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Finite rank approximation for random phase approximation calculations with Skyrme interactions: An application to Ar isotopes

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Starting from an effective interaction of Skyrme type, a finite rank separable approximation is proposed for the residual particle-hole interaction with the aim to allow one to perform structure calculations in very large particle-hole spaces. The approximation is checked on a specific example by calculating isoscalar quadrupole and isovector dipole modes in a finite nucleus using the random phase approximation. It is found that the finite rank approximation is very accurate in the isoscalar channel and it reproduces reasonably well the isovector channel. The use of the finite rank interaction is illustrated by calculating the evolution of the dipole strength distribution along the Ar isotope chain, from $A=32$ to $A=52$. [S0556-2813(98)02003-2]

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I. INTRODUCTION

Among the great variety of microscopic nuclear models aiming at a description of the properties of nuclear excitations one can distinguish essentially two types of approaches. In one approach the emphasis is put on the consistency of the picture by employing an effective interaction which must describe, throughout the periodic table, the ground states in the framework of the Hartree-Fock (HF) approximation and the excited states in time-dependent HF, or random phase approximation (RPA), or approximations beyond. To this class belong the Gogny's interaction [1] and the Skyrme-type interactions [2]. This approach is quite successful not only for predicting accurately nuclear ground state properties [3,4] but also for calculating the main features of giant resonances in closed-shell nuclei [5,6] and single-particle strengths near closed shells [7]. The main difficulty is that the complexity of giant resonance calculations beyond standard RPA (e.g., for studying damping mechanisms of collective excitations) increases rapidly with the size of the configuration space and one has to work within limited spaces. The other approach is more phenomenological and assumes some simple separable form for the residual nucleon-nucleon interaction while the mean field is modelized by an empirical potential well. These are the basic ingredients of the well-known quasiparticle-phonon model (QPM) of Soloviev *et al.* [8]. The practical advantage of this approach is that it allows one to calculate nuclear excitations in very large configuration spaces since there is no need to diagonalize matrices whose dimensions grow with the size of configuration space. Very detailed predictions can be made for nuclei away from closed shells [9].

When the residual particle-hole (p-h) interaction is separable, the RPA problem can be easily solved no matter how many p-h configurations are involved. The RPA eigenvalues are obtained as the roots of a single secular equation and then the corresponding RPA amplitudes can be calculated by performing only summations. If the p-h interaction is a sum of N separable terms (finite rank separable interaction) the roots of the secular equation are those of a $N\times N$ determinant. Since N is considerably smaller than the dimension D of the p-h space, one still gains over the case of a nonseparable interaction where solving the RPA problem would require diagonalizing a two-dimensional 2D×2D matrix. This is the motivation for proposing in this work a finite rank approximation for the p-h interaction resulting from Skyrme-type forces. Thus, the self-consistent mean field can be calculated in the standard way with the original Skyrme interaction whereas the RPA solutions would be obtained with the finite rank approximation to the p-h matrix elements. This would eventually allow one to use consistently Skyrme-type forces to study complicated situations (effects of two- and three-phonon configurations) where only the QPM model is available at present [9].

In the present work, we build a finite rank approximation for p-h interactions of Skyrme type. We check this approximation by comparing RPA results calculated with the original and approximate interactions. As a first application we present the evolution of the collective isovector dipole and isoscalar quadrupole states along the isotopic chain of Ar nuclei calculated in RPA. This paper is organized as follows. In Sec. II we sketch our method for constructing a finite rank interaction. Detailed expressions are gathered in Appendix A, whereas the solution of RPA equations with a finite rank interaction is explained in Appendix B. In Sec. III we apply this interaction to the study of Ar isotopes. A comparison of results obtained with the original and approximate interaction is done in Sec. III A whereas Secs. III B and III C are devoted to a discussion of the calculated isovector dipole and isoscalar quadrupole states, respectively. Conclusions are drawn in Sec. IV.

II. FINITE RANK APPROXIMATION FOR PARTICLE-HOLE MATRIX ELEMENTS

We start from the effective Skyrme interaction [2] which is often used in consistent HF-RPA calculations of nuclear excitations. We adopt the notation of Ref. [10] containing explicit density dependence and all spin-exchange terms rather than the original form of Ref. [2], where density dependence at the HF level was introduced by a three-body contact force and where some spin-exchange terms were dropped. The exact p-h residual interaction $\widetilde{V}_{ph}$ corresponding to the Skyrme force and including both direct and exchange terms can be obtained as the second derivative of the energy density functional with respect to the density [11]. Thus, $\widetilde{V}_{ph}$ has some velocity dependence which makes it cumbersome to use in finite systems but it also has a great advantage, namely, it contains a δ function in coordinate space. First, we shall simplify $\widetilde{V}_{ph}$ by approximating it by its Landau–Migdal form

$$V_{ph}=N_0^{-1}\sum_l [F_l+G_l\sigma_1\cdot\sigma_2+(F_l'+G_l'\sigma_1\cdot\sigma_2)\tau_1\cdot\tau_2]\times\delta(\mathbf{r}_1-\mathbf{r}_2), \tag{1}$$

where σ_i and τ_i are the nucleon spin and isospin operators, and $N_0=2k_Fm^*/\pi^2\hbar^2$ with k_F and m^* standing for the Fermi momentum and nucleon effective mass. For Skyrme interactions all Landau parameters with $l>1$ are zero. Here, we keep only the $l=0$ terms in V_{ph}. The expressions for the parameters F_0, G_0, F_0', G_0' in terms of the Skyrme force parameters can be found in Ref. [10]. Because of the density dependence of the interaction the Landau parameters of Eq. (1) are functions of the coordinate $\mathbf{r}$.

In what follows the indices p and h will refer respectively to unoccupied and occupied single-particle states in the HF spectrum calculated with the original Skyrme interaction. In RPA problems there appear two types of interaction matrix elements, the $A^{(J)}_{ph,p'h'}$ matrix related to forward-going graphs and the $B^{(J)}_{ph,p'h'}$ matrix related to backward-going graphs. Because of the zero range of V_{ph} both matrices can be obtained from the following quantity:

$$H_J(ph'p'h)\equiv\sum_{m,M}(-1)^{j_p-m_p+j_{p'}-m_{p'}}\begin{pmatrix} j_p & j_h & J\\ m_p & -m_h & -M\end{pmatrix}\times\begin{pmatrix} j_{p'} & j_{h'} & J\\ m_{p'} & -m_{h'} & -M\end{pmatrix}\times\langle\phi_{p,m_p}(1)\phi_{h',m_{h'}}(2)|V_{ph}(1,2)|\phi_{h,m_h}(1)\times\phi_{p',m_{p'}}(2)\rangle, \tag{2}$$

where the uncoupled matrix element calculated with the HF single-particle wave functions

$$\phi_{i,m}(1)=\frac{u_i(r_1)}{r_1}\mathcal{Y}^m_{l_i,j_i}(\widehat{r_1},\sigma_1) \tag{3}$$

appears on the right-hand side. The $A^{(J)}$ and $B^{(J)}$ matrices are given by

$$A^{(J)}_{ph,p'h'}=H_J(ph'p'h),$$
$$B^{(J)}_{ph,p'h'}=(-1)^{j_{p'}+j_{h'}}H_J(pp'h'h). \tag{4}$$

Let us explain the procedure for making a finite rank approximation by examining only the contribution of the term $F_0(r)$ of Eq. (1). The complete expressions can be found in Appendix A. The coupled p-h matrix element corresponding to F_0 is

$$H_J(ph'p'h)=\hat{J}^{-2}I_F(ph'p'h)\langle p||Y_J||h\rangle\langle p'||Y_J||h'\rangle, \tag{5}$$

where $\langle p||Y_J||h\rangle$ is the reduced matrix element of the spherical harmonics Y_{JM}, $\hat{J}=\sqrt{2J+1}$, and I_F is the radial integral

$$I_F(ph'p'h)=N_0^{-1}\int_0^\infty F_0(r)u_p(r)u_h(r)u_{p'}(r)u_{h'}(r)\frac{dr}{r^2}. \tag{6}$$

In practice, I_F can be calculated accurately by choosing a large enough cutoff radius R and using a n-point integration Gauss formula with abscissas and weights $\{r_k,w_k\}$:

$$I_F(ph'p'h)\simeq N_0^{-1}R\sum_{k=1}^n w_k\frac{F_0(r_k)}{r_k^2}u_p(r_k)u_h(r_k)u_{p'}(r_k)u_{h'}(r_k). \tag{7}$$

This suggests the introduction of the coefficients

$$\chi^{(k)}=-N_0^{-1}R\hat{J}^{-1}w_k\frac{F_0(r_k)}{r_k^2}, \tag{8}$$

and the p-h matrix elements

$$D^{(k)}(ph)=u_p(r_k)u_h(r_k)\langle p||Y_J||h\rangle, \tag{9}$$

so that H_J is just a sum of separable terms:

$$H_J(ph'p'h)=-\sum_{k=1}^n\chi^{(k)}D^{(k)}(ph)D^{(k)}(p'h'). \tag{10}$$

This can be easily extended to all four terms of Eq. (1) and the angular-momentum coupled matrix element $H_J(ph'p'h)$ is finally expressed as a sum of $N=4n$ separable terms:

$$H_J(ph'p'h)=-\sum_{\alpha=1}^N\chi^{(\alpha)}D^{(\alpha)}(ph)D^{(\alpha)}(p'h'). \tag{11}$$

The expressions for the quantities $\chi^{(\alpha)}$ and $D^{(\alpha)}(ph)$ are given in Appendix A. The explicit solution of the corresponding RPA equations can be found in Appendix B where it is shown that the matrix problems never exceed the dimensions $N\times N$.

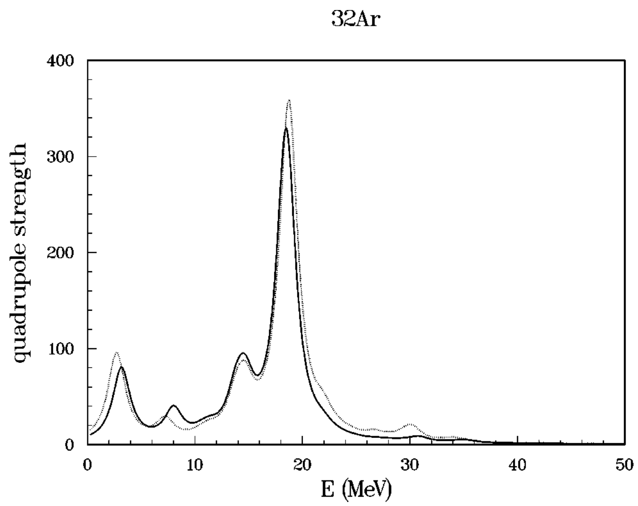

FIG. 1. Comparison of isoscalar quadrupole strengths calculated with the full *S*III force (solid curve) and the finite rank approximation (dotted curve). The strengths are in fm^4 MeV^{-1}.

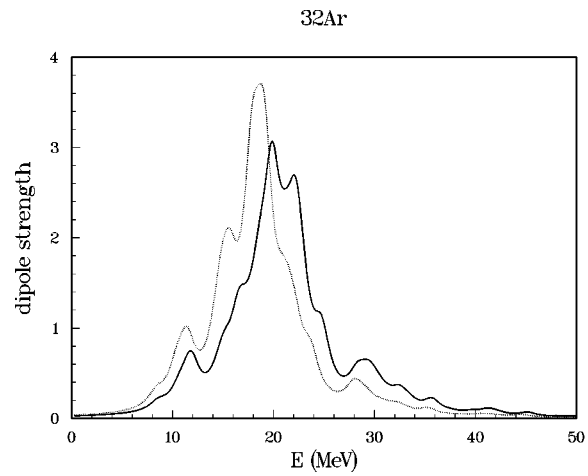

FIG. 2. Comparison of isovector dipole strengths calculated with the full *S*III force (solid curve) and the finite rank approximation (dotted curve). The strengths are in fm^2 MeV^{-1}.

III. APPLICATION TO Ar ISOTOPES

As a first application of the finite rank approximation we study the evolution of the giant quadrupole resonance (GQR) and giant dipole resonance (GDR) in the chain of Ar isotopes from the neutron-poor to the neutron-rich side. The strength distributions are calculated in RPA, starting from the HF mean fields computed in coordinate space with an effective Skyrme interaction. Note that no approximation is made for the interaction at the level of HF calculations. In this work we use the standard parametrization *S*III [12]. Spherical symmetry is assumed for all HF ground states. For nonclosed subshell nuclei we use the filling approximation [12].

In order to perform RPA calculations, the single-particle continuum is discretized by diagonalizing the HF Hamiltonian on a basis of ten harmonic oscillator shells and truncating the single-particle space to three unoccupied levels for each (l,j) value. This is sufficient to exhaust practically all the energy-weighted sum rule. For the sake of presentation the calculated transition strength distributions are smoothed out by folding them with a Lorentzian distribution of width $\Delta = 2$ MeV. In the present calculations we have adopted the value $n=24$ for the finite rank approximation [see Eq. (10)] and we have checked that variations of n around this value do not change significantly the results.

A. Comparison with exact interaction results

First, we check how the RPA results calculated with the finite rank approximation (11) can reproduce the strength distributions obtained with the original p-h interaction $\widetilde{V}_{ph}$. We select ^{32}Ar as an illustrative case. The exact and approximate strength distributions are compared in Fig. 1 for the isoscalar GQR and in Fig. 2 for the isovector GDR. In the isoscalar channel the Landau–Migdal form (1) with the F_0 and G_0 calculated according to Ref. [10] can give an accurate representation of the original p-h Skyrme interaction if we adopt the effective value $k_F=1.8$ fm^{-1}. This value is larger than the nuclear matter value in order to compensate for the effects of the neglected terms F_1 and G_1. From Fig. 1 it can be seen that the original interaction and its approximate form give practically the same results, with a strongly collective GQR around 18 MeV and a low-lying 2^+ state at 3 MeV. On the other hand, the isovector dipole results show that the approximate interaction (1) restricted to $l=0$ terms is slightly less repulsive than the original p-h interaction in the isovector channel. The results of Fig. 2 and all GDR results in the rest of this work have been calculated using the value $k_F=1.3$ fm^{-1} for the Landau parameters F_0' and G_0'. The use of a smaller value of k_F for F_0' and G_0' would probably improve the agreement. From Fig. 2 it can be seen that the approximate interaction reproduces the main features of the dipole strength distribution with a shift of 1–1.5 MeV in the position of the GDR. This shift remains of the same order for the other Ar isotopes. Thus, the finite rank approximation for the isovector p-h interaction is slightly weaker than the original interaction but it leads to strength distributions which present the same essential features as the exact ones.

B. Isoscalar quadrupole states

In Figs. 3 and 4 are shown isoscalar quadrupole strength functions calculated in Ar isotopes from $A=32$ to $A=52$.

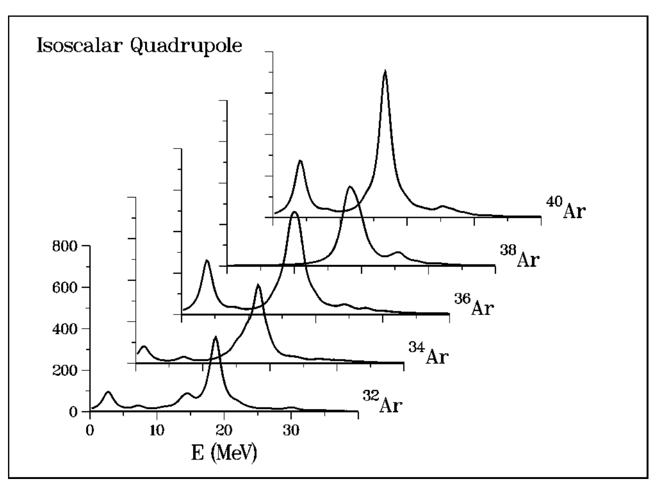

FIG. 3. Isoscalar quadrupole strength distributions (in fm^4 MeV^{-1}) in even isotopes ^{32}Ar to ^{40}Ar.

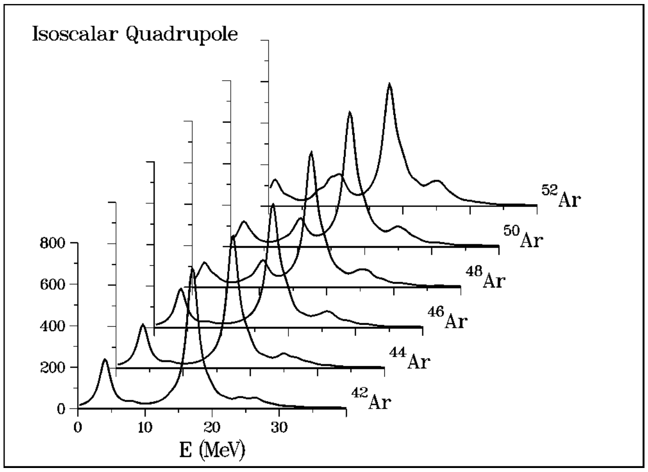

FIG. 4. Isoscalar quadrupole strength distributions (in fm^4 MeV^{-1}) in even isotopes ^{42}Ar to ^{52}Ar.

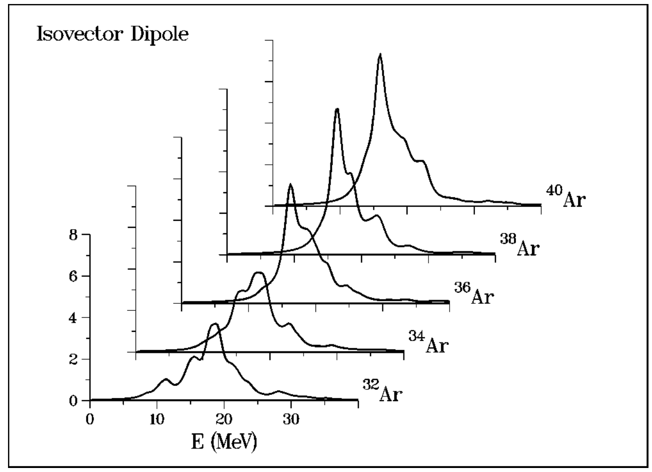

FIG. 5. Isovector dipole strength distributions (in fm^2 MeV^{-1}) in even isotopes ^{32}Ar to ^{40}Ar.

The general evolution when the mass number increases is the existence of a strongly collective giant resonance whose energy is fairly stable (around 17 MeV) and some smaller concentrations of transition strength at lower energies. These low-energy excitations are of noncollective nature as we shall see now.

In these $Z=18$ nuclei the HF reference states correspond to a partly occupied $1d_{3/2}$ subshell for the protons with a well-marked separation to the first unoccupied $1f_{7/2}$ level. For the neutrons the subshell closures occur at $A=32$ ($1d_{5/2}$ filled), $A=34$ ($2s_{1/2}$ filled), $A=38$ ($1d_{3/2}$ filled), $A=46$ ($1f_{7/2}$ filled), $A=50$ ($2p_{3/2}$ filled), and $A=52$ ($2p_{1/2}$ filled). The proton p-h excitations with $J^{\pi}=2^{+}$ correspond to a change of major shell and cannot contribute to low-lying excitations. The only possibilities of having low-energy 2^{+} states are provided by a small number of neutron p-h configurations within the same major shell. Thus, the structure of the states below 5 MeV can easily be identified. In ^{32}Ar it is a $2s_{1/2}(1d_{5/2})^{-1}$ configuration while in ^{34,36}Ar it is a $1d_{3/2}(2s_{1/2})^{-1}$ configuration. In ^{38}Ar the $2s$-$1d$ neutron shell is closed and therefore such low-lying excitation cannot take place. From ^{40}Ar to ^{46}Ar the $2p_{3/2}(1f_{7/2})^{-1}$ configuration is involved, whereas in ^{48,50}Ar one is dealing with $2p_{1/2}(2p_{3/2})^{-1}$. Finally, the very low energy state in ^{52}Ar is due to a $1f_{5/2}(2p_{1/2})^{-1}$ transition. One can also notice in the nuclei ^{48,50,52}Ar an additional bump in the 10 MeV energy region. This bump can be attributed to a $1f_{5/2}(1f_{7/2})^{-1}$ transition which does not appear in lighter nuclei because the $1f_{5/2}$ orbital is yet unbound.

C. Isovector dipole states

In the long chain of Ar isotopes one evolves from a situation where the neutron and proton distributions in the ground state are similar, to a situation where there is a distinct neutron skin in the nuclei having a large neutron excess. In a fluid dynamical picture of the isovector dipole mode where the neutron and proton fluids would oscillate against each other, one would expect a slow lowering down of the giant resonance energy as well as the appearance in the neutron-rich isotopes of a weaker excitation at lower energy due to the oscillations of the neutron skin against the protons. This qualitative general behavior is indeed observed in the calculated isovector dipole strength distributions which are shown in Figs. 5 and 6. In the lighter isotopes up to $A=36$ the dipole strength shows less collectivity than in heavier isotopes. The peak energy of the GDR goes slowly down from about 18 MeV in ^{32}Ar to 16 MeV in ^{52}Ar. As discussed above, these peak energies are probably underestimated by about 1 MeV as compared to those of RPA calculations done with the original interaction *S*III. In the three heaviest isotopes ^{48,50,52}Ar one can observe the appearance of smaller bumps around 7.5 MeV. They correspond to neutron p-h configurations where the hole is in the $2p$ subshell while the particle is in a $l=2$ low-lying single-particle resonance (in the present discretized calculation such a single-particle resonance appears as a discrete state at positive energy whose position does not depend sensitively on the discretization method adopted).

In a recent work [13] the isovector dipole strengths in Ar isotopes were calculated using a relativistic RPA model. The GDR peak energies thus obtained are very close to the present results. Moreover, it was found that in the three isotopes ^{48,50,52}Ar there are smaller bumps around 6 MeV excitation energy whereas these bumps do not exist in lighter isotopes. It would be interesting to explore experimentally

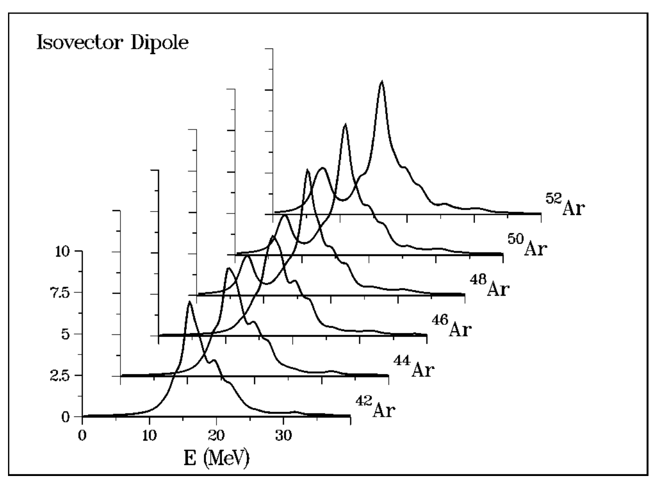

FIG. 6. Isovector dipole strength distributions (in fm^2 MeV^{-1}) in even isotopes ^{42}Ar to ^{52}Ar.

the low energy tail of the dipole strength in the heavy Ar isotopes since such pigmy-type resonances would have important consequences on photoabsorption cross sections.

IV. CONCLUSION

We have shown that by approximating the original p-h interaction derived from a Skyrme force by its Landau-Migdal expansion truncated at the $l=0$ terms it is possible to calculate accurately the isoscalar RPA modes and also to reproduce reasonably well the isovector RPA modes. The benefit of having a Landau-Migdal form is that it allows one to construct a finite rank p-h interaction and thus to combine the advantages of consistency (the mean field and the residual interaction of RPA are determined from the same effective interaction) and simplicity (the size of the RPA problem does not increase with increasing configuration space). It would be possible to improve the finite rank interaction in the isovector channel by using an effective value of the Fermi momentum k_F smaller than that adopted here, in order to account for the extra repulsion due to the neglected F_1' and G_1' Landau parameters. Thus, future large scale RPA calculations with Skyrme type interactions can be envisaged.

As an illustration of the method we have used the finite rank p-h interaction derived from the Skyrme force SIII to calculate the evolution of dipole strength distribution along the chain of Ar isotopes. It is found that, with increasing mass number A the giant dipole resonance becomes more collective but its peak energy varies little from $A=38$ to $A=52$. In addition, a low-lying component around 7 MeV appears in the $A=48-52$ isotopes due to single-particle transitions.

ACKNOWLEDGMENTS

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APPENDIX A

Here we generalize the expressions of $\chi^{(k)}$ and $D^{(k)}$ given in Sec. II to the case where the p-h interaction contains $\sigma_1 \cdot \sigma_2$ and/or $\tau_1 \cdot \tau_2$ terms. In addition to the spherical harmonics Y_{LM} there will appear also operators of the type

$$T^M_{J,L}(\hat{r},\sigma)=[Y_L \times \sigma]^M_J . \tag{A1}$$

To handle the $\tau_1 \cdot \tau_2$ terms we simply have to attach to each p-h pair $\{ph\}$ an extra index $q=\pm 1$ which specifies if it is a neutron p-h pair or a proton p-h pair. Then, the matrix element of $\tau_1 \cdot \tau_2$ between a $\{ph;q\}$ and a $\{p'h';q'\}$ configuration is proportional to qq', i.e., $+1$ if one connects two neutron pairs or two proton pairs and -1 otherwise.

We shall distinguish natural parity states $(L=J)$ and unnatural parity states $(L=J\pm 1)$. For the natural parity case, the $\chi^{(\alpha)}$ coefficients are

$$\begin{aligned}
\chi^{(\alpha)} = & -N_0^{-1} R \hat{J}^{-1} w_k F_0(r_k)/r_k^2 \quad \text{if} \quad \alpha = k,\\
& -N_0^{-1} R \hat{J}^{-1} w_k F_0'(r_k)/r_k^2 \quad \text{if} \quad \alpha = n+k,\\
& -N_0^{-1} R \hat{J}^{-1} w_k G_0(r_k)/r_k^2 \quad \text{if} \quad \alpha = 2n+k,\\
& -N_0^{-1} R \hat{J}^{-1} w_k G_0'(r_k)/r_k^2 \quad \text{if} \quad \alpha = 3n+k,
\end{aligned} \tag{A2}$$

where the index k runs from 1 to n. The corresponding $D^{(\alpha)}(ph;q)$ factors are

$$\begin{aligned}
D^{(\alpha)}(ph;q) = & \, u_p(r_k) u_h(r_k) \langle p || Y_J || h \rangle \quad \text{if} \quad \alpha = k,\\
& q u_p(r_k) u_h(r_k) \langle p || Y_J || h \rangle \quad \text{if} \quad \alpha = n+k,\\
& u_p(r_k) u_h(r_k) \langle p || T_{J,J} || h \rangle \quad \text{if} \quad \alpha = 2n+k,\\
& q u_p(r_k) u_h(r_k) \langle p || T_{J,J} || h \rangle \quad \text{if} \quad \alpha = 3n+k.
\end{aligned} \tag{A3}$$

For the unnatural parity case, the $\chi^{(\alpha)}$ coefficients are

$$\begin{aligned}
\chi^{(\alpha)} = & -N_0^{-1} R \hat{J}^{-1} w_k G_0(r_k)/r_k^2 \quad \text{if} \quad \alpha = k,\\
& -N_0^{-1} R \hat{J}^{-1} w_k G_0'(r_k)/r_k^2 \quad \text{if} \quad \alpha = n+k,\\
& -N_0^{-1} R \hat{J}^{-1} w_k G_0(r_k)/r_k^2 \quad \text{if} \quad \alpha = 2n+k,\\
& -N_0^{-1} R \hat{J}^{-1} w_k G_0'(r_k)/r_k^2 \quad \text{if} \quad \alpha = 3n+k,
\end{aligned} \tag{A4}$$

whereas the $D^{(\alpha)}(ph;q)$ factors are given by

$$\begin{aligned}
D^{(\alpha)}(ph;q) = & \, u_p(r_k) u_h(r_k) \langle p || T_{J,J+1} || h \rangle \quad \text{if} \quad \alpha = k,\\
& q u_p(r_k) u_h(r_k) \langle p || T_{J,J+1} || h \rangle \quad \text{if} \quad \alpha = n+k,\\
& u_p(r_k) u_h(r_k) \langle p || T_{J,J-1} || h \rangle \quad \text{if} \quad \alpha = 2n+k,\\
& q u_p(r_k) u_h(r_k) \langle p || T_{J,J-1} || h \rangle \quad \text{if} \quad \alpha = 3n+k.
\end{aligned} \tag{A5}$$

The angular-momentum coupled matrix elements defined in Eq. (2) take the general form

$$H_J(ph'p'h;qq') = -\sum_{\alpha=1}^{N} \chi^{(\alpha)} D^{(\alpha)}(ph;q) D^{(\alpha)}(p'h';q'), \tag{A6}$$

with $N=4n$.

APPENDIX B

Here, we show for completeness how the finite rank form (11) of the p-h matrix elements can simplify the resolution of RPA equations. This is a simple generalization of the well-known rank one case. Denoting by $\{X_{ph}, Y_{ph}\}$ the RPA amplitudes corresponding to an RPA eigenvalue ω and by

E_{ph} the unperturbed p-h energies, the RPA equations are

$$(E_{ph}-\omega)X_{ph}=\sum_{\alpha=1}^{N}\chi^{(\alpha)}D^{(\alpha)}(ph)\sum_{p'h'}D^{(\alpha)}(p'h')X_{p'h'}+\sum_{\alpha=1}^{N}\chi^{(\alpha)}D^{(\alpha)}(ph)\sum_{p'h'}D^{(\alpha)}(p'h')Y_{p'h'},$$

$$(E_{ph}+\omega)Y_{ph}=\sum_{\alpha=1}^{N}\chi^{(\alpha)}D^{(\alpha)}(ph)\sum_{p'h'}D^{(\alpha)}(p'h')Y_{p'h'}+\sum_{\alpha=1}^{N}\chi^{(\alpha)}D^{(\alpha)}(ph)\sum_{p'h'}D^{(\alpha)}(p'h')X_{p'h'}. \tag{B1}$$

In the N-dimensional space we can define a vector $\mathbf{T}$ by its components

$$\mathbf{T}_{\alpha}=\sum_{p'h'}D^{(\alpha)}(p'h')(X_{p'h'}+Y_{p'h'}). \tag{B2}$$

The solution of Eq. (B1) can be expressed as

$$X_{ph}=\frac{1}{E_{ph}-\omega}\sum_{\alpha=1}^{N}\chi^{(\alpha)}D^{(\alpha)}(ph)\mathbf{T}_{\alpha},$$

$$Y_{ph}=\frac{1}{E_{ph}+\omega}\sum_{\alpha=1}^{N}\chi^{(\alpha)}D^{(\alpha)}(ph)\mathbf{T}_{\alpha}. \tag{B3}$$

Combining Eqs. (B2) and (B3) we obtain the result

$$(\mathbf{W}-\mathbf{1})\mathbf{T}=0, \tag{B4}$$

where $\mathbf{W}$ is the $N\times N$ matrix

$$\mathbf{W}_{\alpha\beta}=\sum_{ph}D^{(\alpha)}(ph)\left(\frac{1}{E_{ph}-\omega}+\frac{1}{E_{ph}+\omega}\right)D^{(\beta)}(ph)\chi^{(\beta)}. \tag{B5}$$

Thus, the RPA eigenvalues ω are the roots of the secular equation

$$\det(\mathbf{W}-\mathbf{1})=0. \tag{B6}$$

Once an eigenvalue ω is known the corresponding amplitudes $\{X_{ph},Y_{ph}\}$ can be determined in the following way. Using Eq. (B4) we can express $\{\mathbf{T}_{\alpha},\alpha=1,2,\ldots,N-1\}$ in terms of $\mathbf{T}_N$. It is convenient to define the $(N-1)$-dimensional vectors $\mathbf{t}$, $\mathbf{s}$ and the $(N-1)\times(N-1)$ matrix $\hat{\mathbf{w}}$ by

$$\mathbf{t}_{\alpha}=\mathbf{T}_{\alpha},\quad \mathbf{s}_{\alpha}=\mathbf{W}_{\alpha N},\quad \hat{\mathbf{w}}_{\alpha\beta}=(\mathbf{W}-\mathbf{1})_{\alpha\beta}, \tag{B7}$$

where $1\leqslant\alpha,\beta\leqslant N-1$. Then, the solution of Eq. (B4) is

$$\mathbf{t}=-\hat{\mathbf{w}}^{-1}\mathbf{s}\mathbf{T}_N, \tag{B8}$$

or, more explicitly,

$$\mathbf{T}_{\alpha}=-\mathbf{T}_N\sum_{\beta=1}^{N-1}\hat{\mathbf{w}}^{-1}_{\alpha\beta}\mathbf{s}_{\beta}\qquad \alpha=1,2,\ldots,N-1. \tag{B9}$$

Thus, the RPA amplitudes are determined by Eqs. (B3) and (B9) up to a factor $\mathbf{T}_N$ which is fixed by the normalization condition.

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