Dynamical effects of deformation in the coupled two-rotor system

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The coupling between the rotational and shape degrees of freedom in soft nuclei is studied using a constrained time-dependent Hartree-Fock approach. The dynamical equations are derived using the time-dependent variational principle on a restricted trial manifold consisting of cranked squeezed states. This procedure is applied to describe the low-lying isovector magnetic excitations observed recently in the γ-soft nucleus ^{134}Ba. The role of shape softness in the fragmentation mechanism of the low-lying $M1$ strength is emphasized. [S0556-2813(98)03303-2]

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I. INTRODUCTION

The prediction of isovector angular rotational oscillations [1] (scissors vibrations) in deformed nuclei has been particularly stimulating for the experimental research on the nuclear magnetism, leading to the discovery of low-lying $M1$ states. These states were observed in high resolution (e,e') and (γ,γ') scattering experiments on rare earths [2], fp shell nuclei [3], and actinides [4]. The apparent weak excitation of these states in intermediate energy proton scattering [5] has supported the orbital character predicted by the two rotor model (TRM), but the highly fragmented structure [6] has generated a long standing debate about their real origin. Thus, the phenomenological TRM predicts one strong $M1$ state, which may be splitted in two by triaxiality [7,8], while the microscopic random-phase approximation (RPA) or quasi-RPA (QRPA) calculations show the occurrence of several $M1$ excitations produced by only few quasiparticle pairs [9]. The comparison between these results requires a reliable procedure to find a geometrical interpretation of the RPA excitations, but this fundamental problem of the many-body theory has not yet been completely solved. In the case of $M1$ states the main difficulty concerns the appropriate choice of the ''angle operators'' [10,11], required beside the angular momenta to construct the 1^+ quasiboson excitation operator. Depending on this choice, the scissors vibration may appear as a rigid angular oscillation [12–14], or as occurring by a shear motion [15]. An alternative approach to the TRM dynamics, avoiding the definition of the angle operators, is provided by algebraic models as IBA-II [16], based on pure boson generators, or SU(3) models [17,18] and pseudo-SU(3) models [19], where the generators can be expressed in terms of the fermion creation and annihilation operators.

The interest for the scissors modes has been renewed during the last few years by the increasing amount of data and results obtained in the recent experimental and theoretical investigations. The measurements of the $M1$ strength along a chain of even Sm [20] and Nd [21] isotopes has shown a quadratic dependence on the ground state quadrupole deformation. This effect is considered a strong argument supporting the TRM origin of the low-lying $M1$ states, being encountered in all semiclassical models [22]. The energy weighted sum rule for the $M1$ operator indicates also a splitting of the $B(M1)$ strength in low and high energy components if $\Delta N=2$ shell-model configurations are allowed [23].

The assumption of a fixed intrinsic deformation is justified for the nuclei known to be good rotors, but for soft nuclei, the shape degrees of freedom should be considered as dynamical variables rather than as fixed parameters. The measurements of the $B(E2)$ values along the chain of the $A=124$–132 Ba isotopes [24] have shown an increase in the triaxial shape asymmetry with the angular momentum. Also, an increase of the β deformation with the rotational frequency was observed in ^{126}Ba [25] for spins between $4\hbar$ and $10\hbar$.

Complementary to these results are the recent measurements of the low-lying $M1$ spectrum of the γ-soft nucleus ^{134}Ba in a high resolution photon scattering experiment [26]. These new data may shed some light on the dynamical interplay between the rotational and the shape degrees of freedom not only during isoscalar rotations, but also during the rotational oscillations of the protons against neutrons.

In this work the coupling between the angular oscillations and the shape dynamics is studied within a restricted time-dependent Hartree-Fock (TDHF) approach. The constrained dynamical equations are obtained using the time-dependent variational principle on trial manifolds constructed by cranking. This formalism ensures a clear geometrical interpretation of the time-dependent solutions particularly when the one-body component of the nuclear Hamiltonian is approximated by a harmonic oscillator term, and it will be presented in Sec. II. The numerical results concerning the coupling between the scissors vibrations and the shape degrees of freedom in ^{134}Ba are presented in Sec. III. The main results and the conclusions are summarized in Sec. IV.

II. RESTRICTED TDHF DYNAMICS FOR DEFORMED NUCLEI

Let us assume that $\mathcal{L}$ is the many-body Hilbert space, H is the microscopic Hamiltonian, and $\mathcal{M}=\{|Z(X)\rangle\}$ is a $2N$-dimensional trial manifold of normed functions, parametrized by the variables $X=\{x^i\}$, $i=1,2N$. If the antisymmetric matrix $\omega=[\omega_{ij}(X)]$, $\omega_{ij}(X)=2\,\mathrm{Im}\langle\partial_i Z|\partial_j Z\rangle$ defines a symplectic form on $\mathcal{M}$ (Poisson bracket), then the functional

$$\mathcal{J}[X]=\int\langle Z|i\partial_t - H|Z\rangle dt \tag{1}$$

is stationary at small variations δX when X is the solution of the quasiclassical Hamilton equations

$$\sum_{j=1}^{2N} \dot{x}^j \omega_{jk}(X) = \frac{\partial \langle Z|H|Z\rangle}{\partial x^k}. \tag{2}$$

The Hartree-Fock equations are obtained when $\mathcal{M}$ is the manifold $\mathcal{S}$ of all Slater determinants which are generated from a given determinant $|\Psi_0\rangle$ by a unitary change of the single-particle (SP) basis. This manifold has a complicated structure, accounting for many degrees of freedom of the nucleus. Therefore, in the present study $\mathcal{S}$ will be restricted to a submanifold parametrized only by the phase space coordinates relevant for the rotational and shape dynamics, constructed by the cranking procedure [27].

The static deformation parameters δ and γ are introduced both in the collective and microscopic models of the nucleus [28], being used to characterize the spontaneous breaking of the rotational symmetry of the nuclear mean field. If $|Z_0\rangle$ denotes a symmetry breaking ground state, and

$$Q_\mu = \sum_{i=1}^{A} (r^2 Y_{2\mu})_i \tag{3}$$

are the quadrupole operators defined with

$$r^2 Y_{20} = \sqrt{\frac{5}{16\pi}}(2z^2 - x^2 - y^2),$$

$$r^2 Y_{21} = -r^2 (Y_{2-1})^* = -\sqrt{\frac{15}{8\pi}} z(x+iy),$$

$$r^2 Y_{22} = r^2 (Y_{2-2})^* = \sqrt{\frac{15}{32\pi}}(x+iy)^2,$$

then the deformation parameters can be expressed in terms of the expectation values $\langle Q_\mu\rangle = \langle Z_0|Q_\mu|Z_0\rangle$ by using the relations [28]

$$\delta = 3\frac{\langle Q_0\rangle}{\langle r^2\rangle}, \quad \tan\gamma = \sqrt{2}\frac{\langle Q_2\rangle}{\langle Q_0\rangle}. \tag{4}$$

Let us denote by $\mathcal{G}_x$ the group of rotations around the X axis and by L_x the orbital angular momentum operator for protons or neutrons. Then, the intrinsic ground state of a nucleon system rotating around the X axis with constant angular velocity ω is given by the eigenstate $|Z\rangle_\omega$ of the cranking Hamiltonian $H - \omega L_x$,

$$(H - \omega L_x)|Z\rangle_\omega = E_\omega |Z\rangle_\omega. \tag{5}$$

The set of functions $|Z\rangle_\omega$ contains $|Z_0\rangle$ and represents a curve in $\mathcal{L}$ parametrized by the Lagrange multiplier ω or, implicitly, by the expectation value of the angular momentum $\langle L_x\rangle = {}_\omega\langle Z|L_x|Z\rangle_\omega$. The action of $\mathcal{G}_x$ shifts this curve over a surface in $\mathcal{L}$ which contains the states

$$|Z(\phi,\omega)\rangle = e^{-i\phi L_x}|Z\rangle_\omega \tag{6}$$

and defines a trial manifold $\mathcal{M}_r = \{|Z(\phi,\omega)\rangle\}$.

This trial manifold may be easily constructed in the harmonic oscillator approximation. This approximation was proved to be relevant for the microscopic description of the fusion-fission reactions of light nuclei [29], being successfully used in the Harvey model [30]. Moreover, it provides the microscopic framework for the Elliott's nuclear SU(3) model [31].

Let us assume that $|Z_0\rangle$ is an eigenstate of the linearized many-body Hamiltonian

$$H_L = \sum_{\psi,\psi'} (h_L)_{\psi\psi'} c_\psi^\dagger c_{\psi'}, \tag{7}$$

where h_L is the SP anisotropic harmonic oscillator Hamiltonian, and $c_\psi^\dagger$ (c_ψ) is the creation (annihilation) operator for a nucleon in the SP state ψ. If $h_0 = \Sigma_{\xi=x,y,z}\omega_0(\hat{n}_\xi + 1/2)$, denotes the SP spherical oscillator Hamiltonian, $\hat{n}_\xi = b_\xi^\dagger b_\xi$ the number operator of the oscillator quanta along the ξ axis, $b_\xi^\dagger = \sqrt{m\omega_0/2}(\xi - ip_\xi/m\omega_0)$, and k_ξ is the dimensionless potential operator $k_\xi = m\omega_0\xi^2/2$, then the Hamiltonian

$$h_L = h_0 - \frac{2}{3}\,\omega_0 \sum_{\xi=x,y,z} \delta_\xi k_\xi \tag{8}$$

corresponds to an anisotropic oscillator with frequencies $\omega_\xi = \omega_0\sqrt{1-2\delta_\xi/3}$. When in Eq. (5) $H = H_L$, the solutions $|Z\rangle_\omega$ can be related by unitary transformations to the eigenstates of a spherical harmonic oscillator [12,13]. Thus, if $l_x = i(b_y b_z^\dagger - b_z b_y^\dagger)$ is the SP angular momentum and ω is not larger than $\omega_s\sqrt{3}/2$, $\omega_s = \sqrt{(\omega_y^2 + \omega_z^2)/2}$, then

$$h_L - \omega l_x = \mathcal{C}_\omega h_s \mathcal{C}_\omega^{-1} \tag{9}$$

with

$$\mathcal{C}_\omega = e^{-i\lambda c_x} e^{\Sigma_{\xi=x,y,z}\theta_\xi(d_\xi^\dagger - d_\xi)/2} \tag{10}$$

an unitary transformation generated by the operators c_x, $d_\xi^\dagger$, and d_ξ,

$$c_x = b_y^\dagger b_z + b_z^\dagger b_y, \quad d_\xi^\dagger = b_\xi^\dagger b_\xi^\dagger, \quad d_\xi = b_\xi b_\xi. \tag{11}$$

In the right-hand side of Eq. (9) $h_s = \Sigma_\xi \Omega_\xi(\tilde{b}_\xi^\dagger \tilde{b}_\xi + 1/2)$ with $\tilde{b}_\xi^\dagger = \sqrt{m\omega_s/2}(\xi - ip_\xi/m\omega_s)$ while the parameters λ, θ_ξ of $\mathcal{C}_\omega$ and Ω_ξ are given by $\tan 2\lambda = 2\omega/\omega_s\eta$, $\sinh 2\theta_\xi = \omega_s(1-\omega_\xi^2/\omega_s^2)/2\Omega_\xi$, $\eta = (\omega_y^2 - \omega_z^2)/2\omega_s^2$, $\Omega_x = \omega_x$, $\Omega_{y,z}^2 = (\omega_s + \epsilon_{y,z})^2 - (\omega_s\eta/2)^2$, with $\epsilon_y = -\epsilon_z = \omega_s\eta/2\cos 2\lambda$.

The operator c_x acts within a single oscillator shell, while $d_\xi^\dagger$, d_ξ change the number of oscillator quanta by 2. If $\omega = 0$, then $\Omega_\xi = \omega_\xi$ and the operator $\mathcal{C}_0$ generates the transition from the eigenstates of a spherical oscillator with frequency ω_s to an eigenstate of h_L. When $\omega > 0$, $\mathcal{C}_\omega$ produces in addition a shift from the static intrinsic frame to a frame rotating with the angular velocity ω. This shift is generated by c_x, which appears as an ''angle'' operator conjugate to l_x. Analog operators, c_y, c_z are associated to l_y and l_z, and the eight operators $c_x, c_y, c_z, l_x, l_y, l_z, \hat{n}_x - \hat{n}_y, \hat{n}_y - \hat{n}_z$, generate the su(3) algebra. It is interesting to note that this algebra is the same as $\{\vec{L}, Q^a\}$, generated by the angular momentum and the five algebraic quadrupole operators [19]

$$Q^a_\mu = \sqrt{\frac{4\pi}{5b^4}}[r^2 Y_{2\mu}(\vec{r}/|\vec{r}|) + b^4 p^2 Y_{2\mu}(\vec{p}/|\vec{p}|)]. \quad (12)$$

The explicit relation between the generators is provided by the equations

$$c_x = -i\frac{1}{\sqrt{6}}(Q^a_1 - Q^a_{-1}), \quad c_y = \frac{1}{\sqrt{6}}(Q^a_1 + Q^a_{-1}), \quad (13)$$

$$c_z = -i\frac{1}{\sqrt{6}}(Q^a_2 - Q^a_{-2}), \quad \hat{n}_x - \hat{n}_y = \frac{1}{\sqrt{6}}(Q^a_2 + Q^a_{-2}), \quad (14)$$

$$\hat{n}_y - \hat{n}_z = -\frac{1}{2}\left[Q^a_0 + \frac{1}{\sqrt{6}}(Q^a_2 + Q^a_{-2})\right], \quad (15)$$

while the relationship between the relative angle of the proton and neutron axes and the labels of the Clebsch-Gordan series for the product of two su(3) irreducible representations was discussed in [18].

According to Eqs. (6), (9), and (10), the rotational motion around the x axis can be accounted using the trial function manifold $\mathcal{M}_r$ defined by

$$|Z(\phi,\omega)\rangle = e^{-i\phi L_x} e^{-i\lambda C_x} \mathcal{D}|\Psi_0\rangle, \quad (16)$$

where $\mathcal{D}$ is the unitary operator

$$\mathcal{D} = e^{\sum_{\xi=y,z}\theta_\xi (D^\dagger_\xi - D_\xi)/2}. \quad (17)$$

The generators C_x and D_ξ are expressed by

$$C_x = \sum_{\psi,\psi'} (c_x)_{\psi\psi'} c^\dagger_\psi c_{\psi'}, \quad D_\xi = \sum_{\psi,\psi'} (d_\xi)_{\psi\psi'} c^\dagger_\psi c_{\psi'}, \quad (18)$$

and $|\Psi_0\rangle$ is a Slater determinant constructed with eigenstates of the spherical harmonic oscillator with frequency ω_s.

The frequencies ω_y, ω_z of the anisotropic oscillator potential and the related deformations δ_y, δ_z, have been assumed above as fixed parameters. To include them as dynamical variables in Eq. (2) it is necessary to extend the manifold $\mathcal{M}_r$ with new states, parametrized by the corresponding conjugated momentum coordinates [32]. Such states may be constructed similarly to $|Z\rangle_\omega$, by the cranking of h_L with the ''velocity operators'' $i[h_L, k_\xi]$.

This problem may be easily solved by noticing that the set of three operators $\{s_{1,\xi}, s_{2,\xi}, s_{3,\xi}\}$ defined by $s_{1,\xi} = (d^\dagger_\xi + d_\xi)/4$, $s_{2,\xi} = i(d^\dagger_\xi - d_\xi)/4$ and $s_{3,\xi} = (\hat{n}_\xi + 1/2)/2$, generate the su(1,1) algebra [33], and the velocity operator is

$$i[h_L, k_\xi] = 2i\omega_0 [s_{3,\xi}, s_{1,\xi} + s_{3,\xi}] = 2\omega_0 s_{2,\xi}. \quad (19)$$

The cranked Hamiltonian $h_L - \lambda_s s_{2,\xi}$ can be related to h_0 by a unitary transformation generated by $s_{1,\xi}$, and therefore the additional ''shape'' momentum variables can be accounted simply by making the factor θ_ξ of $D^\dagger_\xi$ in Eq. (17) complex, $\theta_y = s_2 \exp(-2i\Phi_2)$ and $\theta_z = s_3 \exp(-2i\Phi_3)$. The extended trial manifold, denoted $\mathcal{M}$, is parametrized by $X \equiv \{\phi, \lambda, s_2, \Phi_2, s_3, \Phi_3\}$, and contains the states

$$|Z(X)\rangle = \mathcal{U}_X |\Psi_0\rangle, \quad (20)$$

where

$$\mathcal{U}_X = e^{-i\phi L_x} e^{-i\lambda C_x} e^{\sum_{\xi=y,z}(\theta_\xi D^\dagger_\xi - \theta^*_\xi D_\xi)/2}. \quad (21)$$

The manifold $\mathcal{M}$ is endowed with the symplectic form

$$\omega_{ij}(X) = 2\,\mathrm{Im}\langle\Psi_0|\partial_i \mathcal{U}^{-1}_X \partial_j \mathcal{U}_X|\Psi_0\rangle. \quad (22)$$

Using the notations $u_k = \cosh s_k$, $v_k = \sinh s_k$, and

$$N_0 = \left\langle \Psi_0 \left| \sum_\psi c^\dagger_\psi c_\psi \right| \Psi_0 \right\rangle,$$

$$\Sigma_\xi = \left\langle \Psi_0 \left| \sum_{\psi,\psi'} (2\hat{n}_\xi + 1)_{\psi\psi'} c^\dagger_\psi c_{\psi'} \right| \Psi_0 \right\rangle, \quad (23)$$

the nonvanishing elements $\omega_{ij}(X) = -\omega_{ji}(X)$ are

$$\omega_{\phi\lambda} = \cos 2\lambda[(u_3^2 + v_3^2)\Sigma_z - (u_2^2 + v_2^2)\Sigma_y],$$

$$\omega_{\phi s_2} = -2u_2 v_2 \Sigma_y \sin 2\lambda,$$

$$\omega_{\phi s_3} = 2u_3 v_3 \Sigma_z \sin 2\lambda,$$

$$\omega_{s_2\Phi_2} = -2u_2 v_2 \Sigma_y, \quad \omega_{s_3\Phi_3} = -2u_3 v_3 \Sigma_z, \quad (24)$$

and can be checked that they satisfy the closing relation

$$\partial_i \omega_{kl} + \partial_l \omega_{ik} + \partial_k \omega_{li} = 0. \quad (25)$$

The matrix $[\omega]$ can be inverted analytically, and for a single rotor the system of Eq. (2) becomes

$$\dot{x}^j = \sum_{k=1}^{6} \rho_{kj}(X) \frac{\partial\langle\psi|H|\psi\rangle}{\partial x^k}, \quad (26)$$

where the only elements of ρ different of zero are

$$\rho_{\phi\lambda} = -1/\omega_{\phi\lambda},$$

$$\rho_{\lambda\Phi_k} = \frac{\omega_{\phi s_k}}{\omega_{\phi\lambda}\omega_{s_k\Phi_k}}, \quad k = 2,3,$$

$$\rho_{s_i\Phi_k} = -\frac{\delta_{ik}}{\omega_{s_i\Phi_i}}, \quad i,k = 2,3. \quad (27)$$

In the case of two soft rotors in interaction, one corresponding to protons $(\tau = p)$ and another to neutrons $(\tau = n)$, the trial function manifold is chosen as a product between $|Z^p(X_p)\rangle$ and $|Z^n(X_n)\rangle$, denoted

$$|Z(X_p, X_n)\rangle = \mathcal{U}^p_{X_p}\mathcal{U}^n_{X_n}|\Psi_0\rangle, \quad (28)$$

such that the Poisson bracket of the proton and neutron variables vanishes. In this case, the dynamics of the whole system is given by two sets of six equations:

$$\dot{x}^j_p = \sum_{k=1}^{6} \rho^p_{kj} \frac{\partial\mathcal{H}}{\partial x^k_p}, \quad \dot{x}^j_n = \sum_{k=1}^{6} \rho^n_{kj} \frac{\partial\mathcal{H}}{\partial x^k_n}, \quad j = 1,\ldots,6 \quad (29)$$

coupled only by the proton-neutron interaction terms of the Hamilton function $\mathcal{H}=\langle Z|H|Z\rangle$.

Let us consider now a nuclear Hamiltonian of the form

$$H=\sum_{\psi,\psi'}(h_0^p)_{\psi\psi'}c^\dagger_{\psi,p}c_{\psi',p}+\sum_{\psi,\psi'}(h_0^n)_{\psi\psi'}c^\dagger_{\psi,n}c_{\psi',n}-\frac{\chi}{2}\sum_{\mu=-2}^{2}(-1)^{2-\mu}(Q_{p,\mu}Q_{p,-\mu}+Q_{n,\mu}Q_{n,-\mu}+2fQ_{p,\mu}Q_{n,-\mu}), \quad (30)$$

where $c^\dagger_{\psi,\tau}$ creates a proton ($\tau=p$) or a neutron ($\tau=n$) in the SP state ψ, and

$$Q_{\tau,\mu}=\sum_{\psi,\psi'}(r^2Y_{2\mu})_{\psi\psi'}c^\dagger_{\psi,\tau}c_{\psi',\tau} \quad (31)$$

is the one-body quadrupole operator. This Hamiltonian consists of two separate one-body harmonic oscillator terms, for protons and for neutrons, and a two-body quadrupole-quadrupole (QQ) interaction with proton-proton, neutron-neutron, and proton-neutron components. The parameters of H are χ and f, related to the strengths of the isoscalar and isovector components of the QQ interaction χ_0, respectively, χ_1. If b denotes the ratio $b=\chi_1/\chi_0$ [34], then $\chi=(1+b)\chi_0$ and $f=(1-b)/(1+b)$.

When the SP basis is $|\psi\rangle=|r\rangle|s\rangle$, with $|r\rangle\equiv|n_x^r n_y^r n_z^r\rangle$ an eigenstate of h_0 in Cartesian representation, and $|s\rangle$ the $j=1/2$ spinor, then the SP energy matrix $\langle\psi|h_0|\psi'\rangle$ is diagonal. Moreover, the calculus of the averages $\langle Z|H|Z\rangle$ can be performed analytically, and the Hamiltonian on the right-hand side of Eq. (29) is

$$\mathcal{H}(X_p,X_n)=\mathcal{H}_{0,p}(X_p)+\mathcal{H}_{0,n}(X_n)+\mathcal{H}_{qq,p}(X_p)+\mathcal{H}_{qq,n}(X_n)+\mathcal{H}_{qq,pn}(X_p,X_n), \quad (32)$$

with

$$\mathcal{H}_{0,\tau}(X_\tau)=\frac{\omega_0}{2}[3(v_2^2+v_3^2)N_0+(u_2^2+v_2^2)\Sigma_y+(u_3^2+v_3^2)\Sigma_z]_\tau,$$

$$\mathcal{H}_{qq,\tau}(X_\tau)\approx-\frac{\chi}{2}(2|\langle Q_{\tau,2}\rangle|^2+2|\langle Q_{\tau,1}\rangle|^2+|\langle Q_{\tau,0}\rangle|^2),$$

$$\mathcal{H}_{qq,pn}(X_p,X_n)\approx-f\frac{\chi}{2}[2\,\mathrm{Re}(\langle Q_{p,2}\rangle\langle Q_{n,-2}\rangle-\langle Q_{p,1}\rangle\langle Q_{n,-1}\rangle)+\langle Q_{p,0}\rangle\langle Q_{n,0}\rangle].$$

The average values $\langle Q_{\tau,\mu}\rangle$ appearing here are given by

$$\langle Q_{\tau,2}\rangle=c_0\sqrt{\frac{3}{32}}(\Sigma_x-|f_2|^2\Sigma_y-|g_2|^2\Sigma_z)_\tau,$$

$$\langle Q_{\tau,1}\rangle=-ic_0\sqrt{\frac{3}{32}}[2(g_3^*f_2-f_3^*g_2)N_0+(g_3f_2^*+g_3^*f_2)\times(\Sigma_y-N_0)-(f_3g_2^*+f_3^*g_2)(\Sigma_z-N_0)]_\tau,$$

$$\langle Q_{\tau,0}\rangle=c_0[2(|g_3|^2\Sigma_y+|f_3|^2\Sigma_z)-\Sigma_x-|f_2|^2\Sigma_y-|g_2|^2\Sigma_z)_\tau,$$

where $c_0=(5/4\pi)^{1/2}\hbar/m\omega_0$ and f_2, f_3, g_2, g_3 depend on the coordinates X according to

$$f_2=\alpha u_2+\alpha^*v_2e^{-2i\Phi_2},\quad f_3=\alpha^*u_3+\alpha v_3e^{-2i\Phi_3}, \quad (33)$$

$$g_2=\zeta u_3+\zeta^*v_3e^{-2i\Phi_3},\quad g_3=\zeta^*u_2+\zeta v_2e^{-2i\Phi_2}, \quad (34)$$

with α and ζ the complex functions of λ and ϕ given by Eq. (A4) in the Appendix.

III. NUMERICAL RESULTS

The $B(M1)$ strength measured for the γ-soft nucleus ^{134}Ba in a recent high resolution photon scattering experiment [26] is distributed on several fragments at energies between 2.5 and 4 MeV. The total $M1$ strength summed over all states is $0.56\pm0.004\mu_N^2$, and has the centroid located at $E_{\mathrm{av}}=2.99$ MeV. The strongest transitions appear from the states located at 2.571, 2.939, and 3.334 MeV to the 0_1^+ ground state. Decays with lower intensities of these three states occur to the low-lying $2^+_{1,2}$ states, placed at the excitation energies $E(2_1^+)=0.6$ MeV and $E(2_2^+)=1.17$ MeV. For the neutron-deficient isotopes of Ba the ground state energy as a function of deformation has a double-well shape [36–38], with a stable and isomeric minimum at oblate, respectively, prolate deformations.

In the present calculation we considered a core of 40 protons and 40 neutrons, distributed over the $N=0,1,2,3$ shells of the harmonic oscillator potential. The state $|\Psi_0\rangle\equiv|\Psi_0^p\rangle|\Psi_0^n\rangle$ of Eq. (28) contains the remaining 16 protons and 38 neutrons, distributed on energy levels from the $N=4$ and $N=5$ oscillator shells. The level ordering was determined assuming that $\omega_x<\omega_y<\omega_z$. The 16 protons of $|\Psi_0^p\rangle$ are placed on the five levels (n_x,n_y,n_z) of the $N=4$ shell having $n_z=0$, and on (3,0,1), (2,1,1), (5,0,0). The 38 neutrons of $|\Psi_0^n\rangle$ are distributed on the 12 levels of the $N=4$ shell having $n_z=0,1,2$, on two levels with $n_z=3$, (1,0,3), (0,1,3) and on five levels of the $N=5$ shell, (5,0,0), (4,1,0), (2,3,0), (3,2,0), (1,4,0). The contribution of the core levels is accounted by adding 40 to N_0 and 100 to Σ_ξ given by Eq. (23). The oscillator frequency is $\omega_0=41A^{-1/3}$ MeV, the interaction strength χ_0 was fixed such that $\chi_0c_0^2=55.16A^{-5/3}$ MeV [36], and $b=-0.4$, close to the value given in Ref. [34].

The ground state $|Z_0\rangle$ of the system was obtained by the method of frictional cooling [39] consisting in solving Eq. (29) with artificial dissipative terms on the right-hand side. If the trajectory is calculated for long enough time, then the asymptotic state $|Z_0\rangle=\mathcal{D}_0^p\mathcal{D}_0^n|\Psi_0\rangle$ is independent on the initial conditions and close to the true ground state.

This method shows that the Hamiltonian of Eq. (32) has no bounded minimum, because at large deformations the contribution of the QQ interaction energy is large and negative, making the system unstable. This instability is related to the volume conservation condition [40], not considered here. However, in the physical region of oblate deformations there is a well-defined metastable ground state (MGS). For this state the deformation parameters defined by Eq. (4) are $\delta_p=-0.265$, $\gamma_p=-25°$, $\delta_n=-0.225$, $\gamma_n=-16.2°$. The expected value of the total quadrupole moment in the MGS $|Z_0\rangle$ is $\langle Q_0\rangle=\langle Q_{p,0}+Q_{n,0}\rangle=322$ fm^2, close to the value

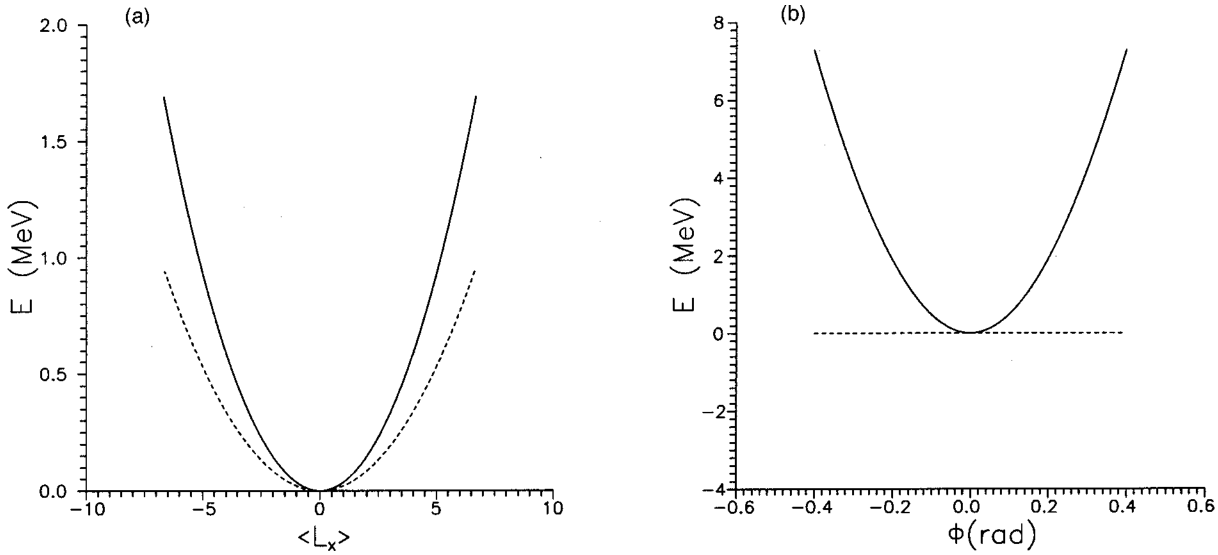

FIG. 1. The energy as a function of the angular momentum for protons [(a), solid] and neutrons [(a), dashed], and on the angular variable ϕ when $\phi_p=-\phi_n=\phi/2$ [(b), solid], and $\phi_p=\phi_n=\phi$ [(b), dashed].

~ 310 fm^2 given by the average formula $\langle Q_0\rangle_{\rm av}=\sqrt{5/36\pi}A\langle r^2\rangle_0\delta_0=0.36\delta_0 A^{5/3}$ fm^2 [35] for $A=134$ and $\delta_0=(\delta_p+\delta_n)/2$. Also, for $|Z_0\rangle$ the self-consistency equation

$$\frac{\delta_0}{3}=\frac{\chi_0 c_0}{\hbar\omega_0}\langle Q_0\rangle \tag{35}$$

is fulfilled with good accuracy.

If the deformation parameters are fixed to the MGS values, then $|Z(X_p,X_n)\rangle$ and the Hamiltonian $\mathcal{H}(X_p,X_n)$ become functions of only four variables, denoted by $|Z^r(\phi_p,\lambda_p,\phi_n,\lambda_n)\rangle$ and $\mathcal{H}^r(\phi_p,\lambda_p,\phi_n,\lambda_n)$. The parameters λ_τ appearing here may be further expressed as functions of the angular momentum average by using the implicit equation

$$\langle Z|L_x^\tau|Z\rangle=\frac{\sin 2\lambda_\tau}{2}[(u_3^2+v_3^2)\Sigma_z-(u_2^2+v_2^2)\Sigma_y]_\tau. \tag{36}$$

After this change of parametrization $\mathcal{H}^r(0,\lambda_p,0,0)$ and $\mathcal{H}^r(0,0,0,\lambda_n)$ become functions of $L_\tau=\langle Z|L_x^\tau|Z\rangle$, $\tau=p,n$. These functions are represented in Fig. 1(a) by solid, respectively, dashed lines, with the zero point of the energy scale fixed at $\mathcal{H}^r(0,0,0,0)$. This functional dependence results as

$$\mathcal{H}^r(0,\lambda_p,0,\lambda_n)\approx\frac{L_p^2}{2I_p}+\frac{L_n^2}{2I_n} \tag{37}$$

with the "dynamical" moments of inertia $I_p\approx 16.4$ MeV^{-1}, $I_n\approx 27$ MeV^{-1}. These values are close to the ones provided by the Inglis formula [28]: $I_p^{\rm Inglis}=17.4$ MeV^{-1}, $I_n^{\rm Inglis}=31.5$ MeV^{-1}.

The functions $V(\phi)\equiv\mathcal{H}^r(\phi/2,0,-\phi/2,0)$ and $\mathcal{H}^r(\phi,0,\phi,0)$ are represented in Fig. 1(b) by solid, respectively, dashed lines. The function $V(\phi)$ is close to a harmonic potential $C\phi^2/2$ with $C=96$ MeV, interpreted as the elastic constant of the restoring force between the rotor axes determined by the isovector QQ interaction. The dashed line in Fig. 1(b) shows that despite the occurrence of a deformed vacuum defining the intrinsic frame, the Hamiltonian $\mathcal{H}^r$ remains invariant to arbitrary rotations around the x axis in the laboratory frame.

According to these results, at fixed deformation the Hamilton function $\mathcal{H}^r$ describes a system of two rigid rotors having the moments of inertia I_p,I_n, and interacting by a restoring elastic potential $C(\phi_p-\phi_n)^2/2$. For this system the shape degrees of freedom are frozen, and Eq. (29) for 12 variables reduces to only four equations:

$$\dot\phi_p=\rho^p_{\lambda,\phi}\frac{\partial\mathcal{H}^r}{\partial\lambda_p},\quad \dot\lambda_p=\rho^p_{\phi,\lambda}\frac{\partial\mathcal{H}^r}{\partial\phi_p}, \tag{38}$$

and

$$\dot\phi_n=\rho^n_{\lambda,\phi}\frac{\partial\mathcal{H}^r}{\partial\lambda_n},\quad \dot\lambda_n=\rho^n_{\phi,\lambda}\frac{\partial\mathcal{H}^r}{\partial\phi_n}. \tag{39}$$

Moreover, from the definitions of ω_{ij} and ρ_{ij} one can easily see that

$$\rho^2_{\lambda,\phi}=\frac{\partial\lambda_\tau}{\partial L_\tau}, \tag{40}$$

and therefore by changing the coordinates λ_τ to L_τ, Eqs. (38) and (39) take the canonical form with the Hamiltonian

$$\mathcal{H}^r(\phi_p,L_p,\phi_n,L_n)=\frac{L_p^2}{2I_p}+\frac{L_n^2}{2I_n}+\frac{C}{2}(\phi_p-\phi_n)^2. \tag{41}$$

The isoscalar and the isovector degrees of freedom can be further separated by the change of coordinates [41]

$$L_{\rm is}=L_p+L_n,\quad \phi_{\rm is}=\frac{I_p\phi_p+I_n\phi_n}{I_p+I_n} \tag{42}$$

and

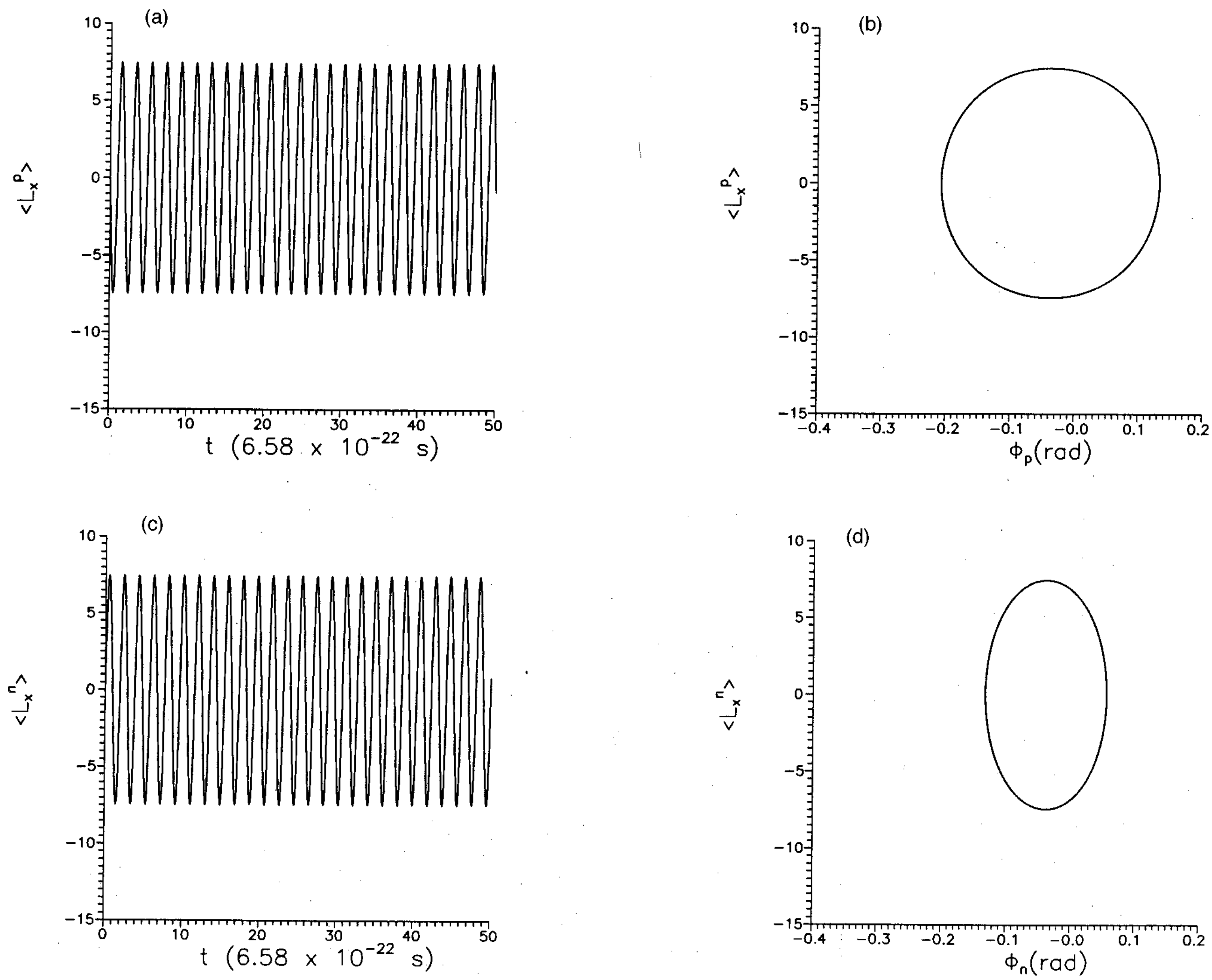

FIG. 2. The expectation value of the angular momentum L_τ as a function of time (a), (c), and the phase-space orbits $\{\phi_\tau(t), L_\tau(t)\}$ (b), (d) for the BWS quantized orbit of scissors vibration in the rigid two-rotor system.

$$L_{\rm iv}=\frac{I_n L_p - I_p L_n}{I_p+I_n}, \quad \phi_{\rm iv}=\phi_p-\phi_n. \tag{43}$$

The frequency Ω of the small amplitude isovector oscillations is determined by the constant C and by the reduced moment of inertia $I_r=I_p I_n/(I_p+I_n)$,

$$\Omega=\sqrt{\frac{C}{I_r}}, \tag{44}$$

and has the value $\Omega=3.1$ MeV.

According to the requantization formalism [42], the closed vibrational orbits $\mathcal{O}_n$ selected by the Bohr-Wilson-Sommerfeld (BWS) integrality condition

$$\left| \oint L_{\rm iv} d\phi_{\rm iv} \right| = 2\pi n, \quad n=1,2,3,\ldots, \tag{45}$$

are related to the eigenstates of the many-body system. For $n=1$ and small amplitudes this condition corresponds to the normalization of the RPA quasiboson operators [27]. In the present case it selects an orbit $\mathcal{O}$ with the excitation energy $E_{\mathcal{O}}=3.27$ MeV. This orbit was determined by the numerical integration of Eqs. (38) and (39) with the initial conditions $\phi_p=-\phi_n=0.132$ rad, and $L_p=L_n=0$. The time-evolution of the average angular momenta L_τ, and the phase-space orbits $\{\phi_\tau(t), L_\tau(t)\}$ are represented in Figs. 2(a), 2(b) for protons and in Figs. 2(c), 2(d) for neutrons.

In the system of Eqs. (38) and (39) the shape variables are frozen at the MGS values, and the dynamics was restricted only to the rotational degrees of freedom. If the initial conditions are the same as used to calculate the orbit $\mathcal{O}$, but instead of Eqs. (38) and (39), the whole system of Eq. (29) is integrated, then the angular and shape variables have the time evolution presented in Figs. 3 and 4, respectively. These results indicate the occurrence of a complex dynamical pattern of anharmonic oscillations both for the rotational and the shape degrees of freedom. Thus, for the calculation of the excitation energies and of the $B(M1)$ spectrum a separation of the normal modes of oscillations becomes necessary.

When the oscillation amplitude for the parameters X_p, X_n is small the time-dependent state $|Z\rangle$ in Eq. (28) should take the form

$$|Z(t)\rangle = e^{\Sigma_\nu e^{-i\Omega_\nu t} B_\nu^\dagger - e^{i\Omega_\nu t} B_\nu} |Z_0\rangle, \tag{46}$$

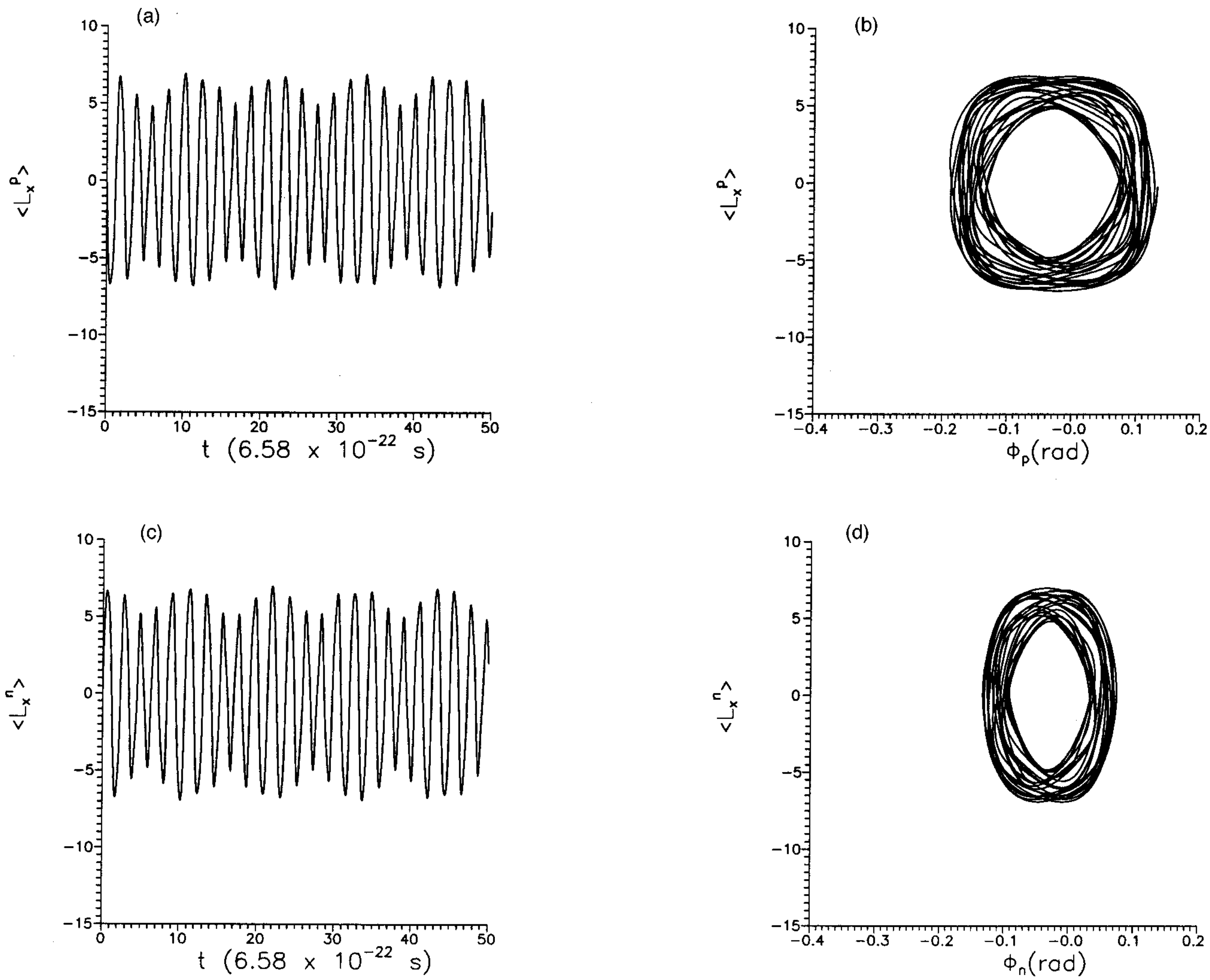

FIG. 3. The expectation value of the angular momentum L_τ as a function of time (a), (c), and the phase-space orbits $\{\phi_\tau(t), L_\tau(t)\}$ (b), (d) for the scissors vibration of the soft TRM system. The initial conditions are the same as used for the orbits of Fig. 2.

with Ω_ν the frequencies of the normal modes, and $B_\nu^\dagger$ the corresponding RPA-like excitation operators. Therefore, if $|\widetilde{Z}_0\rangle$ denotes the correlated ground state defined by $B_\nu|\widetilde{Z}_0\rangle = 0$, then the $B(M1)$ strength of the state $B_\nu^\dagger|\widetilde{Z}_0\rangle$ is

$$B(M1)_{\nu,x} = \frac{|\langle \widetilde{Z}_0 | M_{\mathrm{iv},x} B_\nu^\dagger | \widetilde{Z}_0 \rangle|^2}{|\langle \widetilde{Z}_0 | [B_\nu, B_\nu^\dagger] | \widetilde{Z}_0 \rangle|}, \tag{47}$$

with

$$M_{\mathrm{iv},x} = (g_p - g_n)\sqrt{\frac{3}{16\pi}}(L_x^p - L_x^n)\mu_N \tag{48}$$

the x component of the isovector $M1$ operator. The matrix element $\langle \widetilde{Z}_0 | (L_x^p - L_x^n) B^\dagger | \widetilde{Z}_0 \rangle$ and the normalization factor $|\langle \widetilde{Z}_0 | [B, B^\dagger] | \widetilde{Z}_0 \rangle|$ can be expressed in terms of the Fourier amplitudes $\mathcal{M}_\Omega$ and $\mathcal{N}_\Omega$ of the time-dependent functions

$$\mathcal{M}(t) = \langle Z_0 | (L_x^p - L_x^n) | Z(t) \rangle$$
$$\approx \sum_\nu \langle Z_0 | (L_x^p - L_x^n)(e^{-i\Omega_\nu t} B_\nu^\dagger - e^{i\Omega_\nu t} B_\nu) | Z_0 \rangle \tag{49}$$

and

$$\mathcal{N}(t) = \ln \langle Z(0) | Z(t) \rangle$$
$$\approx \sum_\nu \langle Z_0 | B_\nu B_\nu^\dagger | Z_0 \rangle e^{-i\Omega_\nu t} + \langle Z_0 | B_\nu^\dagger B_\nu | Z_0 \rangle e^{i\Omega_\nu t}. \tag{50}$$

Therefore, if one defines the Fourier series

$$\mathcal{M}(t) = \sum_{\omega > 0} \mathcal{M}_\omega^+ e^{-i\omega t} + \mathcal{M}_\omega^- e^{i\omega t} \tag{51}$$

and

$$\mathcal{N}(t) = \sum_{\omega > 0} \mathcal{N}_\omega^+ e^{-i\omega t} + \mathcal{N}_\omega^- e^{i\omega t} \tag{52}$$

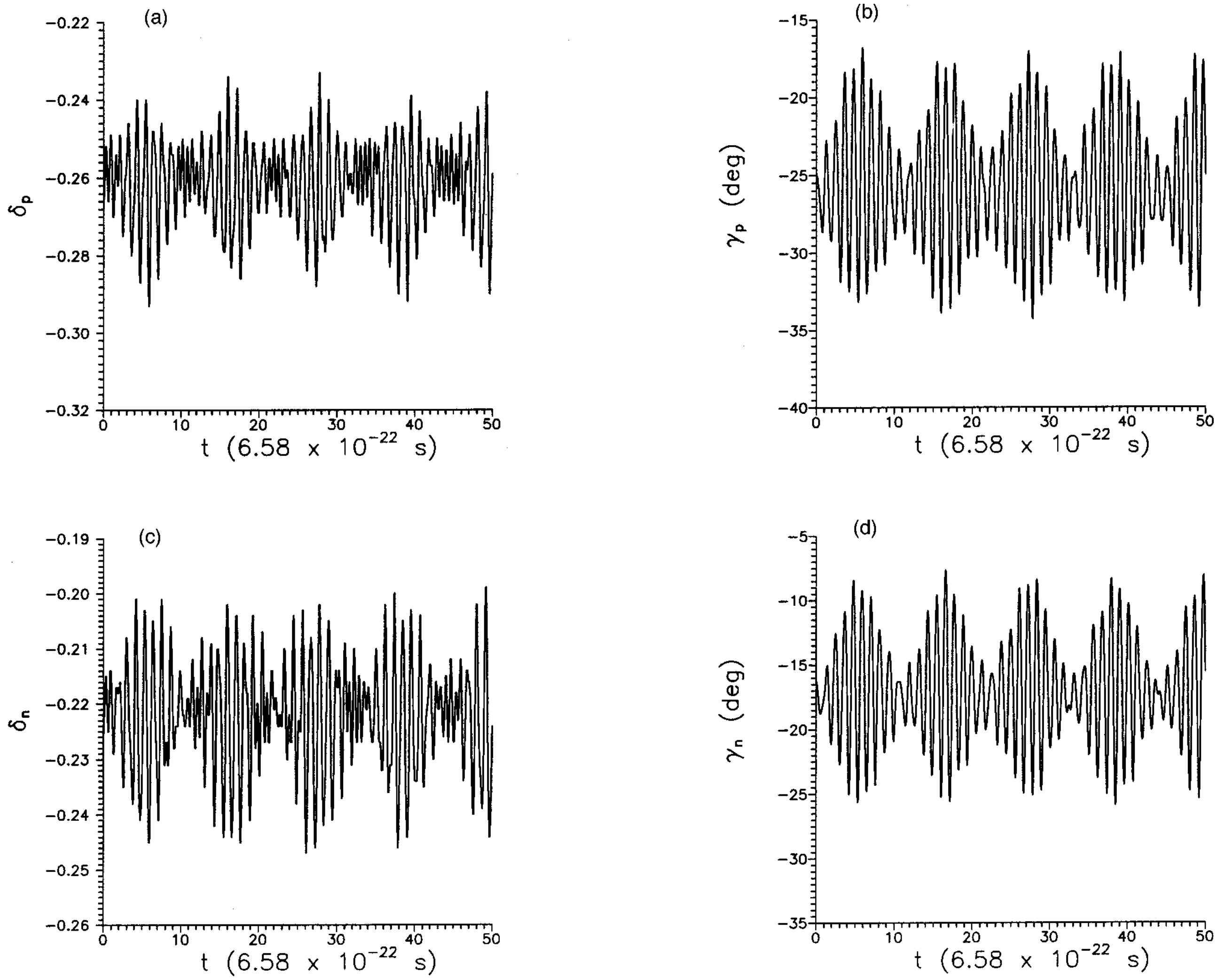

FIG. 4. The deformation variables δ_τ and γ_τ as functions of time for protons (a), (b) and neutrons (c), (d), in the soft TRM system. The initial conditions are the same as used for the orbits of Fig. 2.

then

$$\langle\widetilde{Z}_0|(L_x^p-L_x^n)B_\nu^\dagger|\widetilde{Z}_0\rangle\approx\mathcal{M}^+_{\Omega_\nu} \tag{53}$$

and

$$|\langle\widetilde{Z}_0|[B_\nu,B_\nu^\dagger]|\widetilde{Z}_0\rangle|\approx|\mathcal{N}^+_{\Omega_\nu}-\mathcal{N}^-_{\Omega_\nu}|. \tag{54}$$

Using these amplitudes, and assuming that the frequencies and the $B(M1)$ values for the angular oscillations around the y axis are the same, the total strength for the ν mode with the frequency Ω is

$$B(M1)=2B(M1)_{\nu,x}=\frac{3}{8\pi}(g_p-g_n)^2\frac{|\mathcal{M}^+_\Omega|^2}{|\mathcal{N}^+_\Omega-\mathcal{N}^-_\Omega|}\mu_N^2. \tag{55}$$

The functions $\mathcal{M}(t)$ and $\mathcal{N}(t)$ of Eqs. (51) and (52) can be calculated as averages in the many-fermion state $|\Psi_0\rangle$ constructed with SP spherical harmonic oscillator eigenstates, such that

$$\begin{aligned}\mathcal{M}(t)&=\langle\Psi_0|(\mathcal{D}_0^p\mathcal{D}_0^n)^{-1}(L_x^p-L_x^n)\mathcal{U}_p(t)\mathcal{U}_n(t)|\Psi_0\rangle\\&=\langle\Psi_0^p|\mathcal{D}_0^{p-1}L_x^p\mathcal{U}_p(t)|\Psi_0^p\rangle\langle\Psi_0^n|\mathcal{D}_0^{n-1}\mathcal{U}_n(t)|\Psi_0^n\rangle\\&\quad-\langle\Psi_0^n|\mathcal{D}_0^{n-1}L_x^n\mathcal{U}_n(t)|\Psi_0^n\rangle\langle\Psi_0^p|\mathcal{D}_0^{p-1}\mathcal{U}_p(t)|\Psi_0^p\rangle,\end{aligned} \tag{56}$$

and

$$\mathcal{N}(t)=\langle\Psi_0^p|\mathcal{U}_p^{-1}(0)\mathcal{U}_p(t)|\Psi_0^p\rangle\langle\Psi_0^n|\mathcal{U}_n^{-1}(0)\mathcal{U}_n(t)|\Psi_0^n\rangle. \tag{57}$$

Using the overlap coefficients given in the Appendix, all these averages can be expressed as analytical functions of the coordinates X_p,X_n.

The $B(M1)$ spectrum obtained with $g_p-g_n=1/2$ for a system of two rigid rotors is presented in Fig. 5(a), and for two soft rotors in Fig. 5(b). The finite width of the lines and the background peaks are due to the existence of an upper limit for the time interval on which the functions $\mathcal{M}(t)$ and $\mathcal{N}(t)$ are calculated, chosen here of $500\hbar/\mathrm{MeV}$. In Fig. 5(a) appears a single maximum located at 3.27 MeV, with $0.56\mu_N^2$ at the peak, close to the total $B(M1)$ value observed

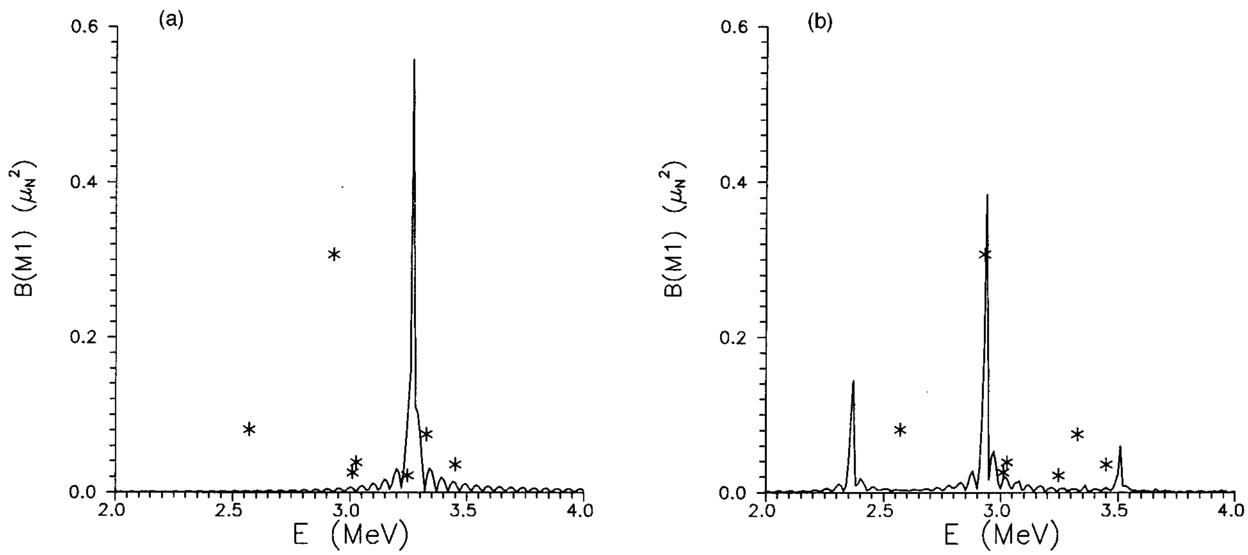

FIG. 5. The calculated $B(M1)$ spectrum for the rigid (a) and soft (b) two-rotor system (solid curves). The values measured in the experiment [26] for the low-lying 1^+ states of ^{134}Ba are represented by asterisks.

in experiment. The results presented in Fig. 5(b) show that the shape softness leads to a fragmentation of the $M1$ strength in three dominant states, located at 2.37, 2.94, and 3.51 MeV. The corresponding $B(M1)$ values are 0.145, 0.384, and $0.06\mu_N^2$, respectively, in agreement with the measured strengths of the main $M1$ transitions.

IV. SUMMARY AND CONCLUSIONS

The series of measurements of the low-lying $M1$ excitation spectrum in deformed nuclei with γ-soft triaxiality was extended recently [26] to ^{134}Ba by a high resolution photon scattering experiment. This experiment confirms the previous observation [43] of the scissors vibrations as an excitation mode of γ-soft deformed nuclei. However, in the Ba region this mode is of particular interest because it should provide indications about the effect of the deformation dynamics on the rotational properties.

In this work the coupling between the rotational and the shape degrees of freedom is described using a semimicroscopical model derived by a constrained TDHF calculation. The dynamical equations [Eq. (29)] are obtained using the time-dependent variational principle on a restricted trial manifold consisting of cranked squeezed states. The trial parameters which take into account the rotational motion are the angular momentum and the rotation angle around the x axis, while the shape degrees of freedom are introduced using the generators of the squeezing transformations along the y and z directions [Eq. (21)]. The model Hamiltonian [Eq. (30)] consists of the one-body harmonic oscillator term and the quadrupole-quadrupole two-body interaction. When the shape degrees of freedom are frozen at the ground state configuration, the energy function defined by this Hamiltonian on the trial manifold corresponds to a system of two rotors interacting by a restoring force linear in the relative angle. The elastic constant and the moments of inertia obtained for this system lead to an excitation energy [Eq. (44)] of the scissors mode which is near to the energy of the dominant $M1$ fragment observed in experiment. The phase-space orbits are closed, and can be quantized using the Bohr-Wilson-Sommerfeld formula. These quantized orbits are shown in Fig. 2(b) for protons, and Fig. 2(d) for neutrons. The spectrum obtained in this case consists of a single line [Fig. 5(a)] with the peak close to the total measured $B(M1)$ strength.

When the shape variables are coupled dynamically to the rotational motion the scissors vibrations become anharmonic (Fig. 3). The $M1$ spectrum in this case contains three main states, in reasonable agreement with the group of three dominant states appearing in experiment [Fig. 5(b)]. The occurrence of other weak fragments may be due to the pairing and spin-orbit interactions, or to the triaxiality effects, not considered in the present calculations.

These results show that the dynamical coupling between the isovector angular oscillations and the shape degrees of freedom lead to fragmentation and shift the dominant 1^+ state to lower energy. This coupling may have an important role in the fragmentation mechanism of the low-lying $M1$ strength in soft nuclei, being worth of further theoretical and experimental investigations.

ACKNOWLEDGMENTS

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APPENDIX

The action of the unitary operator

$$U=e^{-i\phi l_x}e^{-i\lambda c_x}e^{\Sigma_{\xi=y,z}(\theta_\xi d_\xi^\dagger-\theta_\xi^* d_\xi)/2} \tag{A1}$$

on the spherical boson operators $b_2^\dagger$, $b_3^\dagger$ may be written as a linear transformation to the new boson operators $a_k^\dagger=U^{-1}b_k^\dagger U$, $k=2,3$, such that

$$a_2^\dagger=\alpha \cosh s_2 b_2^\dagger+\alpha \sinh s_2 e^{2i\Phi_2} b_2-\zeta \cosh s_3 b_3^\dagger -\zeta \sinh s_3 e^{2i\Phi_3} b_3 \tag{A2}$$

and

$$a_3^\dagger=\alpha^* \cosh s_3 b_3^\dagger+\alpha^* \sinh s_3 e^{2i\Phi_3} b_3+\zeta^* \cosh s_2 b_2^\dagger +\zeta^* \sinh s_2 e^{2i\Phi_2} b_2 \tag{A3}$$

with

$$\alpha=\cos\lambda\ \cos\phi-i\ \sin\lambda\ \sin\phi,$$

$$\zeta=\cos\lambda\ \sin\phi-i\ \sin\lambda\ \cos\phi. \tag{A4}$$

This linear transformation can be written in the general form

$$a_j^\dagger=\sum_{k=2,3} x_{kj} b_k^\dagger+y_{kj} b_k, \tag{A5}$$

where x and y are the complex coefficients determined by Eqs. (A2) and (A3).

Let us denote by $\tilde{\psi}_{00}$ and ψ_{00} the vacuum states defined by

$$a_k\tilde{\psi}_{00}=0, \quad b_k\psi_{00}=0, \quad k=2,3, \tag{A6}$$

and by $\tilde{\psi}_{nm}$ and ψ_{nm} the states

$$\tilde{\psi}_{nm}=\frac{1}{\sqrt{m!n!}}(a_2^\dagger)^n(a_3^\dagger)^m\tilde{\psi}_{00}, \tag{A7}$$

and

$$\psi_{nm}=\frac{1}{\sqrt{m!n!}}(b_2^\dagger)^n(b_3^\dagger)^m\psi_{00}. \tag{A8}$$

Then, according to [33], the overlap coefficient $q_{m_2m_3,n_2n_3}=\langle\psi_{m_2m_3}|\tilde{\psi}_{n_2n_3}\rangle$ is expressed by

$$q_{m_2m_3,n_2n_3}=(n_2!n_3!m_2!m_3!)^{1/2}\sum_{k_2=0}^{n_2}\sum_{k_3=0}^{n_3}\sum_{(j_1,j_2,l_1,l_2)(j_1',j_2',l_1',l_2')} g^{2k_2n_2}_{j_1j_2l_1l_2} g^{3k_3n_3}_{j_1'j_2'l_1'l_2'} \delta_{j_1+j_2+l_1+l_2,k_2}\delta_{j_1'+j_2'+l_1'+l_2',k_3} \times\frac{q^0_{m_2+j_1+j_1'-j_2-j_2',m_3+l_1+l_1'-l_2-l_2'}}{\sqrt{(m_2+j_1+j_1'-j_2-j_2')!(m_3+l_1+l_1'-l_2-l_2')!}}. \tag{A9}$$

The coefficient $q^0_{n_2,n_3}=\langle\psi_{n_2n_3}|\tilde{\psi}_{00}\rangle$ has nonzero values only if n_2,n_3 are both even or odd, and is given by

$$q^0_{n_2,n_3}=(-i)^{\sigma_2}\left[\frac{n_2!n_3!\sqrt{u_2u_3}}{(u_2+1)(u_3+t+1)2^{n_2+n_3-2}}\right]^{1/2} \times\sum_{k=\sigma_2}^{n_2}\frac{(-1)^{(k-\sigma_2)/2}[(1-u_2)/(1+u_2)]^{(n_2-k)/2}[s/(1+u_2)]^k}{k![(n_2-k)/2]!} \times\sum_{m=(k+I_k)/2}^{(n_3+k)/2}\frac{(2m)!(-1)^{(n_3+k)/2-m}}{m!(2m-k)![(n_3+k)/2-m]![(s+t+1)/2]^m(1+u_3)^m}. \tag{A10}$$

Here $I_n=[1-(-1)^n]/2$, $\sigma_2\equiv I_{n_2}$, $t=s^2/(u_2+1)$, and

$$u_k=\frac{1}{1+2(|y_{k2}|^2-x_{k2}y_{k2}^*+|y_{k3}|^2-x_{k3}y_{k3}^*)}, \tag{A11}$$

$$s=2\ \mathrm{Im}\frac{x_{22}y_{23}-x_{23}y_{22}}{(x_{23}-y_{23})(x_{32}-y_{32})-(x_{22}-y_{22})(x_{33}-y_{33})}, \tag{A12}$$

$$g^{ikn}_{j_1j_2l_1l_2}=\sum_{j_3,l_3=even}\delta_{j_3+l_3,n-k}(-1)^{(j_3+l_3)/2}\frac{y_{2i}^{j_1+j_3/2}x_{2i}^{j_2+j_3/2}y_{3i}^{l_1+l_3/2}x_{3i}^{l_1+l_3/2}}{2^{(j_3+l_3)/2}j_1!j_2!(j_3/2)!(l_3/2)!l_1!l_2!}. \tag{A13}$$

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