Microscopic description of the scissors mode and its fragmentation

T. Beuschel and J. P. Draayer
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803-4001

D. Rompf
Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, 35392 D-Gießen, Germany

J. G. Hirsch
Departamento de Física, CINVESTAV del IPN, A.P. 14-740 México, 07000 D.F., Mexico
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The $M1$ transition and ground state spectra of the even-even $^{160-164}$Dy and $^{156-160}$Gd isotopes are calculated within the framework of the microscopic pseudo-SU(3) shell model. A reasonable description of the $M1$ strength function, including its fragmentation, is obtained with an interaction that includes collective as well as single-particle and pairing terms with the parameters of the theory fit to the energy spectrum and $E2$ transition strengths. [S0556-2813(98)04403-3]

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In recent years there have been several theoretical [1–6] and experimental [7,8] studies of low-lying $M1$ transition strengths in deformed nuclei (see also references in [9]). A feature that remains unclear is the fragmentation of this so-called scissors mode, that is, the break-up of the $M1$ strength among several levels closely packed and clustered around a few strong transition peaks in the energy region between 2 and 4 MeV. When the scissors mode was predicted by Lo Iudice and Palumbo [1] in 1978, six years before its was detected [10], it was interpreted as a collective magnetic-dipole state of two spheriods, one representing protons and the other for neutrons, exercising rotational oscillations relative to one another. However, this picture cannot explain the fragmentation of the $M1$ mode. In this article, the microscopic pseudo-SU(3) shell model is used to describe such nuclei, specifically their low-lying rotational bands and the observed fragmentation of their $M1$ strength. A key ingredient, missing in other studies, is the SU(3) symmetry breaking induced by single-particle and pairing terms in the interaction.

Since its introduction in the late 1960s [11,12], the pseudospin concept has been successfully applied to various properties of heavy deformed nuclei [13–15]. However, the nucleon-nucleon interaction used in these investigations was schematic because of difficulties related to the evaluation of matrix elements of general interactions in an SU(3) basis. Recently these limitations were lifted so that it is now possible to calculate matrix elements of generic one-body and two-body interactions [16], including pairing [17] terms used in this study.

The pseudo-SU(3) model is a microscopic theory that takes full advantage of pseudospin symmetry [18,19], which is manifest in the near degeneracy of orbital pairs $[(l-1)_{j=l+1/2},(l+1)_{j=l-1/2}]$, as well as full account of the Pauli principle. Group theoretical methods are used for the construction of the basis states and the calculation of matrix elements [12,15]. In terms of the space $(\mathrm{U}(N)\leftrightarrow[f])$, shape $[\mathrm{SU}(3)\leftrightarrow(\lambda,\mu)]$, orbital $[\mathrm{SO}(3)\leftrightarrow L]$, spin (S), and total angular momentum (J), as well as various multiplicities [α in the $\mathrm{U}(N)\supset\mathrm{SU}(3)$ reduction, ρ for multiplicities in the product of the proton and neutron SU(3) irreducible representations (irreps), and κ for the $\mathrm{SU}(3)\supset\mathrm{SO}(3)$ reduction], the basis states have the form (π for protons, ν for neutrons)

$$|\{m_\pi[f_\pi]\alpha_\pi(\lambda_\pi,\mu_\pi),m_\nu[f_\nu]\alpha_\nu \times(\lambda_\nu,\mu_\nu)\}\rho(\lambda,\mu)\kappa L\{S_\pi,S_\nu\}S;JM\rangle. \quad (1)$$

A general form for the Hamiltonian is

$$H=-(a_2+a_{\mathrm{sym}})C_2+a_3C_3+bK_J^2+cJ^2+D_\pi\sum_{i_\pi} l_{i_\pi}^2 + D_\nu\sum_{i_\nu} l_{i_\nu}^2-G_\pi H_P^\pi-G_\nu H_P^\nu, \quad (2)$$

where C_2 and C_3 are the second and third order invariants of SU(3), which are related to the axial and triaxial deformation of the nucleus, and J^2 and K_J^2 are the square of the total angular momentum and its projection on the intrinsic body-fixed symmetry axis, which generate rotational bands and K-band splitting. The parameter a_{sym} is introduced to shift

TABLE I. Deformation and occupancies for the Gd and Dy isotopes. In each case the deformation was determined by the minimum of the summed single-particle energies of a Nilsson Hamiltonian, and is in agreement with experiment. The distribution of the valence protons and neutrons into normal and unique parity determines the pseudo-SU(3) wave function.

Nucleus	β	n_N^π	n_A^π	n_N^ν	n_A^ν
^{156}Gd	0.30	8	6	6	4
^{158}Gd	0.31	8	6	6	6
^{160}Gd	0.29	8	6	8	6
^{160}Dy	0.31	10	6	6	6
^{162}Dy	0.28	10	6	8	6
^{164}Dy	0.28	10	6	10	6

TABLE II. Hamiltonian parameters (MeV) derived from the fitting procedure. An effective charge was used in the calculation of $B(E2)$ transition strengths.

Nucleus	a_2	$a_{\rm sym}$	a_3	b	c	D_π	D_ν	$G_{\pi,\nu}$	$q_{\rm eff}$
^{156}Gd	0.0230	0.0008	77.2×10^{-6}	0.0121	0.1435	0.0756	−0.0724	0.1052	1.3119
^{158}Gd	0.0245	0.0006	80.4×10^{-6}	0.0080	0.2259	−0.0738	0.0478	0.0685	1.3634
^{160}Gd	0.0224	0.0004	39.4×10^{-6}	0.0085	0.1871	0.0271	−0.0817	0.1096	1.2361
^{160}Dy	0.0212	0.0008	9.1×10^{-6}	0.0127	0.0517	0.0798	−0.1134	0.1386	1.2000
^{162}Dy	0.0218	0.0005	36.3×10^{-6}	0.0070	0.1421	−0.0835	−0.0470	0.1245	1.2486
^{164}Dy	0.0233	0.0001	46.2×10^{-6}	0.0083	0.1005	−0.1116	−0.1309	0.0879	1.2053

TABLE III. Total $B(M1)$ transition strengths ($[\mu_N^2]$) as given by experiment [7] and our calculation using a pure SU(3) Hamiltonian and a Hamiltonian that includes mixing. Experimental and theoretical values ($[e^2b^2]$) for the ground band $B(E2,0_1^+\rightarrow2_1^+)$ transition strengths are also given.

	$\Sigma B(M1)[\mu_N^2]$			$B(E2,0_1^+\rightarrow2_1^+)$ $[e^2b^2]$	
Nucleus	Expt.	Theory SU(3)	Theory mix	Expt.	Theory mix
^{156}Gd	3.40	3.52	2.91	4.66	4.79
^{158}Gd	4.32	3.52	3.02	5.02	5.23
^{160}Gd	4.21	4.23	3.29	5.19	5.00
^{160}Dy	2.48	3.52	3.20	4.98	4.87
^{162}Dy	3.29	4.23	3.19	5.22	5.14
^{164}Dy	5.63	4.36	3.38	5.57	5.37

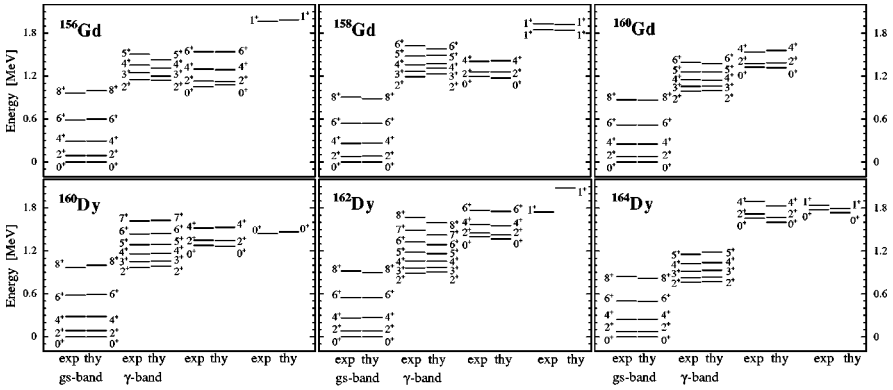

FIG. 1. Excitation spectra for energies up to 2 MeV. The left-hand columns of each band are the experimental values that were input for a fitting procedure that gave the parameters of the Hamiltonian. The theoretical energies given by this Hamiltonian are in the right-hand columns.

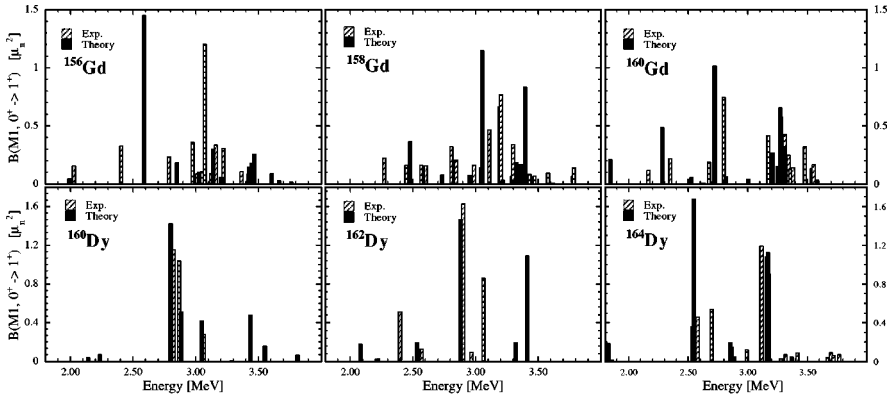

FIG. 2. A comparison of experimental (crosshatched bars) [7] and theoretical (solid bars) $M1$ strength distributions. In each case the eigenstates were determined by fitting parameters in the Hamiltonian to the experimental energy spectrum, Fig. 1, and associated $B(E2)$ transitions.

SU(3) irreps with either λ or μ odd relative to those with λ and μ even, for which $a_{\rm sym}$ is zero, as the former belong to different symmetry types (B_α, $\alpha=1,2,3$, rather than A) of the intrinsic Vierergruppe (D_2) [20]. The one-body proton and neutron angular momentum terms $l^2_{i_\pi}$ and $l^2_{i_\nu}$, together with the two-body pairing terms H^π_P and H^ν_P, are SU(3)-symmetry-breaking interactions. Since the quadrupole-quadrupole interaction $Q\cdot Q=4C_2-3L^2$ dominates for deformed nuclei, only basis states with C_2 larger than a certain value are expected to give a significant contribution in the low-energy region.

To select an appropriate set of SU(3) basis functions, one first determines the proton and neutron occupancies by filling pairwise from below the single-particle levels of the appropriate Nilsson Hamiltonian [21]:

$$h=h_{\rm osc}+C\mathbf{l}\cdot\mathbf{s}+D\mathbf{l}^2-m\omega^2r^2\beta Y^2_0. \qquad (3)$$

The deformation β that gives the lowest total energy of the combined proton and neutron system determines the number of valence-space nucleons of each type in their respective normal and unique parity levels, where the latter are intruder states pushed down into the valence space from the next higher shell by the strong spin-orbit interaction. A simplifying assumption made in most pseudo-SU(3) model calculations is that the relevant dynamics can be described by taking into account the nucleons in normal parity sector only [22]; nucleons in intruder states (unique parity sector) are assumed to follow in an adiabatic manner the motion of the nucleons in the normal parity sector with their effect represented through a reparametrization of the theory. For the nuclei investigated here, the occupation numbers and the corresponding deformation β are given in Table I. All proton and neutron SU(3) irreps consistent with these occupancies and having $C_2\geqslant C_{2_{\rm min}}$, where $C_{2_{\rm min}}$ was set so as to include all irreps lying below ≈ 6 MeV, were included in the analysis. The basis was then built by taking all possible SU(3) couplings of these proton and neutron irreps.

The parameters for the Hamiltonian were determined through a fitting procedure that included all $J\leqslant 8$ levels up through 2 MeV and their respective $B(E2)$ transition strengths (Table II). This procedure gave very good agreement between theory and experiment (Fig. 1 and Table III), and served to confirm our use of the model in this mass and energy region.

The $M1$ strength distributions derived from the eigenvectors [15] are shown, along with the corresponding experimental results [7], in Fig. 2. Key features of these strength distributions are easy to understand within the framework of the pseudo-SU(3) model. The basic structure of the strength distribution is determined by the SU(3)-symmetry-preserving part of the Hamiltonian,

$$H_{\rm SU(3)}=-(a_2+a_{\rm sym})C_2+a_3C_3+bK^2_J+cJ^2, \qquad (4)$$

which embodies strong selection rules [15]. Specifically, in this limit there are no couplings between different SU(3) irreps and there are at most four $M1$ transitions between 1^+ states and the 0^+ ground state. For the nuclei under investigation, these are identified in Table IV. From this pure SU(3) limit, which allows for an interpretation in terms of collective degrees of freedom of a two-rotor model [23,24], one expects three clusters of transitions, one doubly degenerate due to a multiplicity in the SU(3) coupling, for ^{160}Gd, ^{162}Dy, and ^{164}Dy, and two for the other nuclei, a prediction that seems to be in agreement with the experimental results.

TABLE IV. $B(M1)$ transition strengths ($[\mu^2_N]$) in the pure pseudo-SU(3) limit. The strong coupled pseudo-SU(3) irrep $(\lambda,\mu)_{\rm g.s.}$ for the ground state and the irreps associated with the 1^+ states, $(\lambda',\mu')_{1^+}$, to which $M1$ transitions are possible are also given for each nuclei. A superscript denotes the multiplicity with which the irrep occurs. Note that when there is a multiplicity, the transition strength to the second ($\rho=2$) irrep is much smaller than to the first one ($\rho=1$). This demonstrates, for the first time, physics associated with a resolution of the outer SU(3) multiplicity.

Nucleus	$(\lambda,\mu)_{\rm g.s.}$	$(\lambda',\mu')_{1^+}$	$B(M1,0^+_1\rightarrow 1^+)[\mu^2_N]$
^{156,158}Gd,^{160}Dy	(28,4)	(26,5)	1.91
		(27,3)	1.61
^{160}Gd,^{162}Dy	(28,8)	(26,9)	1.77
		$(27,1)^1$	1.82
		$(27,1)^2$	0.083
		(29,6)	0.56
^{164}Dy	(30,8)	(28,9)	1.83
		$(27,1)^1$	1.88
		$(27,1)^2$	0.090
		(31,6)	0.56

The underlying SU(3) structure of the scissors mode is confirmed by the fact that the total $M1$ transition strength is close to the experimental result (Table III). However, because the calculated $M1$ strengths are concentrated in only two or three states, it fails to reproduce the observed fragmentation of the strength.

By including SU(3)-symmetry-breaking terms in the Hamiltonian, namely, the one-body proton and neutron angular momentum operators $l^2_{i_{\pi,\nu}}$ and the two-body pairing terms $H^{\pi,\nu}_P$, this simple theory gives way to one that includes a breakup of the $M1$ strength into relatively closely packed levels centered around the sharp peaks of the pure SU(3) limit of the theory. Within the pseudo-SU(3) model, fragmentation of the $M1$ strength is caused by symmetry-breaking terms in the interaction. In particular, it seems that pairing is essential for a proper description of the fragmentation of the $M1$ strength. A noteworthy feature is that the rotational structure of the low-energy spectrum given by the pure-SU(3) model survives the mixing induced by the pairing.

As a consequence of the symmetry breaking, one finds a number of transitions that in general are close to the experimentally observed ones, varying from 5 for ^{162} Dy to 11 for ^{156} Gd. Also, for most of the nuclei considered, the centroid of the experimental and theoretical $M1$ transition strength distribution lies at about the same energy, and so good agreement between theory and experiment is obtained. The total $M1$ strength, which for the full Hamiltonian is a bit lower then for its pure SU(3) limit due to interference generated by the mixing (Table III), also shows a reasonable reproduction of the experimental data, in most cases slightly underestimat-

ing them. A possible explanation for this discrepancy is missing spin-1 admixtures in the wave functions, as these are known [25] to play an important role, especially for ^{164}Dy where the largest deviation from the experimental value is found. This shortcoming will be addressed in a future publication since the model is currently being extended to include explicitly spin-1 configurations.

To summarize, the collective properties of strongly deformed nuclei, as seen through their rotational spectra and enhanced $E2$ transitions, and the structure of their $M1$ transition strength distributions are modified, but not destroyed, by including noncollective one-body and two-body parts in the interaction. In particular, the observed fragmentation of the $M1$ strength seems to demand pairing, even though the amount required does not wipe out the collective rotational features of these nuclei. The pseudo-SU(3) model gives a good description of the dynamics. An extension of the theory to the case when spin-1 contributions [6] play an important role and for odd-A nuclei (half-integer spin) is underway.

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[1] N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. **41**, 1532 (1978).
[2] D. Zawischa and J. Speth, Z. Phys. A **339**, 97 (1991).
[3] C. de Coster and K. Heyde, Nucl. Phys. **A529**, 507 (1991).
[4] R. Nojarov, Nucl. Phys. **A571**, 93 (1994).
[5] P. von Neumann-Cosel, J. N. Ginocchio, H. Bauer, and A. Richter, Phys. Rev. Lett. **75**, 4178 (1995)
[6] V. G. Soloviev, A. V. Sushkov, N. Yu. Shirikova, and N. Lo Iudice, Nucl. Phys. **A600**, 155 (1996).
[7] U. Kneissl, H. H. Pitz, and A. Zilges, Prog. Part. Nucl. Phys. **37**, 349 (1996).
[8] W. Ziegler, N. Huxel, P. von Neumann-Cosel, C. Rangacharyulu, A. Richter, C. Spieler, C. de Coster, and K. Heyde, Nucl. Phys. **A564**, 366 (1993).
[9] A. Richter, Prog. Part. Nucl. Phys. **34**, 261 (1995).
[10] D. Bohle, A. Richter, W. Steffen, A. E. L. Dieperink, N. Lo Iudice, F. Palumbo, and O. Scholten, Phys. Lett. **137B**, 27 (1984).
[11] K. T. Hecht and A. Adler, Nucl. Phys. **A137**, 129 (1969).
[12] R. D. Ratna Raju, J. P. Draayer, and K. T. Hecht, Nucl. Phys. **A202**, 433 (1973).
[13] J. Hirsch, O. Castaños, P. O. Hess, and O. Civitarese, Phys. Rev. C **51**, 2252 (1995).
[14] D. Troltenier, W. Nazarewicz, Z. Szymański, and J. P. Draayer, Nucl. Phys. **A567**, 591 (1994).
[15] O. Castaños, J. P. Draayer, and Y. Leschber, Ann. Phys. (N.Y.) **180**, 290 (1987).
[16] C. Bahri and J. P. Draayer, Comput. Phys. Commun. **83**, 59 (1994).
[17] D. Troltenier, C. Bahri, and J. P. Draayer, Nucl. Phys. **A586**, 53 (1995).
[18] C. Bahri, J. P. Draayer, and S. A. Moszkowski, Phys. Rev. Lett. **68**, 2133 (1992).
[19] A. L. Blokhin, T. Beuschel, J. P. Draayer, and C. Bahri, Nucl. Phys. **A612**, 163 (1997).
[20] Y. Leschber, Hadronic J. Suppl. **3**, 1 (1987).
[21] S. G. Nilsson, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. **29**, No. 16, 1 (1955).
[22] J. Escher, J. P. Draayer, and A. Faessler, Nucl. Phys. **A586**, 73 (1995).
[23] D. Rompf, J. P. Draayer, D. Troltenier, and W. Scheid, Z. Phys. A **354**, 359 (1996).
[24] D. Rompf, Ph.D. thesis, Inst. für Theoretische Physik der Justus-Liebig-Universität, Gießen, Germany, 1997.
[25] D. Frekers, D. Bohle, A. Richter, R. Abegg, R. E. Azuma, A. Celler, C. Chan, T. E. Drake, K. P. Jackson, J. D. King, C. A Miller, R. Schubank, J. Watson, and S. Yen, Phys. Rev. Lett. **58**, 658 (1989).