Microscopic quasiparticle-phonon description of odd-mass $^{127-133}$Xe isotopes and their β decay

J. Toivanen* and J. Suhonen†
Department of Physics, University of Jyväskylä, POB 35, SF-40351 Jyväskylä, Finland
(Received 18 August 1997)

Quasiparticle-phonon equations of motion are solved starting from a microscopic realistic many-body Hamiltonian. In this microscopic quasiparticle-phonon model (MQPM) the relevant part of the three-quasiparticle Hilbert space may possibly be taken into account even in calculations using large single-particle bases. As an example, the MQPM is applied to the calculation of energy levels and Fermi and Gamow-Teller beta-decay transition amplitudes for transitions between odd-mass $^{127-133}$Xe, $^{127-133}$I, and $^{127-133}$Cs isotopes. Considering the fully microscopic nature of the MQPM, comparison of its results and data indicates a rather satisfactory agreement between theory and experiment.
[S0556-2813(98)05403-X]

PACS number(s): 21.60.Jz, 23.40.Hc, 27.60.+j

I. INTRODUCTION

Beta-decay and excitation spectra of odd-mass (odd-A for short) nuclei have traditionally been described using phenomenological or semiphenomenological models, such as quasiparticle-phonon models or particle-vibration models, using simple pairing-plus-quadrupole interactions [1,2]. In many cases these models are in good qualitative agreement with experiments, when their parameters, such as the pairing strengths, quadrupole-force parameters, and parameters of the quasiparticle-phonon coupling Hamiltonian, are properly fitted. They are also simple to solve because of the simplicity of the interactions, and they allow simple interpretation of the states of the odd-A systems in terms of low-lying one-phonon and one-quasiparticle states of neighboring even-even (reference) nuclei. For this reason they have widely been used to describe selected observed features of odd-A nuclei. On the other hand, more microscopic particle-phonon or quasiparticle-phonon models have been developed in the past, but they have not been applied in a systematic way to the beta decay of odd-A nuclei.

The other extreme is presented by the shell-model type of calculations for odd-A nuclei, such as the so-called cluster-phonon models [3,4], where all three-quasiparticle components are explicitly constructed to diagonalize the Hamiltonian of the odd-A system. They give quite accurate results in principle, but the resulting states are difficult to interpret in terms of elementary excitations of the neighboring even-even nuclei and, in addition, they require huge computational effort as compared to the two-step diagonalization method of the quasiparticle-phonon models. Therefore, unified approaches that are microscopic but still simple to use and to interpret are called for.

In our approach, the microscopic quasiparticle-phonon model (MQPM), one strives for self-consistent treatment of all for three parts of the Hamiltonian, namely the quasiparticle, phonon, and quasiparticle-phonon terms. This is possible by starting from a microscopic Hamiltonian with two-body matrix elements derived from a G matrix. The G-matrix method enables a systematic way of deriving both the proton-neutron and the like-nucleon two-body interactions. The H_{22}, H_{04}, and H_{40} parts of the quasiparticle Hamiltonian are treated in the quasiparticle random-phase approximation (QRPA) framework. They lead to definition of the excitation (phonon) spectrum of the even-even reference nucleus. The whole residual interaction is then diagonalized in the quasiparticle-phonon basis coupled from the BCS quasiparticles and QRPA phonons.

The simplifying trick in the MQPM is to use the predetermined lowest two-quasiparticle excitations (phonons) of the nearby even-even reference nucleus as basic building blocks for the odd-A states, and use the properties of the pp-nnQRPA equations [5,6] for the even-even nucleus to simplify the emerging one- and three-quasiparticle equations of motion. The low-lying three-quasiparticle states constructed in this way are composed of various quasiparticles coupled to the few lowest and most important RPA excitations such that their coupling with the higher RPA excited states is weak and can be neglected. This truncation of the three-quasiparticle model space can be controlled by looking at the convergence of the odd-A spectrum as a function of the number of the included RPA excitations of different multipolarity.

As an example of the use and power of the MQPM we apply our model to the calculation of the energy levels of the xenon isotopes $^{127-133}$Xe, and to Fermi and Gamow-Teller beta decays between odd-A Xe, I, and Cs nuclei in the mass range $A=127-133$. This is a continuation of our previous work [7] discussing the same nuclei. In the first version of the MQPM [7] the coupling part of the microscopic Hamiltonian, H_{31}, does not emerge from the equations-of-motion method (EOM) [8]. The EOM method introduces an additional term into the quasiparticle-phonon matrix elements, not taken into account in [7]. In the present article we use the EOM form of the coupling part of the Hamiltonian. In addition to being microscopically more justified, this form of the coupling Hamiltonian yields results closer to the experimental data and thus improves the quantitative predictibility of the MQPM.

This article is organized as follows. In Sec. II we will describe the model and, in Sec. III we will present the results, and, finally, in Sec. IV we will summarize the results and draw the conclusions.

*Electronic address: toivanen@jyfl.jyu.fi
†Electronic address: suhonen@jyfl.jyu.fi

II. THEORETICAL FRAMEWORK

A. Energies and wave functions of the odd-mass nucleus

The microscopic quasiparticle-phonon model, MQPM, treats the structure of the odd-*A* nuclei in four steps. First, the neighboring even-even nucleus, or nuclei, can be used to study the properties of the chosen mass region and to fix the possible free parameters of the model Hamiltonian. In the present case we have used the Bonn-A G matrix [9] and a subsequent phenomenological renormalization of the two-body-interaction matrix elements was done. This Hamiltonian is used to generate the phonons which are excitations of the even-even nuclei. In the MQPM the monopole part of the same Hamiltonian is used to generate the quasiparticles, basic building blocks of the odd-*A* excitations, through the BCS procedure. Second, the phonons are derived by the use of the quasiparticle random-phase approximation (QRPA) procedure [5,6,10]. As the third step, the two basic excitations, QRPA phonons and BCS quasiparticles, are coupled to form a basis for a realistic treatment of the odd-*A* nucleus. As the last step, the residual Hamiltonian, containing the interaction of the odd nucleon with the even-even reference nucleus [the $H_{31}+H_{13}$ part of the Hamiltonian in Eq. (3) below] is diagonalized in this (overcomplete) basis.

As mentioned already above, we start our nuclear-structure calculation from a realistic *A*-fermion Hamiltonian containing a diagonal one-body part (the mean-field single-particle part) and a two-body residual interaction part containing antisymmetrized two-body matrix elements. In occupation-number representation it reads

$$H=\sum_{\alpha}\ \varepsilon_{\alpha}c_{\alpha}^{\dagger}c_{\alpha}+\frac{1}{4}\sum_{\alpha\beta\gamma\delta}\ \bar{v}_{\alpha\beta\gamma\delta}c_{\alpha}^{\dagger}c_{\beta}^{\dagger}c_{\delta}c_{\gamma}, \tag{1}$$

where we have used the convention that in the creation and annihilation operators greek indices denote all (harmonic-oscillator) single-particle quantum numbers $\alpha=\{a,m_a\}$, and roman indices, when used, denote all single-particle quantum numbers except the magnetic ones, i.e., $a=\{n_a,l_a,j_a\}$. The antisymmetrized two-body-interaction matrix element is defined as $\bar{v}_{\alpha\beta\gamma\delta}=\langle\alpha\beta|v|\gamma\delta\rangle-\langle\alpha\beta|v|\delta\gamma\rangle$.

The approximate ground state of the even-even reference nucleus is obtained from a BCS calculation, where quasiparticle energies and occupation factors u_a and v_a are obtained from the Bogoliubov-Valatin transformation to quasiparticles

$$a_{\mu}^{\dagger}=u_{\mu}c_{\mu}^{\dagger}-v_{\mu}\tilde{c}_{\mu},$$

$$\tilde{a}_{\mu}^{\dagger}=u_{\mu}\tilde{c}_{\mu}^{\dagger}+v_{\mu}c_{\mu}, \tag{2}$$

where $\tilde{a}_{\mu}^{\dagger}=a_{-\mu}^{\dagger}(-1)^{j+m}$ and $\tilde{c}_{\mu}^{\dagger}=c_{-\mu}^{\dagger}(-1)^{j+m}$. After this transformation the Hamiltonian can be written in its quasiparticle representation as

$$H=\sum_{\alpha}\ E_a a_{\alpha}^{\dagger}a_{\alpha}+H_{22}+H_{40}+H_{04}+H_{31}+H_{13}, \tag{3}$$

where E_a are the quasiparticle energies and other terms of the Hamiltonian are normal-ordered parts of the residual interaction labeled according to the number of quasiparticle creation and annihilation operators which they contain [6,10].

In our method of calculation we use in Eq. (1), as a starting point, the Coulomb-corrected Woods-Saxon single-particle energies ε_{α} with the parametrization of [11]. By performing the BCS calculation in this basis and comparing the resulting quasiparticle spectrum with the low-energy spectrum of the neighboring proton-odd and neutron-odd nuclei we obtain information about the validity of the Woods-Saxon parametrization of the mean field. If necessary, the proton or neutron single-particle energies can be adjusted in the vicinity of the proton and neutron fermi surfaces in order to achieve a more realistic description of the one-quasiparticle properties of the neighboring odd nuclei. The monopole matrix elements of the two-body interaction are scaled by pairing-strength parameters (separately for protons and neutrons) whose values can be determined [6] by comparison with the semiempirical pairing gaps obtained from the proton and neutron separation energies.

In the next step a correlated ground state and the excited states of the even-even reference nucleus are constructed by use of the QRPA. In the QRPA the creation operator for an excited state (QRPA phonon) has the form

$$Q_{\omega}^{\dagger}=\sum_{a\leq a'}[X_{aa'}^{\omega}A^{\dagger}(aa';J_{\omega}M)-Y_{aa'}^{\omega}\tilde{A}(aa';J_{\omega}M)], \tag{4}$$

where the quasiparticle pair creation and annihilation operators are defined as $A^{\dagger}(aa';JM)=\sigma_{aa'}^{-1}[a_a^{\dagger}a_{a'}^{\dagger}]_{JM}$, $\tilde{A}(aa';JM)=\sigma_{aa'}^{-1}[\tilde{a}_a\tilde{a}_{a'}]_{JM}$ and $\sigma_{aa'}=\sqrt{1+\delta_{aa'}}$. Here the greek indices ω denote phonon spin J_{ω} and parity π_{ω}. Furthermore, they contain an additional quantum number k_{ω} enumerating the different QRPA roots for the same angular momentum and parity. Thus $\omega=\{J_{\omega},\pi_{\omega},k_{\omega}\}$.

For each value of the angular momentum and parity the spectrum of the even-even nucleus is constructed by diagonalizing the QRPA matrix containing the usual submatrices *A* (a QTDA matrix in two-quasiparticle basis) and *B* (induced by correlations of the ground state) [5]. The two-body matrix elements of multipolarity J^{π}, occurring in the *A* and *B* matrices, are multiplied by two phenomenological scaling constants, namely the particle-hole strength, $g_{\rm ph}$, and the particle-particle strength $g_{\rm pp}$ [12]. In the present calculation the bare G-matrix value of $g_{\rm pp}=1.0$ has been used for all multipolarities. The value of $g_{\rm ph}$ can be set by the experimental value of the energy of the first J^{π} state and/or the electromagnetic decay rate of the first J^{π} state to the ground state.

The basis states in our quasiparticle-phonon calculation are constructed from the previously determined BCS quasiparticles (2) and the QRPA phonons (4) of the studied even-even reference nucleus. In the MQPM the creation operators of states in an odd-*A* nucleus have the following form in terms of BCS quasiparticles and QRPA phonons

$$\Gamma_i^{\dagger}(jm)=\sum_n\ C_n^i a_{njm}^{\dagger}+\sum_{a\omega}\ D_{a\omega}^i[a_a^{\dagger}Q_{\omega}^{\dagger}]_{jm}. \tag{5}$$

The equations of motion for the eigenstates of the odd-*A* nucleus are found by using the double-commutator techniques introduced in [8]. They have the form

$$\langle -|\{[a_a^\dagger Q_\omega^\dagger]_{jm}^\dagger ,\hat{H},\Gamma_i^\dagger(jm)\}|-\rangle = \Omega_i\langle -|\{[a_a^\dagger Q_\omega^\dagger]_{jm}^\dagger ,\Gamma_i^\dagger(jm)\}|-\rangle, \tag{6}$$

$$\langle -|\{a_{njm},\hat{H},\Gamma_i^\dagger(jm)\}|-\rangle = \Omega_i\langle -|\{a_{njm},\Gamma_i^\dagger(jm)\}|-\rangle, \tag{7}$$

where the double anticommutator is defined as $2\{A,B,C\} = \{A,[B,C]\}+\{[A,B],C\}$ and the single commutators and anticommutators have their usual definitions. The state $|-\rangle$ above is the unperturbed BCS vacuum of the reference nucleus. Equations (6) and (7) lead to a generalized real symmetric eigenvalue problem

$$\begin{pmatrix} A & B \\ B^T & A' \end{pmatrix}\begin{pmatrix} C^i \\ D^i \end{pmatrix} = \Omega_i \begin{pmatrix} 1 & 0 \\ 0 & N \end{pmatrix}\begin{pmatrix} C^i \\ D^i \end{pmatrix}, \tag{8}$$

where the overlap between the one-quasiparticle and the quasiparticle-phonon states is always zero. However, the overlap between quasiparticle-phonon states can be nonzero and thus the quasiparticle-phonon states form a nonorthogonal, usually overcomplete basis set. Equation (8) is solved to find the odd-A states and their energies by standard techniques described below. Taking only terms to the order $|X|^2$ and $|Y|^2$ into account in Eqs. (6) and (7), the matrices A, B, N and A' become (below we have suppressed the trivial quantum number m, the z-projection of j)

$$A(aa';j) = \langle -|\{a_a,\hat{H},a_{a'}^\dagger\}|-\rangle = E_a\delta_{aa'}, \tag{9}$$

$$\begin{aligned} A'(\omega a\omega' a';j) &= \langle -|\{[a_a^\dagger Q_\omega^\dagger]_j^\dagger ,\hat{H},[a_{a'}^\dagger Q_{\omega'}^\dagger]_j\}|-\rangle \\ &= \frac{1}{2}(\hbar\Omega_\omega + E_a + \hbar\Omega_{\omega'} + E_{a'})N(\omega a\omega' a';j) - \frac{1}{2}\hat{J}_\omega\hat{J}_{\omega'}\sum_b \begin{Bmatrix} j_{a'} & j_b & J_\omega \\ j_a & j & J_{\omega'} \end{Bmatrix}(\hbar\Omega_\omega + E_a + \hbar\Omega_{\omega'} + E_{a'} - 2E_b) \\ &\quad\times \overline{X}_{ba'}^{\omega}\overline{X}_{ba}^{\omega'}\sigma_{ba}^{-1}\sigma_{ba'}^{-1} + \frac{1}{2}\hat{J}_\omega\hat{J}_{\omega'}\sum_b \frac{\delta_{jj_b}}{\hat{j}^2}(-\hbar\Omega_\omega - E_a - \hbar\Omega_{\omega'} - E_{a'} - 2E_b)\overline{Y}_{ba}^{\omega}\overline{Y}_{ba'}^{\omega'}\sigma_{ba}^{-1}\sigma_{ba'}^{-1}, \end{aligned} \tag{10}$$

where the overlap matrix element between two three-quasiparticle states are

$$N(\omega a\omega' a';j) = \langle -|\{[a_a^\dagger Q_\omega^\dagger]_j^\dagger ,[a_{a'}^\dagger Q_{\omega'}^\dagger]_j\}|-\rangle = \delta_{\omega\omega'}\delta_{aa'} + K(\omega a\omega' a';j). \tag{11}$$

Here $\hbar\Omega_\omega$ denote the QRPA-phonon energies, and the matrix K in Eq. (11) reads

$$K(\omega a\omega' a';j) = \hat{J}_\omega\hat{J}_{\omega'}\sum_b \left[\begin{Bmatrix} j_{a'} & j_b & J_\omega \\ j_a & j & J_{\omega'} \end{Bmatrix}\overline{X}_{ba'}^{\omega}\overline{X}_{ba}^{\omega'} - \frac{\delta_{jj_b}}{\hat{j}^2}\overline{Y}_{ba}^{\omega}\overline{Y}_{ba'}^{\omega'}\right]\sigma_{ba}^{-1}\sigma_{ba'}^{-1}. \tag{12}$$

Here $\overline{X}_{aa'}^{\omega} \equiv X_{aa'}^{\omega} - (-1)^{j_a + j_{a'} - J_\omega}X_{a'a}^{\omega}$. The same definition holds for $\overline{Y}$.

The interaction matrix elements between the one-quasiparticle and quasiparticle-phonon states have the following form:

$$\begin{aligned} \langle -|[Q_\omega a_a]_j\hat{H}a_{a'}^\dagger|-\rangle &= \frac{1}{3}\frac{\hat{J}_\omega}{\hat{j}_{a'}}\sum_{b\le b'} H_{\rm pp}(bb'aa'J_\omega)(u_bu_{b'}X_{bb'}^{\omega} - v_bv_{b'}Y_{bb'}^{\omega})\sigma_{bb'}^{-1} - \frac{1}{3}\frac{\hat{J}_\omega}{\hat{j}_{a'}}\sum_{b\le b'} H_{\rm hh}(bb'aa'J_\omega)(v_bv_{b'}X_{bb'}^{\omega} \\ &\quad - u_bu_{b'}Y_{bb'}^{\omega})\sigma_{bb'}^{-1} + \frac{1}{3}\frac{\hat{J}_\omega}{\hat{j}_{a'}}\sum_{b\le b'} H_{\rm ph}(bb'aa'J_\omega)(u_bv_{b'}X_{bb'}^{\omega} + v_bu_{b'}Y_{bb'}^{\omega})\sigma_{bb'}^{-1} \\ &\quad - \frac{1}{3}\frac{\hat{J}_\omega}{\hat{j}_{a'}}\sum_{b\le b'} H_{\rm hp}(bb'aa'J_\omega)(v_bu_{b'}X_{bb'}^{\omega} + u_bv_{b'}Y_{bb'}^{\omega})\sigma_{bb'}^{-1}, \end{aligned} \tag{13}$$

where $j_{a'} = j$ and

$$H_{\rm pp}(bb'aa'J) = 2v_bu_{b'}G(bb'aa'J), \tag{14}$$

$$H_{\rm hh}(bb'aa'J) = 2u_bv_{b'}G(bb'aa'J), \tag{15}$$

$$H_{\rm ph}(bb'aa'J) = 2v_bv_{b'}F(bb'aa'J) + 2u_bu_{b'}F(b'baa'J)(-1)^{j_b + j_{b'} + J}, \tag{16}$$

$$H_{\rm hp}(bb'aa'J) = 2u_bu_{b'}F(bb'aa'J) + 2v_bv_{b'}F(b'baa'J)(-1)^{j_b + j_{b'} + J}. \tag{17}$$

In the previous version of our model [7] the second term in the quasiparticle-phonon matrix elements of Eq. (10) was missing. This additional term stems from the use of theequations-of-motion (EOM) method of Eq. (6) when deriving the eigenvalue equation (8). It has an important effect on the location of the three-quasiparticle-type states relative to the one-quasiparticle-type ones, and it is essential for yielding theoretical results in agreement with data.

To solve the rather involved eigenvalue problem of Eq. (8) we adopt the method where we first solve the eigenvalue equation for the overlap matrix **N**:

$$\sum_j N_{ij} u_j^{(k)} = n_k u_i^{(k)} . \tag{18}$$

The eigenvectors can be written in the basis $|i\rangle$ $|i\rangle \equiv \Gamma_i^\dagger |\mathrm{QRPA}\rangle$ of Eq. (5) ($|\mathrm{QRPA}\rangle$ is the correlated ground state of the even-even reference nucleus) as

$$|\tilde{k}\rangle = \frac{1}{\sqrt{n_k}} \sum_i u_i^{(k)} |i\rangle . \tag{19}$$

They have the property of being mutually orthogonal, have a norm equal to unity and form a complete set after removing states having eigenvalue $n_k = 0$ (this removes the overcompleteness of the set $\{|i\rangle\}$).

Using the new orthogonal complete set of states (19) we can transform (8) to an ordinary real and symmetric eigenvalue problem of the form

$$\sum_l \langle \tilde{k} |H| \tilde{l} \rangle g_l^{(n)} = \lambda_n g_k^{(n)} , \tag{20}$$

where

$$\langle \tilde{k} |H| \tilde{l} \rangle = \frac{1}{\sqrt{n_k n_l}} \sum_{ij} u_i^{(k)*} \langle i|H|j\rangle u_j^{(l)} . \tag{21}$$

The coefficients C_i^n of the eigenstates are calculated from the g coefficients in the following way:

$$C_i^n = \sum_k n_k^{-1/2} g_k^{(n)} u_i^{(k)} . \tag{22}$$

In practice one omits states having eigenvalue n_k less than some set upper limit ϵ.

B. Expressions for allowed beta decay

In the calculation of beta-decay transition amplitudes one has to know the matrix elements of the charge-changing transition densities (CCTD) between the initial and final states. The transition amplitude is expressed in terms of matrix elements of the CCTD and the single-particle matrix elements of the transition operator [12]. In the following equations we write explicit expressions of all the needed reduced matrix elements.

In our formalism we employ the following reduced matrix elements of the CCTD between one-quasiparticle states

$$(-||a_n[c_{n'}^\dagger, \tilde{c}_{p'}]_L a_p^\dagger||-) = \hat{L} u_n u_p \delta_{nn'} \delta_{pp'} , \tag{23}$$

$$(-||a_p[c_{n'}^\dagger, \tilde{c}_{p'}]_L a_n^\dagger||-) = \hat{L} v_p v_n \delta_{pp'} \delta_{nn'} (-1)^{j_{p'}+j_{n'}+L} , \tag{24}$$

$$(-||a_p[c_{p'}^\dagger, \tilde{c}_{n'}]_L a_n^\dagger||-) = \hat{L} u_p u_n \delta_{pp'} \delta_{nn'} , \tag{25}$$

$$(-||a_n[c_{p'}^\dagger, \tilde{c}_{n'}]_L a_p^\dagger||-) = \hat{L} v_n v_p \delta_{nn'} \delta_{pp'} (-1)^{j_{n'}+j_{p'}+L} . \tag{26}$$

These equations describe β^+(EC) transitions between particle states, β^+(EC) transitions between hole states, β^- transitions between particle states and β^- transitions between hole states, respectively. Above the indices n (p) denote (harmonic-oscillator) single-particle states of a neutron (proton).

Reduced matrix elements of the CCTD between a quasiparticle-phonon state and a one-quasiparticle state are

$$(-||[Q_\omega a_n]_j [c_{n'}^\dagger, \tilde{c}_{p'}]_L a_p^\dagger||-) = \hat{J}_\omega \hat{L} \hat{j} \left[\begin{Bmatrix} j_p & j & L \\ j_n & j_{p'} & J_\omega \end{Bmatrix} \overline{X}_{pp'}^{\omega} u_{n'} v_{p'} \sigma_{pp'}^{-1} \delta_{nn'} (-1)^{j_p+j+L} + \frac{\delta_{jj_{n'}}}{\hat{j}^2} \overline{Y}_{nn'}^{\omega} v_{n'} u_{p'} \sigma_{nn'}^{-1} \delta_{pp'} \right] , \tag{27}$$

$$(-||[Q_\omega a_p]_j [c_{n'}^\dagger, \tilde{c}_{p'}]_L a_n^\dagger||-) = -(-1)^{j_{n'}+j_{p'}+L} \hat{J}_\omega \hat{L} \hat{j} \left[\begin{Bmatrix} j_n & j & L \\ j_p & j_{n'} & J_\omega \end{Bmatrix} \overline{X}_{nn'}^{\omega} v_{n'} u_{p'} \sigma_{nn'}^{-1} \delta_{pp'} (-1)^{j_n+j+L} + \frac{\delta_{jj_{p'}}}{\hat{j}^2} \overline{Y}_{pp'}^{\omega} u_{p'} v_{n'} \sigma_{pp'}^{-1} \delta_{nn'} \right] . \tag{28}$$

In Eqs. (27) and (28) only the matrix elements corresponding to Eqs. (23) and (24) are shown. The other two are obtained by interchanging the proton and neutron indices.

The reduced matrix elements of the CCTD between two quasiparticle-phonon states are

$$(-||[Q_\omega a_n]_j[c^\dagger_{n'}\tilde{c}_{p'}]_L[a^\dagger_p Q^\dagger_{\omega'}]_{j'}||-)=-\Bigg[s(j_nJ_\omega j)\begin{Bmatrix} j & L & j' \\ j_{p'} & J_\omega & j'_n\end{Bmatrix}\delta_{nn'}\left(\frac{1}{2}\delta_{pp'}\delta_{\omega\omega'}+K(p'\omega p\omega';j')\right)$$
$$+s(j_{n'}J_{\omega'}j)\begin{Bmatrix} j & L & j' \\ j_{p'} & J_{\omega'} & j'_n\end{Bmatrix}\delta_{pp'}\left(\frac{1}{2}\delta_{nn'}\delta_{\omega\omega'}+K(n\omega n'\omega';j)\right)\Bigg]$$
$$\times\hat{j}\hat{L}\hat{j}'(-1)^{j+L+j'}u_{n'}u_{p'}-\left[\begin{Bmatrix} j & L & j' \\ j_n & j'_n & J_{\omega'} \\ J_{\omega'} & j_{p'} & j_p\end{Bmatrix}\bar{X}^{\omega'}_{n'n}\bar{X}^{\omega}_{pp'}s(j_{p'}j_{n'}L)\right.$$
$$\left.+\frac{\delta_{jj_{n'}}\delta_{j'j_{p'}}}{\hat{j}^2\hat{j}'^2}\bar{Y}^{\omega}_{nn'}\bar{Y}^{\omega'}_{p'p}\right]\sigma^{-1}_{pp'}\sigma^{-1}_{nn'}\hat{j}\hat{L}\hat{j}'s(j_nJ_\omega j)\hat{J}_\omega\hat{J}_{\omega'}v_{n'}v_{p'}, \quad (29)$$

where $K(a\omega a'\omega';j)$ is defined in Eq. (12) and $s(j_nJ_\omega j)\equiv(-1)^{j_n+J_\omega-j}$. This matrix element again corresponds to Eq. (23). The other three matrix elements are obtained by interchanging the proton and neutron indices and/or making the substitution $u_a\rightarrow v_a$, $v_a\rightarrow -u_a$. The final expressions for the Fermi and Gamow-Teller transition amplitudes, for the comparative half-life [log(ft)] and the expressions for the reduced single-particle matrix elements can be found, e.g., in [6,12].

III. RESULTS AND DISCUSSION

The neutron single-particle basis in our calculations consisted of the complete oscillator major shells $4\hbar\omega$ and $5\hbar\omega$, while for protons we adopted the major shells $3\hbar\omega$ and $4\hbar\omega$ with the intruder orbital $0h_{11/2}$ from the $5\hbar\omega$ major shell. Basically, the single-particle energies correspond to the Woods-Saxon energies with the Bohr-Mottelson parametrization [11]. The pairing strength parameters were adjusted by the odd-even mass differences and the particle-hole strength to the lowest J^π phonon energy of the even-even reference nucleus. The natural-parity RPA phonons 2^+, 3^-, 4^+, 5^-, and 6^+ were used in the MQPM calculations. After the Hamiltonian parameters are fixed in the BCS and QRPA calculations, the MQPM calculations are parameter-free and thus have good predictive power.

The three lowest states of the odd-N $^{127-133}$Xe nuclei are one-quasiparticle-like states emerging from the neutron single-particle orbitals $2s_{1/2}$, $1d_{3/2}$, and $0h_{11/2}$. Our BCS calculation could reproduce qualitatively, after slight adjustment of the Woods-Saxon single-particle energies near the Fermi level, the experimental neutron one-quasiparticle energies in all the nuclei under discussion, except in ^{129}Xe where it is does not seem possible to reproduce the correct ordering of the lowest one-quasineutron states. This happens because of the strong influence of the neutron orbit $0h_{11/2}$ on the low-lying one-quasiparticle spectrum due to its high statistical weight emerging from its large degeneracy. Furthermore, comparing the MQPM level energies, which closely resemble the BCS one quasiparticle energies, to experimental level energies [13] (Fig. 1) one can see that in ^{127}Xe and in ^{129}Xe the experimental $11/2^-_1$ state lies somewhat higher than the pure BCS $0h_{11/2}$ one-quasiparticle state, which affects also the position of the $9/2^-_1$ states (belong to the $0h_{11/2}\otimes 2^+_1$ multiplet) in these isotopes. It is not possible to correct this by adjusting the single-particle energies or pairing strengths alone. Despite these properties the lowest neutron one-quasiparticle energies behave smoothly in $^{127-133}$Xe which makes it possible to describe both the neutron-particle and neutron-hole nuclei with respect to each even-even $^{128-134}$Xe nucleus.

In the present work we have chosen the $^{127-133}$Xe nuclei to be hole nuclei with respect to the even-even $^{128-134}$Xe reference nuclei. The choice of a hole- or a particle-type odd-A nucleus only makes a difference in the immediate vicinity of a shell closure. In the MQPM calculation the energies of the lowest one-quasiparticle-like states drop about 0.1–0.2 MeV from the BCS one-quasiparticle energies. The energies of these states also converge rapidly, thus justifying their description by perturbation theory [1]. It is worth noting that the MQPM is able to push the $11/2^-_1$ state energy upwards from the BCS quasiparticle energy in the isotopes ^{127}Xe and ^{129}Xe, but the correction is too small to have any practical significance.

Because of the intruder orbital $0h_{11/2}$ near the neutron Fermi level, there exists one low-lying negative-parity one-quasiparticle state in the spectra of the odd Xe isotopes. In addition, the low-lying negative-parity three-quasiparticle states of odd-A xenon isotopes have 90–95 % overlap with quasiparticle-phonon states composed of this orbital coupled to the lowest 2^+ RPA phonon of the even-even reference nucleus. This coupling yields a multiplet of five states, whose spins extend from $j=7/2$ to $j=15/2$. This multiplet of states is a good test for the quality of theoretical models for odd nuclei, in particular of their ability to produce low-lying negative-parity three-quasiparticle states. In our calculations these negative-parity levels converged very well already when four lowest RPA phonons of each multipolarity were used. Despite the good convergence, the energy of the lowest state of the multiplet, $9/2^-$, does not go as low in energy as the experimental state energies in any of the isotopes. This is the most unsatisfactory property of our model and is due to missing the next-order many-body contributions in the quasiparticle-phonon matrix elements of Eq. (13). Still the

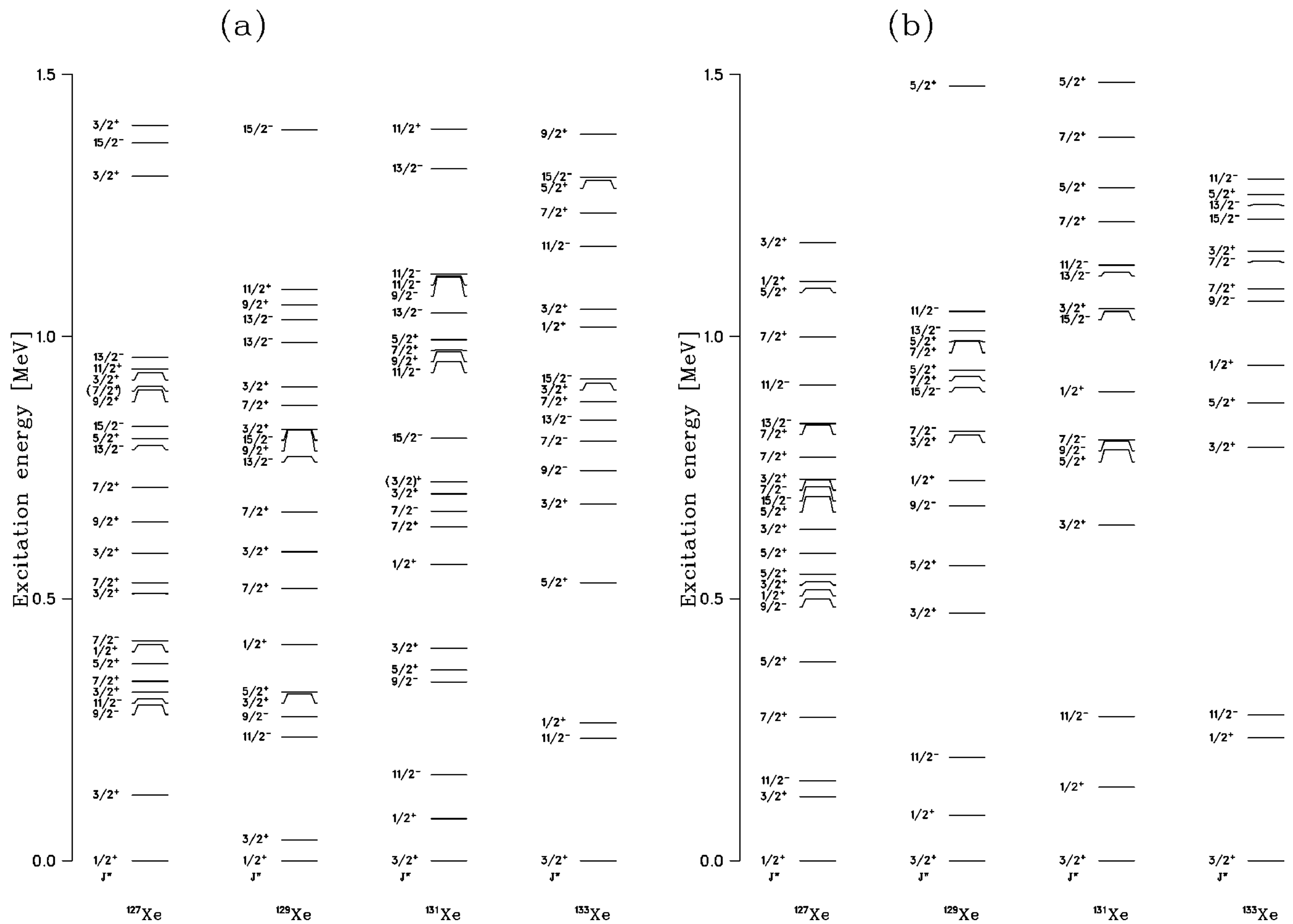

FIG. 1. (a) Experimental spectrum of $^{127-133}$Xe. Data is taken from [13]. (b) Theoretical MQPM spectrum of $^{127-133}$Xe. In this calculation eight phonons of multipolarity 2^+, 3^-, 4^+, 5^-, and 6^+ were used.

agreement is generally better than in the previous version of our model [7].

The positive-parity states of $^{127-133}$Xe did not show convergence until eight phonons of each multipole were used. This can be seen in Fig. 2 where the calculated spectrum of ^{131}Xe is shown for four different sets of QRPA phonons used in the test calculations. From the figure one can see that by coupling the 2_1^+ phonon alone with quasiparticles does not produce enough configurations. When eight 2^+ phonons or eight 2^+ and 3^- phonons are taken into account, the results are almost as good as in the complete calculation, except that in the complete calculation the one-quasiparticle-like states become lower in energy, which gives the false impression that the three-quasiparticle energies rise. Another aspect of the complete calculation is that the energies of the $9/2_1^-$ and $11/2_1^-$ states are substantially lowered as compared to the more restricted calculations. It is also worth noting that in the case of xenon isotopes the 3^- phonons seem not to play a role in the spectrum of the neighboring odd-A nucleus. One could expect that in some other cases, i.e., when the 3^- phonons would be very collective, the role of them would be more important.

For the positive-parity states the agreement between theory and experiment is better than in the case of negative-parity states and much better than in our previous work [7]. The six lowest-lying unperturbed positive-parity quasiparticle-phonon states in the xenon isotopes can be formed by coupling the 2_1^+ phonon to quasiparticles $1d_{3/2}$ and $2s_{1/2}$. According to the rules of angular-momentum coupling there is one state with angular momentum $j=1/2$, two $j=3/2$ states, two $j=5/2$ states and one $j=7/2$ state. The diagonalization of Eq. (8) mixes these components and spreads their energies. As the energy of the 2_1^+ phonon is about 0.5 MeV in ^{128}Xe and rises gradually to about 0.8 MeV in ^{134}Xe the energy of the unperturbed quasiparticle-phonon multiplets, compared to the lowest one-quasiparticle state, grows from 0.5 MeV to 0.8 MeV when going from ^{127}Xe to ^{133}Xe.

By looking at Fig. 1(b) one can see that diagonalization of Eq. (8) rises or lowers the state energies from the unperturbed quasiparticle-phonon energies and also relative to the one-quasiparticle-like state energies who are lowered due to H_{31} part of (3) by about 0.1–0.3 MeV. From the six resulting states the $1/2_2^+$ and $3/2_2^+$ states have the most interesting structure. In ^{133}Xe the state $1/2_2^+$ has as the dominating components $0.96\times 1d_{3/2}\otimes 2_1^+$, $0.14\times 1d_{3/2}\otimes 2_2^+$, and $0.10\times 2s_{1/2}$ where we have indicated also the amplitudes of the components. In ^{131}Xe the $1/2_2^+$ state has almost the same structure, but in ^{129}Xe the $1d_{3/2}\otimes 2_1^+$ amplitude is 0.77, and in ^{127}Xe the $1/2^+$ state is almost pure $1d_{3/2}\otimes 2_1^+$ quasiparticle-phonon state. The $3/2_2^+$ state of ^{133}Xe is mainly

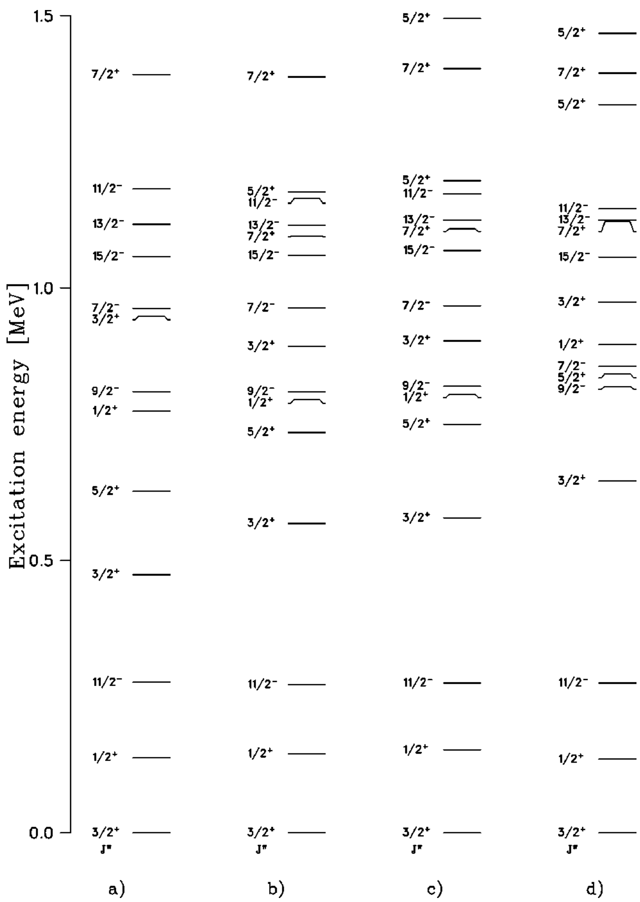

FIG. 2. Comparison of the theoretical energy spectra of the nucleus ^{131}Xe for different sets of QRPA phonons used in the calculations. In spectrum (a) only the 2_1^+ RPA phonon was used, in spectrum (b) the eight lowest 2^+ phonons were used, and in spectrum (c) the eight lowest 2^+ and 3^- phonons were used. Spectrum (d) is the result of the complete calculation where eight lowest phonons of multipolarities 2^+, 3^-, 4^+, 5^-, and 6^+ were used.

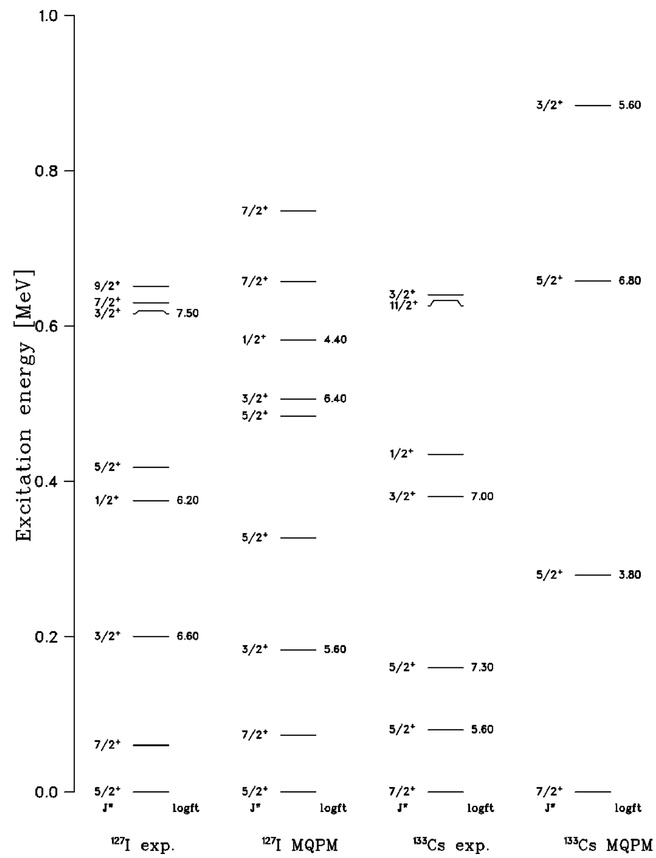

FIG. 3. Comparison of the theoretical and experimental energy spectra and β^--decay logft values (to the right of the energy levels) for the decays $^{127}\mathrm{Xe}\rightarrow{}^{127}\mathrm{I}$ and $^{133}\mathrm{Xe}\rightarrow{}^{133}\mathrm{Cs}$.

$0.65\times 1d_{3/2}\otimes 2_1^+$, $0.27\times 2s_{1/2}\otimes 2_1^+$, $0.32\times 1d_{3/2}\otimes 2_2^+$, and $0.12\times 2s_{1/2}\otimes 2_2^+$. The three-quasiparticle components of the state $3/2_2^+$ remain approximately the same in $^{127-133}$Xe, but the $1d_{3/2}$ one-quasiparticle component grows gradually to 0.32 when going from ^{133}Xe to ^{127}Xe. This indicates that in the case of the $3/2^+$ states the mixing of one-quasiparticle components with three-quasiparticle components increases towards the closed neutron shell.

The states $5/2_1^+$ and $7/2_1^+$ seem to have in all xenon isotopes quasiparticle-phonon components $2s_{1/2}\otimes 2_1^+$ and $1d_{3/2}\otimes 2_1^+$ as the main components, except in the case of ^{127}Xe, where the main components are the $1d_{5/2}$ and $0g_{7/2}$ one-quasiparticle states.

It seems that for a fully microscopic theory it is hard to predict completely correctly the relative ordering of the perturbed states emerging from a quasiparticle-phonon multiplet, as is the case above ($1d_{3/2}\otimes 2_1^+$, $2s_{1/2}\otimes 2_1^+$). It appears that the calculated $5/2_1^+$ state should lie much further down and at the same time the calculated $1/2_2^+$ state should shift faster up in energy when going from ^{129}Xe to ^{133}Xe. On the other hand, the theoretical $3/2_2^+$ and $7/2_1^+$ states have the correct qualitative and quantitative behavior when going from ^{127}Xe to ^{133}Xe. In any case, the xenon isotopes are surely not the easiest cases to discuss within a theoretical framework due to their large density of levels below 1 MeV. The advantage of these isotopes is that there is a good amount of experimental data on their energy spectra.

Next we discuss briefly the odd-Z nuclei (with respect to the $^{128-134}$Xe reference nuclei) which are either beta-decay daughter or mother nuclei of $^{127-133}$Xe. It is difficult, if not impossible, to fit the BCS one-quasiparticle energies to the lowest-lying states of the lighter proton-quasiparticle nuclei ^{127,129}Cs because these nuclei have a very complicated low-lying level structure as compared to the previously discussed odd-N xenon nuclei. The problem is that these nuclei have many very low-lying three-quasiparticle states mixed with one-quasiparticle states, thus making the identification of one-quasiparticle-like states difficult. For this reason we have chosen to fit our proton single-particle set to $^{127-133}$I which have much simpler low-lying level structures with energy levels that can be assigned to definite single-particle levels. However, the heavier proton-quasiparticle nuclei ^{131,133}Cs, which are nearer to the closed proton shells, do not have this problem and have quite similar level structure as ^{131,133}I making it possible to describe both proton-particle and -hole nuclei with respect to even-even ^{132,134}Xe. Therefore, considering MQPM calculations for beta decay, we can comfortably describe the six proton-odd nuclei $^{127-133}$I and ^{131,133}Cs as proton-hole and proton-particle nuclei with respect to even-even $^{128-134}$Xe nuclei, respectively. In the proton-hole nuclei $^{127-133}$I the lowest two states are $1d_{5/2}$ and $0g_{7/2}$ one-quasiparticle-like states. These can be roughly

TABLE I. The experimental and theoretical comparative half-lives ($\log ft$ values) of allowed beta-decay transitions for nuclei where the beta-decay daughter or mother is a xenon isotope $^{127-133}$Xe. The additional $\log ft$ values in parentheses are calculated using only Gamow-Teller matrix elements in cases where the isospin mixing in the MQPM produces a non-negligible Fermi matrix element. Also the experimental and theoretical excitation energies of the final states are given.

Transition	Mode	Expt. $\log ft$	Energy	Theor. $\log ft$	Energy
^{127}Xe($1/2_1^+$)→^{127}I($3/2_1^+$)	$\beta+$	6.6	0.200	5.6	0.183
^{127}Xe($1/2_1^+$)→^{127}I($1/2_1^+$)		6.2	0.375	4.4	0.582
^{129}Cs($1/2_1^+$)→^{129}Xe($1/2_1^+$)	$\beta+$	6.3	0.00	5.7(5.8)	0.00
^{129}Cs($1/2_1^+$)→^{129}Xe($3/2_1^+$)		7.3	0.040	8.2	0.122
^{131}Cs($5/2_1^+$)→^{131}Xe($3/2_1^+$)	EC	5.5	0.00	3.7	0.00
^{131}I($7/2_1^+$)→^{131}Xe($5/2_1^+$)	$\beta-$	6.6	0.364	6.6	0.843
^{131}I($7/2_1^+$)→^{131}Xe($5/2_2^+$)		7.0	0.723	8.0	1.337
^{131}I($7/2_1^+$)→^{131}Xe($7/2_1^+$)		7.0	0.637	4.8(5.1)	1.123
^{133}Xe($3/2_1^+$)→^{133}Cs($3/2_1^+$)	$\beta-$	7.0	0.384	5.6(6.1)	0.884
^{133}Xe($3/2_1^+$)→^{133}Cs($5/2_1^+$)	$\beta-$	5.6	0.080	3.8	0.279
^{133}Xe($3/2_1^+$)→^{133}Cs($5/2_2^+$)		7.4	0.161	6.8	0.657
^{133}I($7/2_1^+$)→^{133}Xe($5/2_1^+$)	$\beta-$	6.8	0.530	7.0	0.873
^{133}I($7/2_1^+$)→^{133}Xe($5/2_2^+$)		8.1	1.05	6.8	1.271
^{133}I($7/2_1^+$)→^{133}Xe($7/2_1^+$)		7.6	0.875	4.7(5.0)	1.091

reproduced already at the BCS level modifying slightly the proton single-particle energies from the Woods-Saxon ones. Thus the relative ordering of lowest one-quasiparticle states can be made correct.

To test the ability of the MQPM to describe allowed beta decay we have applied it to beta decays where the initial state is the ground state of a xenon nucleus and the final states belong to a proton-odd nucleus or the initial state is the ground state of the proton-odd nucleus and the final states belong to a xenon nucleus. We will not show all the spectra for the proton-odd nuclei $^{127-133}$I or ^{131,133}Cs, but rather one representative case of both elements is shown in Fig. 3 with the beta-decay $\log ft$ values of the allowed transitions from the $1/2^+$ ground state of ^{127}Xe and the $3/2^+$ ground state of ^{133}Xe. In all but four cases the Fermi transition amplitudes, whenever nonzero, are much smaller than the Gamow-Teller ones. The four transitions where Fermi-type matrix elements have a magnitude comparable to the Gamow-Teller matrix elements are indicated in Table I by additional $\log ft$ values in parenthesis. These values have been obtained by using only Gamow-Teller matrix elements and neglecting the Fermi matrix elements. The unreasonably large Fermi matrix elements have their origin in the violation of the isospin symmetry due to the used proton-neutron formalism.

For ^{127}I the MQPM can reproduce the lowest three-quasiparticle energies reasonably well. In this case the beta-decay mother nucleus is described as a neutron-hole nucleus with respect to the ^{128}Xe reference nucleus. The beta decay to the lowest final states of ^{127}I is qualitatively described by the MQPM. The $3/2_1^+$ state of ^{127}I has the leading components $0.90\times 1d_{5/2}\otimes 2_1^+$ and $0.26\times 1d_{3/2}\otimes 2_1^+$ and the state $1/2_1^+$ the components $0.91\times 1d_{5/2}\otimes 2_1^+$ and $0.20\times 2s_{1/2}$. The state $5/2_2^+$ consists of $0.71\times 0g_{7/2}\otimes 2_1^+$ and $0.50\times 1d_{5/2}$ and the state $3/2_2^+$ is mainly $0.90\times 0g_{7/2}\otimes 2_1^+$ and $0.32\times 1d_{3/2}$.

The decay ^{133}Xe→^{133}Cs is shown on the right half of Fig. 3. Here the beta-decay mother nucleus ^{133}Xe is a neutron-particle nucleus with respect to the reference ^{132}Xe. The MQPM is also here able to describe the lowest final-state energies and beta-decay matrix elements qualitatively, this time overestimating them in magnitude. The main component of the state $5/2_1^+$ is the single-quasiparticle state $1d_{5/2}$, and the state $5/2_2^+$ has the leading components $0g_{7/2}\otimes 2_1^+$ and $0g_{7/2}\otimes 2_2^+$ at about equal weight. The theoretical $\log ft$ value of the state $5/2_1^+$ is too small because in this case the MQPM is not able to mix enough three-quasiparticle components into the state vector. Therefore the beta-decay amplitude coming from the one-quasiparticle to one-quasiparticle transition (25) dominates the total transition amplitude.

Finally, in Table I, we show the $\log ft$ values of a set of allowed beta-decay transitions where the $^{127-133}$Xe nuclei are either beta-decay mother or daughter nuclei. Only those transitions have been shown where there exists experimental data on the $\log ft$ values. Also the theoretical and experimental excitation energies of the final states are depicted. As can be seen from the table, the agreement is good only for transitions from one-quasiparticle-like states to three-quasiparticle-like states. The greatest problems arise in the case of transitions from a one-quasiparticle state to another one, and in this case the theoretical decay rates are too fast and accordingly the $\log ft$ values are small. This means that either in the initial or the final states there is a too weak mixing of the three-quasiparticle components to the one-quasiparticle one. Another problem arises for the case of transitions where Fermi-type matrix elements are nonzero. These transitions tend to be too fast even when the Fermi matrix elements are neglected, as can be seen from Table I. As mentioned earlier, in ^{131}Xe and ^{133}Xe the $5/2_1^+$ and $7/2_1^+$ states are mostly three-quasiparticle states which can make the decay rates from the $7/2_1$ states of ^{131}I and ^{133}I to these states slow. For the $5/2_1^+$ final state this can be seen in the decay rates in Table I but the theoretical decay rate to the

$7/2_1^+$ is still much too fast in comparison with experiment. For the decay of the $3/2_1^+$ state in ^{133}Xe the decay rates to the $5/2^+$ states seem to follow qualitatively the experimental pattern, although the theoretical decay rates are slightly too fast.

IV. SUMMARY AND CONCLUSIONS

The equations-of-motion method is a useful tool to simplify the Hamiltonian equations for odd-mass nuclei, when one- and three-quasiparticle configurations are included. In this way one can derive the eigenvalue equations of the microscopic quasiparticle-phonon model (MQPM) where a consistent truncation of the one- and three-quasiparticle configuration spaces can be performed starting from a realistic microscopic many-body Hamiltonian. Our calculations show that the MQPM model is able to describe many of the observed features of the energy spectra and allowed beta decays of odd-A nuclei at least qualitatively. In our case, where the active neutron major shell consisted of s-d-$g_{7/2}$ positive-parity orbitals and the $h_{11/2}$ negative-parity intruder orbital, the positive-parity three-quasiparticle neutron-odd states and Gamow-Teller beta decay between them and proton-odd states were reasonably well described, as long as the final states were three quasiparticle type. However, the low-lying negative-parity quasiparticle-phonon states emerging from the $h_{11/2}$ intruder orbital were not satisfactory in the MQPM framework. To complete the model in this respect, next-order contributions to the Hamiltonian matrix elements have to be taken into account in the description of three-quasiparticle states built of RPA phonons and intruder quasiparticles. We can conclude that the model we describe in our article is still incomplete and further development is needed to satisfactorily describe the three-quasiparticle negative-parity states of odd-mass nuclei.

[1] J. A. Halbleib and R. A. Sorensen, Nucl. Phys. **A98**, 542 (1967).
[2] L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35**, 853 (1963).
[3] A. Kuriyama, T. Marumori, and K. Matsuyanagi, Prog. Theor. Phys. **47**, 498 (1972).
[4] A. Kuriyama, T. Marumori, K. Matsuyanagi, and R. Okamoto, Prog. Theor. Phys. **53**, 489 (1975).
[5] M. Baranger, Phys. Rev. **120**, 957 (1960).
[6] J. Suhonen, T. Taigel, and A. Faessler, Nucl. Phys. **A486**, 91 (1988).
[7] J. Toivanen and J. Suhonen, J. Phys. G **21**, 1491 (1995).
[8] D. J. Rowe, Rev. Mod. Phys. **40**, 153 (1968).
[9] K. Holinde, Phys. Rep. **68**, 121 (1981).
[10] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980).
[11] A. Bohr and B. R. Mottelson, *Nuclear Structure, Vol. I* (Benjamin, New York, 1969).
[12] J. Suhonen, Nucl. Phys. **A563**, 205 (1993).
[13] P. F. Mantica *et al.*, Phys. Rev. C **42**, 902 (1990).