.26$	$23.3 \pm 3.8$	[22]
$^{166}\text{Er}$	1663	$1.74 \pm 0.07$	$23.7 \pm 2.9$	[22]
$^{168}\text{Er}$	1786	$1.91 \pm 0.05$	$22.4 \pm 2.5$	[22]
$^{170}\text{Er}$	1825	$1.87 \pm 0.09$	$15.0 \pm 2.6$	[22]
$^{172}\text{Yb}$	1599	$1.90 \pm 0.38$	$10.7 \pm 3.2$	[23]
	2210	$1.71 \pm 0.16$	$10.5 \pm 2.0$	[23]
$^{174}\text{Yb}$	1711	$1.55 \pm 0.21$	$14.6 \pm 3.4$	[23]
$^{176}\text{Yb}$	2027	$1.00 \pm 0.14$	$1.05 \pm 0.15$	[11]
	2163	$1.11 \pm 0.05$	$5.87 \pm 0.43$	[11]
$^{178}\text{Hf}$	2248	$1.52 \pm 0.63$	$0.92 \pm 0.38$	[24]
	2334	$1.30 \pm 0.17$	$3.10 \pm 0.38$	[24]
$^{180}\text{Hf}$	2582	$1.84 \pm 0.25$	$2.78 \pm 0.39$	[24]
	2712	$1.99 \pm 0.39$	$1.89 \pm 0.37$	[24]
$^{184}\text{W}$	2056	$1.55 \pm 0.25$	$2.70 \pm 0.30$	[25]
$^{190}\text{Os}$	2297	$1.26 \pm 0.53$	$1.14 \pm 0.11$	[11]
$^{192}\text{Os}$	2478	$1.74 \pm 0.27$	$1.01 \pm 0.08$	[26]

<sup>a</sup>From Ref. [20].

<sup>b</sup>From Ref. [8].

amount to about  $20 \times 10^{-3} e^2 \text{ fm}^2$  corresponding to about 4 milli Weisskopf units (mWu).

Between the maximas a dip is observed around the two isotopes  $^{146}\text{Nd}$  and  $^{148}\text{Sm}$  which both have 86 neutrons. This dip was first observed for the Sm isotopes by Metzger [14,18], see also [17]. Figure 4 points out the neutron dependence of the  $B(E1)$  values for the nuclei with neutron num-

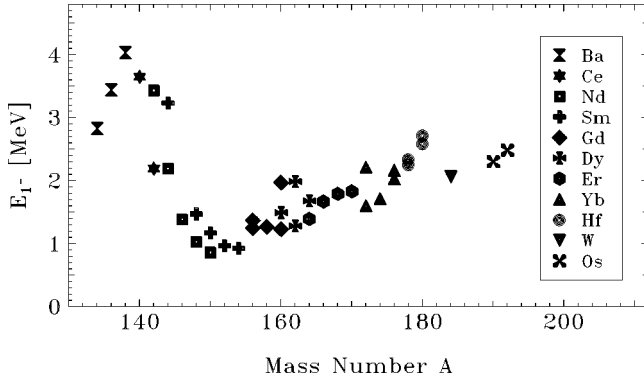


FIG. 2. Energy systematics of the lowest observed  $1^-$  states in nuclei ranging from  $^{134}\text{Ba}$  to  $^{192}\text{Os}$  listed in Table I.

bers  $N=78-94$ . A similar strength reduction is observed in the transition to  $\gamma$ -soft nuclei at the lower and upper end of the examined range of nuclei. In particular for  $^{134}\text{Ba}$  which has four neutron holes with respect to the  $N=82$  shell closure a similar small  $E1$  strength was found as it was observed for the  $N=86$  nucleus  $^{148}\text{Sm}$ .

### III. DISCUSSION

In first order low-lying collective electric dipole transitions in nuclei are forbidden because a nucleus possesses no static electric dipole moment due to parity conservation. Furthermore the  $E1$  giant dipole resonance absorbs nearly all  $E1$  strength. However, if one overlaps in a geometrical picture an octupole vibration with either a static or dynamic quadrupole deformation of the nuclear shape, the nucleus looks similar to a pear and the reflection symmetry in the intrinsic frame of reference is dynamically destroyed. Due to the Coulomb potential the protons tend to concentrate in the tip of the pear and create a dynamic electric dipole moment [30–33]. Because of the proton-neutron interaction and the underlying shell structure other effects may appear, e.g., the formation of a neutron skin, and have to be taken into account as well. This may cancel the  $E1$  moment partly. A quantitative calculation of the resulting electric dipole strength is therefore very difficult in a simple geometrical model.

In deformed nuclei the coupling of the octupole vibration to the quadrupole deformed core leads to rotational bands which can be characterized by their projection of angular momentum on the symmetry axis, the  $K$ -quantum number. In the strongly quadrupole deformed nuclei between  $A=156$  and  $A=172$  the excitation to the  $J^\pi=1^-$  bandhead of the  $K=0$  band carries nearly all the  $E1$  strength. In some cases where the states lie close in energy a mixing of the  $1_{K=0}^-$  and  $1_{K=1}^-$  states occurs and the  $E1$  transition width is shared by both states [23]. It has been shown that the  $K$ -mixing matrix element can be calculated from the observables of the photon scattering experiments alone [34].<sup>1</sup> The decay pattern and absolute transition strength can be reproduced very well in

<sup>1</sup>The usual technique to derive the mixing matrix element is to compare the energies of disturbed and undisturbed levels in the  $K=0$  and  $K=1$  bands.

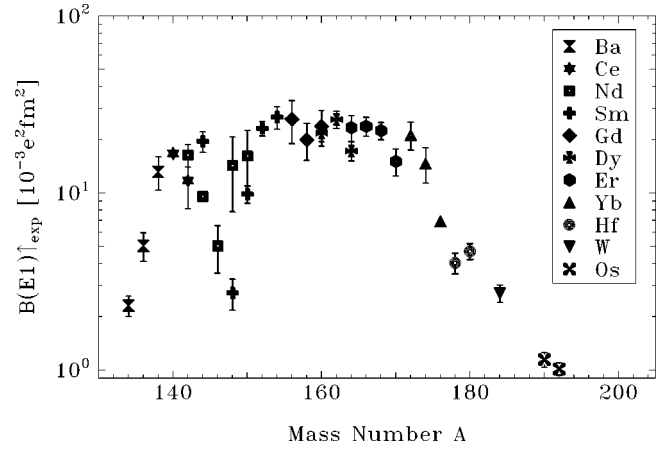


FIG. 3. Systematics of the electric dipole strengths of the lowest (or lowest two)  $1^-$  states in the investigated nuclei listed in Table I.

the framework of the algebraic interacting boson model [35,36]. Microscopic calculations by Soloviev *et al.* with the quasiparticle phonon nuclear model show in general a good agreement with the experimental data but the results are still far away from a one to one correspondence [37]. Especially a more quantitative picture of the mass dependence for the ratio of the  $K=0$  and  $K=1$  electric dipole matrix elements,

$$Z = \frac{\langle E1, \Delta K=1 \rangle}{\langle E1, \Delta K=0 \rangle} \quad (1)$$

is currently investigated [38,39].

In spherical nuclei the simplest models just assume a coupling of harmonic quadrupole and octupole vibrations as a starting point. The  $E1$  strength to the  $1^-$  state is then in first order proportional to the product of the  $B(E2, 2_1^+ \rightarrow 0_1^+)$  and  $B(E3, 3_1^- \rightarrow 0_1^+)$  values. From this point of view one would expect an increase in strengths when going from the spherical nuclei around  $A=140$  to the deformed nuclei around  $A=150$  because the  $B(E3)$  values remain approximately the same whereas the  $B(E2)$  values increase strongly in the tran-

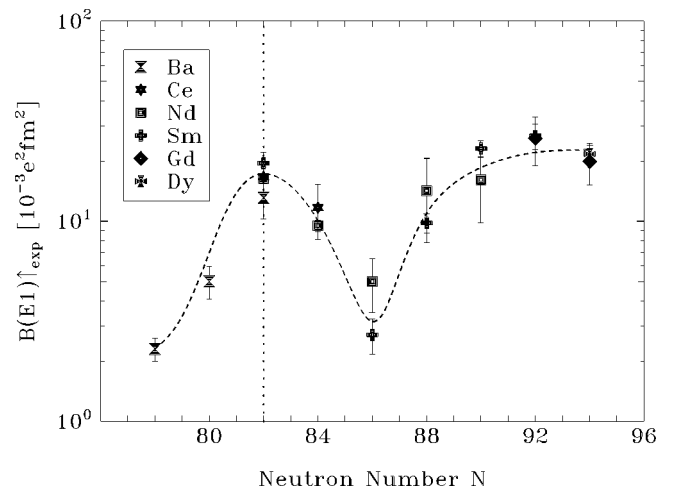


FIG. 4. Systematics of the electric dipole strengths of the lowest  $1^-$  states in nuclei with neutron numbers  $N=78-94$ . The dashed line is drawn to guide the eye. The shell closure at  $N=82$  is marked with a dotted line.

sition to deformed nuclei. As can be seen from Figs. 3 and 4 this is not the case. Actually the minimal  $E1$  strength is observed between these two regions at  $N=86$ .

The observed structure has been the subject of several theoretical works. The dip at  $N=86$  was discussed in the framework of the quasiparticle phonon model and could be reproduced qualitatively [40]. The  $E1$  strength in nuclei near the  $N=82$  shell closure has been discussed in various methods, e.g., in [41]. A recent estimate by Heyde and De Coster showed that the mechanism of including  $1p$ - $1h$  admixtures, at the tail of the giant dipole resonance (GDR), into the otherwise collective two-phonon states may be at the origin of the observed strong  $E1$  rates in spherical nuclei [42]. It is the intention of this paper to present the data in a consistent way to allow further theoretical investigations which are urgently needed in order to obtain a more detailed understanding of the low-lying strong electric dipole excitations.

#### IV. SUMMARY

To summarize, we have presented the systematics of low-lying electric dipole transitions in the mass region  $A \approx 130$ – $200$ . We have found nearly constant values for the

low-lying  $E1$  excitation strength of about  $20 \times 10^{-3} e^2 \text{ fm}^2$  for nuclei at the  $N=82$  shell closure and for well deformed nuclei in the middle of the  $N=82$ – $126$  neutron major shell. In the transitional regions to  $\gamma$ -soft nuclei we have observed a decrease in the  $E1$  strength of about one order of magnitude. An additional dip in the  $E1$  strength appears around  $N=86$ .

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