Binding energy of double Λ hypernuclei in relativistic mean field theory

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We calculate the binding energy of two Λ hyperons bound to a nuclear core within the relativistic mean field (RMF) theory. The starting point is a two-body relativistic equation of the Breit type suggested by the RMF theory, and corrected for the two-particle interaction. We evaluate the 2 Λ correlation energy and estimate the contribution of the σ^* and Φ mesons, acting solely between hyperons, to the bond energy $\Delta B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, and ${}^{13}_{\Lambda\Lambda}$B. Predictions of the $\Delta B_{\Lambda\Lambda}$ A dependence are made for heavier Λ hypernuclei. [S0556-2813(98)00403-8]

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I. INTRODUCTION

Double Λ hypernuclei, nuclear systems containing two Λ hyperons, have been retaining much interest [1]. As the only observed example of a multiply strange system they give us a unique opportunity to study ΛΛ interaction in nuclear medium and to test existing models of the baryon-baryon interaction. Moreover, studies of ΛΛ hypernuclei are closely related to searches for the $S=-2$ dibaryon, known as the H particle [2]. Up to now, only a few events have been identified (${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, ${}^{13}_{\Lambda\Lambda}$B) [3–5] indicating a strong attractive ΛΛ interaction. The analysis of the data yields the ΛΛ bond energy $\Delta B_{\Lambda\Lambda}\approx 4-5$ MeV, where

$$\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z)-2B_{\Lambda}({}^{A-1}_{\Lambda}Z)$$
$$=2M({}^{A-1}_{\Lambda}Z)-M({}^{A}_{\Lambda\Lambda}Z)-M({}^{A-2}Z), \qquad (1)$$

B_{Λ} and $B_{\Lambda\Lambda}$ being the binding energy of Λ and a pair of Λ's, respectively.

Calculations of $B_{\Lambda\Lambda}$ have been performed in various nonrelativistic approaches [6–11], by using effective interactions or G matrices together with cluster or three-body models (a fair list of early works is given in [8]). In particular, the D model of the Nijmegen ΛΛ interactions was shown to yield results in good agreement with experiment.

The purpose of the present work is to investigate $\Delta B_{\Lambda\Lambda}$ within the relativistic mean field (RMF) theory. To some extent, this theory is less appropriate than three-body calculations to the problem of $\Delta B_{\Lambda\Lambda}$, since it replaces the basic two-body interactions by coupling to meson fields. However, our aim is to study the possibility to put the RMF on reasonable grounds, since a sensible estimate of $\Delta B_{\Lambda\Lambda}$ has implications on the calculations of multi-Λ systems within this model.

The RMF approach has been applied to hypernuclear systems containing various amounts of hyperons (see for instance [12–14]). While the original σ-ω model well reproduces spectra of single Λ hypernuclei (in particular, small spin-orbit splitting) its straightforward application to bound systems with two Λ particles failed. It has been found that the σ-ω model cannot provide a sufficiently attractive ΛΛ interaction and, consequently, the binding energy of double-Λ hypernuclei is substantially underestimated. Considering the Lorentz-tensor Λ-ω coupling [15], which allows for stronger couplings of Λ to mesons, could lead to a stronger ΛΛ interaction. Detailed calculations revealed, however, that stronger Λ-meson couplings do not necessarily result in a larger value of $\Delta B_{\Lambda\Lambda}$ [16]. Table I displays a typical example of RMF results for coupling constant ratios $\alpha_{\omega}=g_{\omega\Lambda}/g_{\omega N}=1/3$ and 2/3, compared to the empirical data. Quite recently, Schaffner *et al.* [14] proposed to strengthen the ΛΛ binding by introducing an additional YY interaction mediated by two strange mesons (scalar σ^* and vector Φ) that couple exclusively to hyperons. The coupling of hyperons to the Φ mesons was assumed to satisfy the SU(6) relations, whereas the coupling to σ^* was fixed by fitting to the

TABLE I. $B_{\Lambda\Lambda}$ and $\Delta B_{\Lambda\Lambda}$ (in MeV) (see text for definition) of the double Λ hypernuclei ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, and ${}^{13}_{\Lambda\Lambda}$B. The RMF predictions for the HS parametrization [27] with the coupling constant ratios $\alpha_{\omega}=g_{\omega\Lambda}/g_{\omega N}=1/3$ (a) and $\alpha_{\omega}=2/3$ (b) are compared with the experimental values [1].

	$B_{\Lambda\Lambda}$			$\Delta B_{\Lambda\Lambda}$		
	(a)	(b)	exp.	(a)	(b)	exp.
${}^{6}_{\Lambda\Lambda}$He	4.8	5.3	10.9±0.6	0.8	0.5	4.7±0.6
${}^{10}_{\Lambda\Lambda}$Be	14.8	15.1	17.7±0.4	1.1	0.6	4.3±0.4
${}^{13}_{\Lambda\Lambda}$B	23.5	23.1	27.5±0.7	1.0	0.4	4.8±0.7

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TABLE II. Comparison of ΔS (in MeV) [see Eq. (2) for definition] calculated within the RMF model for the HS parametrization [27] with the experimental values [17] for selected nuclei.

	^{18}O	^{30}Si	^{38}Ar	^{42}Ca	^{92}Zr	^{210}Pb
RMF	0.55	0.21	0.29	0.27	0.13	0.11
exp.	3.90	2.14	3.05	3.12	1.44	1.25

estimated potential well depth for the Λ hyperon in a medium of other Λ hyperons $U_\Lambda^{(\Lambda)} \approx 20$ MeV. The improved RMF model $(\sigma+\omega+\sigma^*+\Phi)$ increases the value $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})$ to about 3 MeV (from the original <1 MeV).

It is clear, however, that the discrepancy between the calculated and empirical values of $\Delta B_{\Lambda\Lambda}$ cannot be attributed entirely to the missing meson exchanges between the two hyperons. To support this argument, we display in Table II

$$\Delta S = S_{2n} - 2S_n \tag{2}$$

calculated for ordinary nuclei with two nucleons outside a spherical core. Here S_n (S_{2n}) is the separation energy of the nucleon (two nucleons). (Note that unlike the binding energies B in case of hypernuclei, separation energies S are used in nuclear mass tables [17].) The RMF results are compared to the experimental data [17]. The model again fails to give the observed ΔS. In this case, possible additional mesons would have to be introduced from the beginning (fit to nuclear matter data) and thus are not expected to change the results. Comparing the RMF and experimental values from Table II it is reasonable to expect a sizable contribution to $\Delta B_{\Lambda\Lambda}$ coming from other effects neglected in the mean field approximation [18]. In the present work, we aim to develop a simple model, which allows us to include the two-Λ correlation energy. This is a first step towards estimating what fraction is left for the contribution from σ^* and Φ mesons. In this way, we are able to extract information about their couplings to the Λ hyperon.

It has to be stressed that the present work neglects ΛN correlations, which could lower the coupling constants and thus affect the ΛΛ correlation energy. Similarly, the lack of spin dependence of the Λ-nucleus potential derived from the RMF may be of importance. We will address these questions in Sec. III, where we show on simple estimates that the present model yields a reasonable limit to the ΛΛ correlation energy.

The model used in our study is described in the following section. In Sec. III, we present results of the calculations. We show that the correlation energy between two Λ's is very sensitive to the RMF parametrization used. Though for quark model inspired values of Λ couplings the correlation energy contributes substantally to $\Delta B_{\Lambda\Lambda}$ it is not sufficient to account for the empirical values. Indeed, extra meson exchanges proposed by Schaffner *et al.*, specific to the hyperon (Λ) sector are required. Taking the SU(6) value for the ΦΛ coupling constant $g_{\Phi\Lambda}$ we determine the $\sigma^*\Lambda$ coupling $g_{\sigma^*\Lambda}$ by fitting the experimental data from observed 2Λ hypernuclei. Predictions are then made for $\Delta B_{\Lambda\Lambda}$ in heavier hypernuclear systems. Conclusions are drawn in Sec. IV.

II. THE MODEL

A starting point to describe the relativistic system of two interacting particles inside the nuclear medium would be either the covariant Bethe-Salpeter equation [19] or the manifestly covariant formalism with constraints [20]. However, due to the complexity of these approaches, most of the works related to this subject are based on the phenomenological equal-time two-body Dirac equation [21–24]. Although not fully covariant, it has proved very useful in understanding two-electron atoms and two-quark systems. We will therefore adopt this approach here, as well.

In order to write down the Dirac equation for two Λ particles in a hypernucleus, we consider these two hyperons moving in the scalar (σ) and vector (ω_0) mean fields brought about by the nucleons. In addition, the hyperons interact with each other. Let us suppose that the interaction is mediated by the exchange of the scalar (σ_Λ) and vector ($\omega_{0\Lambda}$) mesons (for simplicity, we neglect all the other possible meson exchanges). In accordance with the above assumptions the time-independent Dirac equation we propose, neglecting retardation effects, has the following form:

$$\{-i\vec{\alpha}_1\cdot\vec{\nabla}_1 - i\vec{\alpha}_2\cdot\vec{\nabla}_2 + \beta_1[M_\Lambda + \Sigma(\vec{r}_1)] + \beta_2[M_\Lambda + \Sigma(\vec{r}_2)] + \beta_1\beta_2\Sigma_{S\Lambda}(\vec{r}_1,\vec{r}_2) + \Sigma_{0\Lambda}(\vec{r}_1,\vec{r}_2)\}\varphi_\Lambda(\vec{r}_1,\vec{r}_2) = 2E_\Lambda\varphi_\Lambda(\vec{r}_1,\vec{r}_2), \tag{3}$$

where $\Sigma(\vec{r}_i) = \Sigma_S(\vec{r}_i) + \beta_i\Sigma_0(\vec{r}_i)$ represents the self-energy of a Λ particle due to its interaction with the nucleon fields. The scalar (Σ_S) and timelike part of the vector interaction (Σ_0) are given by $\Sigma_S(\vec{r}_i) = g_{\sigma\Lambda}\sigma(\vec{r}_i)$ and $\Sigma_0(\vec{r}_i) = g_{\omega\Lambda}\omega_0(\vec{r}_i)$, respectively. $\Sigma_{S\Lambda}$ and $\Sigma_{0\Lambda}$ are the scalar and timelike vector self-energies of the Λ particles due to their mutual interaction. In terms of σ_Λ and $\omega_{0\Lambda}$ fields, they read $\Sigma_{S\Lambda}(\vec{r}_1,\vec{r}_2) = g_{\sigma\Lambda}\sigma_\Lambda(\vec{r}_1,\vec{r}_2)$, $\Sigma_{0\Lambda}(\vec{r}_1,\vec{r}_2) = g_{\omega\Lambda}\omega_{0\Lambda}(\vec{r}_1,\vec{r}_2)$. Finally, $\varphi_\Lambda(\vec{r}_1,\vec{r}_2)$ is a 16-component spinor (labeled by two indices), representing the two-Λ state. α_i and β_i ($i=1,2$) are the Dirac matrices acting on the ith spinor index. The fields $\sigma(\vec{r})$ and $\omega_0(\vec{r})$ fulfill the Klein-Gordon equations with the nuclear scalar and vector densities as the source terms

$$(\Delta - m_\sigma^2)\sigma = g_\sigma \sum_{i=1,N} \overline{\varphi_{Ni}}(\vec{r})\varphi_{Ni}(\vec{r}), \tag{4}$$

$$(\Delta - m_\omega^2)\omega_0 = -g_\omega \sum_{i=1,N} \overline{\varphi_{Ni}}(\vec{r})\gamma_0\varphi_{Ni}(\vec{r}). \tag{5}$$

Here, the spinors $\varphi_{Ni}(\vec{r})$ represent the one-nucleon states.

As stated above, each Λ particle in addition moves in the σ_Λ and $\omega_{0\Lambda}$ fields whose source is the second hyperon. In the approximation of heavy, static baryons the corresponding Klein-Gordon equations for σ_Λ and $\omega_{0\Lambda}$ acquire the form

$$(\Delta - m_\sigma^2)\sigma_\Lambda = g_{\sigma\Lambda}\delta(\vec{r}_1 - \vec{r}_2), \tag{6}$$

$$(\Delta - m_\omega^2)\omega_{0\Lambda} = -g_{\omega\Lambda}\delta(\vec{r}_1 - \vec{r}_2). \tag{7}$$

If the $\Lambda\Lambda$ interaction is neglected the Dirac equation (1) reduces to two identical Dirac equations, each being equivalent to the mean field approximation for a Λ particle in a hypernucleus.

To proceed further we express the two-Λ spinor in terms of its four large (ψ_Λ), eight medium (θ_Λ, ϑ_Λ), and four small (χ_Λ) components [22,23]:

$$\varphi_\Lambda(\vec{r}_1,\vec{r}_2)=\begin{pmatrix}\psi_\Lambda & \vartheta_\Lambda\\ \theta_\Lambda & \chi_\Lambda\end{pmatrix}. \tag{8}$$

If φ_Λ from Eq. (8) is brought into the Dirac equation (3) and the components θ_Λ, ϑ_Λ, and χ_Λ are eliminated, the following equation for the components ψ_Λ is obtained:

$$\left[-\vec{\sigma}_1\cdot\vec{\nabla}_1\frac{1}{2\overline{M}_{1\Lambda}}\vec{\sigma}_1\cdot\vec{\nabla}_1-\vec{\sigma}_2\cdot\vec{\nabla}_2\frac{1}{2\overline{M}_{2\Lambda}}\vec{\sigma}_2\cdot\vec{\nabla}_2+\Sigma_S(\vec{r}_1)+\Sigma_S(\vec{r}_2)+\Sigma_0(\vec{r}_1)+\Sigma_0(\vec{r}_2)+\Sigma_{S\Lambda}(r)+\Sigma_{0\Lambda}(r)\right]\psi_\Lambda(\vec{r}_1,\vec{r}_2)=2\epsilon_\Lambda\psi_\Lambda(\vec{r}_1,\vec{r}_2), \tag{9}$$

where $2\overline{M}_{i\Lambda}=E_i^*+M_i^*-\Sigma_{0\Lambda}+\Sigma_{S\Lambda}$, with $E_i^*=E_\Lambda-\Sigma_0(\vec{r}_i)$, $M_i^*=M_\Lambda+\Sigma_S(\vec{r}_i)$, $\epsilon_\Lambda=E_\Lambda-M_\Lambda$, and r is the relative distance between two Λ particles, $r=|\vec{r}_1-\vec{r}_2|$.

Equation (9) for $\psi_\Lambda(\vec{r}_1,\vec{r}_2)$ is still rather complicated because $\overline{M}_{i\Lambda}$ depends on $\vec{r}_1$ and $\vec{r}_2$. We simplify the solution by neglecting the radial dependence of $\overline{M}_{i\Lambda}$ and replacing $1/\overline{M}_{i\Lambda}$ by the ground state expectation value $\langle 1/\overline{M}_{i\Lambda}\rangle$. This approximation leads to neglecting the spin-orbit interaction and terms that renormalize somewhat the central potential. However, since the spin-orbit interaction is very small for the Λ hyperon this does not represent a serious drawback. The resulting (Schrödinger-type) equation for ψ_Λ is then

$$\left[-\left\langle\frac{1}{2\overline{M}_{1\Lambda}}\right\rangle\vec{\nabla}_1^2-\left\langle\frac{1}{2\overline{M}_{2\Lambda}}\right\rangle\vec{\nabla}_2^2+\Sigma_S(\vec{r}_1)+\Sigma_S(\vec{r}_2)+\Sigma_0(\vec{r}_1)+\Sigma_0(\vec{r}_2)+\Sigma_{S\Lambda}(r)+\Sigma_{0\Lambda}(r)\right]\psi_\Lambda(\vec{r}_1,\vec{r}_2)=\epsilon_\Lambda\psi_\Lambda(\vec{r}_1,\vec{r}_2). \tag{10}$$

Although at this stage, Eq. (10) can be solved by expanding $\psi_\Lambda(\vec{r}_1,\vec{r}_2)$ in a convenient basis, it is still tedious enough that it is useful to look for further simplifications.

The Λ particle in a hypernucleus spends most of its time in a high density region, where the potential $\Sigma_S(\vec{r}_i)+\Sigma_0(\vec{r}_i)$ can be approximated rather accurately by a spherical harmonic oscillator $W(\vec{r}_i)=-W_0+1/2M_\Lambda\omega^2r_i^2$ [25]. Consequently, we shall use both that the RMF reproduces the hypernuclear spectra with a great accuracy [26] and that the potential seen by the Λ is very close to the harmonic oscillator to get a practical solution of Eq. (10). Note that very similar approximations have been used in nonrelativistic calculations.

The σ_Λ and ω_Λ exchanges between Λ hyperons [Eqs. (6) and (7)] give rise to an effective Λ-Λ potential $U(r)\equiv\Sigma_{S\Lambda}(r)+\Sigma_{0\Lambda}(r)$ ($r=|\vec{r}_1-\vec{r}_2|$) which reduces to a difference of two Yukawa forms:

$$U(r)\equiv\Sigma_{S\Lambda}(r)+\Sigma_{0\Lambda}(r)=-\frac{g_{\sigma\Lambda}^2}{4\pi}\frac{e^{-m_\sigma r}}{r}+\frac{g_{\omega\Lambda}^2}{4\pi}\frac{e^{-m_\omega r}}{r}. \tag{11}$$

The above two approximations lead to the following replacement in the Schrödinger-like equation (10):

$$\Sigma_S(\vec{r}_1)+\Sigma_S(\vec{r}_2)+\Sigma_0(\vec{r}_1)+\Sigma_0(\vec{r}_2)+\Sigma_{S\Lambda}(r)+\Sigma_{0\Lambda}(r)\rightarrow V(\vec{r}_1,\vec{r}_2)=W(r_1)+W(r_2)+U(r). \tag{12}$$

In fact, the oscillator depth W_0 and its frequency ω were not determined by fitting $W(r_i)$ to $\Sigma_S(\vec{r}_i)+\Sigma_0(\vec{r}_i)$ but directly fitted to the experimental energy spectrum of the particular Λ hypernucleus. The two coupling constants $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$ from Eq. (11) were chosen to reproduce the spectroscopic data in the relativistic mean field formalism [26], as accurately as possible, in the whole ensemble of single-Λ hypernuclei known. With the above determined parameters, Eq. (10) allows us to estimate the correlation energy of the two hyperons, which is neglected in the mean field approximation.

Parametrizing the Λ self-energies $\Sigma_S(\vec{r}_i)+\Sigma_0(\vec{r}_i)$ in terms of HO potentials enables us to express the equation of motion (10) in Jacobi coordinates ($\vec{R}$, $\vec{r}$):

$$\vec{r}_1=\frac{1}{\sqrt{2}}(\vec{R}+\vec{r});\quad \vec{r}_2=\frac{1}{\sqrt{2}}(\vec{R}-\vec{r}),$$

and to separate the center-of-mass coordinates from the relative ones. After straightforward manipulations the former equation (10) transforms into the following two equations:

$$\left[\frac{\vec{P}^2}{2m_\Lambda}+\frac{1}{2}m_\Lambda\omega^2R^2+2W_0\right]\psi_R(R)=E_R\psi_R(R), \tag{13}$$

$$\left[\frac{\vec{p}^2}{2m_\Lambda}+\frac{1}{2}m_\Lambda\omega^2r^2+U(\sqrt{2}r)\right]\psi_r(r)=E_r\psi_r(r), \tag{14}$$

where $\vec{P}$, $\vec{p}$ are the Jacobi impulse operators and $m_\Lambda^{-1}=\langle 1/\overline{M}_\Lambda\rangle$.

If the harmonic oscillator parameters are fitted to eigenvalues of a single Λ hypernucleus, (taken either from RMF or from experiments) these two coupled equations yield a first approximation to the two Λ binding energy $B_{\Lambda\Lambda}\simeq-(E_R+E_r)$. The correct value has to include at least two corrections: a modification of the harmonic oscillator parameters due to the additional Λ and the increase of the core energy, the so-called rearrangement energy.

The rearrangement energy can be estimated in the RMF approximation as a difference between the Λ eigenvalue and binding energy

$$\Delta E_{\rm core}=-E_\Lambda-B_\Lambda. \tag{15}$$

TABLE III. The parametrizations used in this work. Meson masses (in MeV) and meson-nucleon coupling constants were adopted from Refs. [27] (HS) and [28] (L1 and L3). Two sets [denoted by (a) and (b)] of the coupling ratios $\alpha_i = g_{i\Lambda}/g_{iN}$ $(i=\sigma,\omega)$ are presented, as well.

	m_σ	$g_{\sigma N}$	m_ω	$g_{\omega N}$	α_σ	
					(a) $(\alpha_\omega=1/3)$	(b) $(\alpha_\omega=2/3)$
HS	520.0	10.481	783.0	13.814	0.342	0.623
L1	550.0	10.30	783.0	12.60	0.334	0.607
L3	492.26	10.692	780.0	14.8705	0.341	0.624

The modification of the harmonic oscillator parameters due to the second Λ can be neglected as it is expected to be negligible in comparison with the above $\Delta E_{\rm core}$. This is because, whereas all the core particles contribute to $\Delta E_{\rm core}$, only the added Λ is affected by the change of the harmonic oscillator parameters.

Furthermore, one can expect that the rearrangement energy of the two Λ hypernucleus is approximately twice the one of the single Λ hypernucleus. Consequently, we obtain

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_\Lambda \simeq -(E_R+E_r)+E_R+E_r(U=0)$$
$$= E_r(U=0)-E_r. \tag{16}$$

The alternative and equivalent way to determine $\Delta B_{\Lambda\Lambda}$ is to fit the harmonic oscillator parameters in such a way that the Λ binding energy $B_\Lambda = -\frac{1}{2}[E_R+E_r(U=0)]$ in the corresponding hypernucleus reproduces the empirical binding energies. Now B_Λ immediately incorporates $\Delta E_{\rm core}$, and similarly $B_{\Lambda\Lambda}=-(E_R+E_r)$ includes the rearrangement of the core caused by the two Λ particles $\simeq 2\Delta E_{\rm core}$. As a result, relation (16) is fulfilled again.

III. THE RESULTS

The model presented in the previous section was applied to calculation of $\Delta B_{\Lambda\Lambda}$ for the following sample of double Λ hypernuclei: ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, ${}^{13}_{\Lambda\Lambda}$B, ${}^{18}_{\Lambda\Lambda}$O, ${}^{42}_{\Lambda\Lambda}$Ca, ${}^{92}_{\Lambda\Lambda}$Zr, and ${}^{210}_{\Lambda\Lambda}$Pb, which includes the measured cases.

Fitting the HO parameters $\hbar\omega$ and W_0 requires two experimental values. Starting from ${}^{13}_{\Lambda}C$ they are given by the $1s$ and $1p$ Λ binding energies. For the three lightest elements, where the $1p$ level is unbound, we extrapolated the sp splitting from the C and O region.

We used three different RMF models, namely, the HS model of Horowitz and Serot [27], and models L1 and L3 of Lee *et al.* [28]. The masses and meson-nucleon coupling constants of σ and ω mesons are presented in Table III. Different parametrizations allowed us to study the dependence of $\Delta B_{\Lambda\Lambda}$ on the mass of the σ meson m_σ. One would expect that the smaller values of m_σ (t.e., model L3) will give larger correlation energy and consequently larger $\Delta B_{\Lambda\Lambda}$.

The couplings of the Λ hyperon to the meson fields are often defined via coupling constant ratios $\alpha_i = g_{i\Lambda}/g_{iN}$, $i=\sigma,\ \omega$. For each of the above RMF parametrizations we used two coupling ratios $\alpha_\omega = 1/3$ (a) and 2/3 (b). Whereas the value of 2/3 is predicted by the constituent quark model, 1/3 ratio was widely used in the pioneering RMF hypernuclear calculations. The corresponding α_σ was then chosen to fit the hypernuclear spectra [29]. The ratios α_i are included in the list of parameters in Table III, as well.

TABLE IV. $\Delta B_{\Lambda\Lambda}$ (in MeV) for the σ-ω model with the parametrizations of Table III as a function of the mass number.

	He	Be	B	O	Ca	Zr	Pb
HS							
(a)	0.96	0.93	0.91	0.78	0.55	0.43	0.20
(b)	1.82	1.69	1.60	1.41	1.01	0.79	0.38
L1							
(a)	0.78	0.76	0.74	0.63	0.44	0.34	0.16
(b)	1.22	1.13	1.07	0.94	0.67	0.53	0.25
L3							
(a)	1.05	1.02	0.99	0.86	0.61	0.48	0.23
(b)	2.33	2.17	2.05	1.82	1.31	1.04	0.51

The $\Delta B_{\Lambda\Lambda}$ corresponding to the different parametrizations of Table III are displayed in Table IV. The results indicate that $\Delta B_{\Lambda\Lambda}$ depends on the model used. The values of $\Delta B_{\Lambda\Lambda}$ are larger for lower values of m_σ as predicted. In addition, $\Delta B_{\Lambda\Lambda}$ is quite sensitive to the coupling ratios α_ω. Whereas for $\alpha_\omega = 1/3$ there is hardly any improvement over the RMF values, 0.5–1.0 MeV is gained with $\alpha_\omega = 2/3$.

The results of Table IV indicate also that including the correlation energy from the σ and ω exchange, though sizable in the case of $\alpha_\omega = 2/3$, cannot by itself account for empirical 4.5 MeV of the $\Delta B_{\Lambda\Lambda}$ in light hypernuclei. Note that the results of Table IV, for light nuclei, cannot be compared directly to those of Table I on a quantitative level, because of small differences used in each calculation. Qualitatively, however, the strong dependence of the correlation energy on the strength of the ω coupling and its large incidence on $\Delta B_{\Lambda\Lambda}$ for 2/3 can be taken for granted. Therefore, according to the chosen parametrization, at least half of the empirical $\Delta B_{\Lambda\Lambda}$ has to come from ''new'' meson exchanges that are not included in the original versions of RMF models.

In order to investigate the range of coupling constants needed to bring the calculated $\Delta B_{\Lambda\Lambda}$ into agreement with experiments, we followed the work of Schaffner *et al.* [14] and assumed scalar σ^* and vector Φ meson fields. We addopted their meson masses, namely, $m_{\sigma^*} = 975.0$ MeV and $m_\Phi = 1020.0$ MeV, respectively. Similarly the Φ coupling is taken from the SU(6) relations, $\alpha_\Phi = g_{\Phi\Lambda}/g_{\omega N} = -\sqrt{2}/3$. Contrary to their work, the σ^* coupling is considered as a free parameter to be fitted to the empirical values of $\Delta B_{\Lambda\Lambda}$. Since σ^* and Φ act only between two Λ's, they simply modify the potential $U(r)$ to be used in Eq. (14).

The calculated $\Delta B_{\Lambda\Lambda}$ as a function of α_{σ^*} are presented in Fig. 1 for ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, and ${}^{13}_{\Lambda\Lambda}$B. Use is made of the HS parametrization of the RMF, which stands roughly in between the two other cases L_1 and L_3 as for the magnitude of the predicted $\Delta B_{\Lambda\Lambda}$. The two sets of curves corresponding to $\alpha_\omega =$ 1/3 and 2/3 intercept the domain defined by the experimental values at α_{σ^*} around 0.71 and 0.79, respectively. In view of the large experimental errors (Table I) these values are only approximates. Nevertheless, it means that in spite of the correlation effects taken into account,

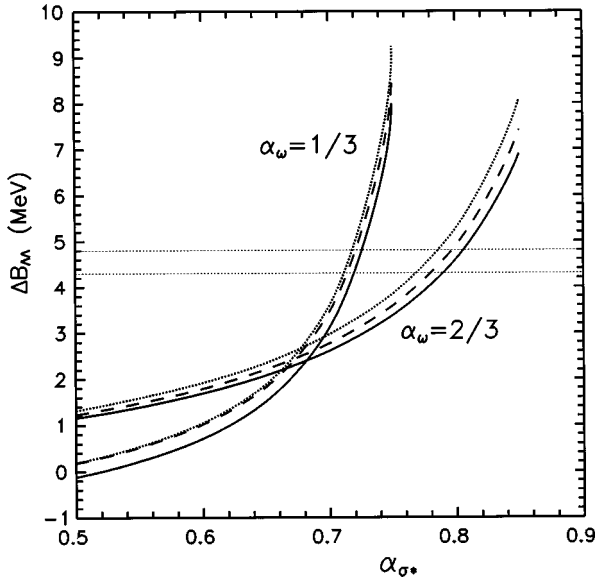

FIG. 1. $\Delta B_{\Lambda\Lambda}$ for ${}^{6}_{\Lambda\Lambda}$He (dotted line), ${}^{10}_{\Lambda\Lambda}$Be (dashed line), and ${}^{13}_{\Lambda\Lambda}$B (solid line) as a function of the coupling ratio $\alpha_{\sigma*}$ calculated for two different Λ-ω coupling ratios ($\alpha_\omega=1/3$ and 2/3) using the HS parametrization. The horizontal dotted lines indicate the spreading of experimental values without errors (see Table I).

which reduce the short range repulsion effect, the largest repulsive α_ω implies the largest attractive $\alpha_{\sigma*}$ coupling.

Having fixed $\alpha_{\sigma*}$ we performed the calculations for the set of double Λ hypernuclei mentioned above. It provides us with the prediction of the A dependence of $\Delta B_{\Lambda\Lambda}$. The results are displayed in Fig. 2. We observed a decrease of the bond energy with A which is roughly the one predicted by a crude perturbative estimate of the $\Lambda\Lambda$ interaction. This result confirms recent calculations by Lanskoy *et al.* [11], based on a Skyrme-Hartree-Fock approach, which show a comparable decrease of $\Delta B_{\Lambda\Lambda}$ with A.

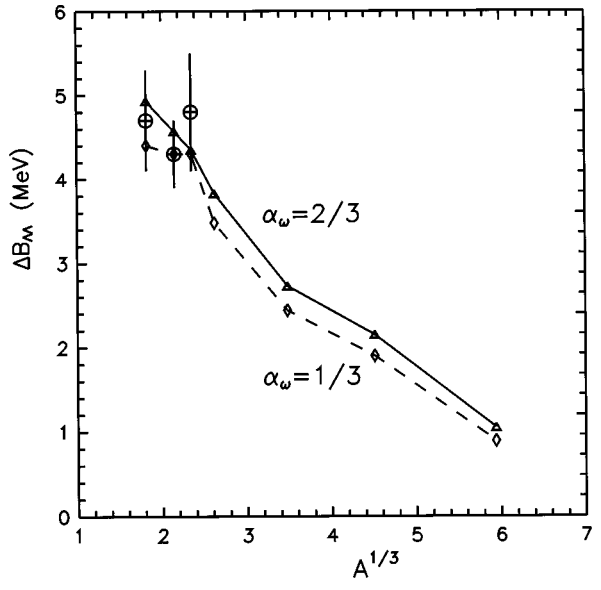

FIG. 2. $\Delta B_{\Lambda\Lambda}$ as a function of A calculated within HS parametrization for two different Λ-ω coupling ratios: $\alpha_\omega=1/3$ (dashed line) and $\alpha_\omega=2/3$ (solid line). Experimental values are also displayed.

We shall end this section by discussing two effects which could qualitatively affect the present conclusions. The first one concerns the spin dependence of the $\Lambda\Lambda$ potential. Whereas the two Λ are in a singlet state, the actual determination of the Λ coupling constants relies on the ΛN spin average. In other words, the $\Lambda\Lambda$ interaction is somewhat underestimated, the singlet potential being known to be more attractive than the triplet one.

In order to get an idea of how much this effect influences the value of the coupling constants, we compared the singlet and triplet effective YNG interactions of Yamamoto and Bando [30]. The ratio of their strengths was then used to determine $V^{\Lambda N}_{\text{singlet}}$ from the spin average $V^{\Lambda N}$ RMF interaction. We left the vector coupling unchanged and modified the scalar coupling constant. The resulting α_σ relevant for the singlet state increased by 2.8 and 4.3% for $\alpha_\omega=2/3$ and 1/3, respectively.

The second effect is acting in the opposite direction; namely, the ΛN interaction determined from hypernuclei contains implicit correlations, whereas the estimate of the $\Lambda\Lambda$ correlation energy should rely on the bare potential. This last should be determined from the RMF interaction by unfolding with an appropriate ΛN correlation function.

To obtain at least a rough estimate of the effect we used the correlation function of Pareño *et al.* [31] and folded the $U(r)$ interaction entering Eq. (14). The α_σ coupling constant ratio appearing in $U(r)$ is then decreased in order to get exactly the same eigenvalue of Eq. (14) as before. We determine in this way the bare scalar coupling constant $g_{\sigma\Lambda}$ while α_ω is kept unchanged. This procedure ends in a decrease of α_σ by 6.3 and 1.7% for $\alpha_\omega=2/3$ and 1/3, respectively. These results have been confirmed by a second estimate based on a correlation function constructed from the approach described in [32], which leads to even slightly lower values.

Adding the two effects we conclude that they tend to cancel each other to a large extent. Consequently the RMF coupling constants might change up to 4%. The uncertainty in the results of Table IV due to the neglecting of these two effects are well within the approximations used in the present model.

IV. CONCLUSIONS

This paper is devoted to the binding energy of double Λ hypernuclei, more precisely to the bond energy $\Delta B_{\Lambda\Lambda}$ as defined by Eq. (1). We show that within the relativistic mean field approach, part of this energy is provided by the short range correlation, the remaining being due to the exchange of σ^* and Φ mesons between the two Λ's. The balance between the two effects depends sensitively on the coupling of the Λ to the ω field. Whereas for a coupling constant $\alpha_\omega=1/3$ the correlation effects are not very efficient, they become sizable at higher values, doubling the RMF results for $\alpha_\omega=2/3$.

The present results have been obtained by reducing a relativistic two-body equation of the Breit type to a Schrödinger equation. Furthermore, advantage has been taken of the fact that the average potential experienced by the Λ in a nucleus is very close to a harmonic oscillator potential. In this way the calculations are considerably simplified.

Although more sophisticated calculations are desirable, they are not expected to change the present results, at least at a semiquantitative level. We recall that for reasons stated in the Introduction our estimate is an upper limit to the Λ-Λ correlation energy.

We found that, according to the ω-Λ coupling, at least half of $\Delta B_{\Lambda\Lambda}$ arises from the meson exchanges specific to the ΛΛ interaction. In such a case one may suspect the argument advocated in the Introduction, stating that for ordinary nuclei ΔS is essentially due to correlation effects. Actually, it is very easy to get convinced from toy models that the gain in binding energy coming from the short-range two body correlation is dominated by the repulsive ω field. Indeed, assuming $\alpha_\omega = 1$. $\Delta B_{\Lambda\Lambda}$ gets close to 3.5 MeV. Thus, the difference between the Λ and the nucleon case reflects the strength of their coupling to the ω field.

We remind the reader that the RMF theory cannot compete with more elaborate three-body (cluster) calculations of $\Delta B_{\Lambda\Lambda}$. In particular for such a light system as ${}^{6}_{\Lambda\Lambda}$He, its application is questionable. In view of extensions to multi-Λ systems, however, it is important to check the constraints it brings on the coupling of the Λ to the various meson fields. In this respect, it would be very desirable to obtain experimental data for heavier nuclear cores than those actually available.

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