Nucleon-nucleon phase shifts and pairing in neutron matter and nuclear matter

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We consider $^1S_0$ pairing in infinite neutron matter and nuclear matter and show that in the lowest order approximation, where the pairing interaction is taken to be the bare nucleon-nucleon interaction in the $^1S_0$ channel, the pairing interaction and the energy gap can be determined directly from the $^1S_0$ phase shifts. This is due to the almost separable character of the nucleon-nucleon interaction in this partial wave. These results put an interaction-independent upper limit on the value of the gap, and on the density where $^1S_0$ superfluidity disappears in neutron matter and nuclear matter. [S0556-2813(98)00303-3]

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Recently, there has been renewed interest in the pairing problem in neutron matter and neutron-rich nuclei. The superfluid properties of neutron matter is of importance in the study of neutron stars [1], while pairing in neutron-rich systems is of relevance for the study of heavy nuclei close to the drip line [2] and the light halo nuclei [3]. Much effort has gone into calculating the superfluid energy gap in dilute neutron matter [4–8]. Most of these studies, e.g., those of Refs. [4,6–8] have been carried out using pairing matrix elements given by the bare nucleon-nucleon (NN) interaction. Many of the same authors have calculated the $^1S_0$ gap in nuclear matter, which has also been the subject of recent relativistic formulations of the pairing problem [9–11].

In this paper, we stay within the simplest treatment of superfluidity in infinite matter, where the pairing interaction is chosen equal to the bare interaction. The next contribution, the so-called induced interaction or polarization term, was first demonstrated to be important by Clark *et al.* [12]. Recent evaluations of polarization effects [13,14], indicate a substantial reduction of the maximum value of the $^1S_0$ energy gap, while the range of densities where it is nonzero stays more or less the same. Our motivation for using the simplest approach to the problem is as follows. First of all, in this lowest-order approximation to the problem it has been found that results for the $^1S_0$ energy gap in neutron matter and in nuclear matter are almost independent of the choice of NN interaction. We aim at explaining how this can be understood directly from the measured properties of the free NN interaction. Although a relation between the pairing gap and NN phase shifts was obtained almost 40 years ago by Emery and Sessler [15] (see also Hoffberg *et al.* [16]), in this work we wish to focus on the near interaction independence of the results for the energy gap at the Fermi level, and try to explain this from the NN scattering data directly. Our investigation is similar in spirit to the work of Refs. [8,17] where the relation between the $^1S_0$ scattering amplitude and the gap function in momentum space was clarified. In this paper, however, the focus is on the size of the energy gap at the Fermi momentum and how well this quantity is determined by NN scattering data. Secondly, with the results from the induced-interaction calculations in mind, we argue that our results give a model-independent upper limit for the maximum value of the $^1S_0$ gap, and also an upper limit for the density where the gap goes to zero.

The energy gap in infinite matter is obtained by solving the BCS equation for the gap function $\Delta(k)$,

$$\Delta(k) = -\frac{1}{\pi}\int_0^\infty dk' k'^2 V(k,k')\frac{\Delta(k')}{E(k')}, \tag{1}$$

where $V(k,k')$ is the bare momentum-space NN interaction in the $^1S_0$ channel, and $E(k)$ is the quasiparticle energy given by $E(k)=\sqrt{[\epsilon(k)-\epsilon(k_F)]^2+\Delta(k)^2}$, where $\epsilon(k)$ is the single-particle energy of a neutron with momentum k, and k_F is the Fermi momentum. Medium effects should be included in $\epsilon(k)$, but we will use free single-particle energies $\epsilon(k)=k^2/2m$, where m is the neutron rest mass, to avoid unnecessary complications. The energy gap is defined as $\Delta_F \equiv \Delta(k_F)$. Equation (1) can be solved by various techniques, some of which are described in Refs. [7,8]. In Fig. 1 we show the results for Δ_F obtained with the CD-Bonn potential (full line) [18] and the Nijmegen I and Nijmegen II potentials (long-dashed line and short-dashed line, respectively) [19]. The results are virtually identical, with the maximum

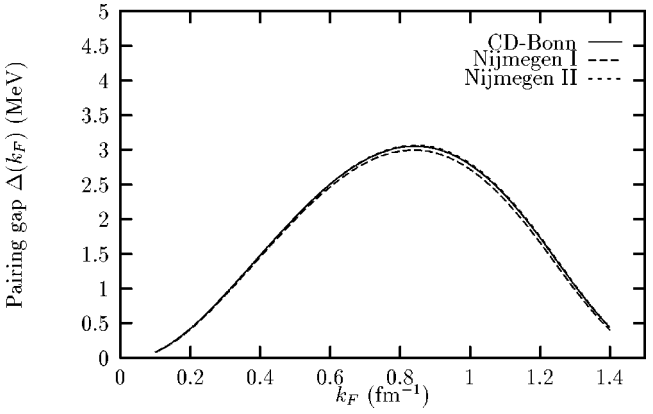

FIG. 1. $^1S_0$ energy gap in neutron matter with the CD-Bonn, Nijmegen I, and Nijmegen II potentials.

value of the gap varying from 2.98 MeV for the Nijmegen I potential to 3.05 MeV for the Nijmegen II potential. The same insensitivity of the energy gap with respect to the choice of *NN* interaction was found in Refs. [4,7,8]. We will now discuss how these results can be understood from the properties of the *NN* interaction in the $^1S_0$ channel.

A characteristic feature of $^1S_0$ *NN* scattering is the large, negative scattering length, indicating the presence of a virtual bound state at $\approx$140 keV scattering energy. This state shows up as a pole in the *NN T* matrix, which then can be written in separable form, and this implies that the *NN* interaction itself to a good approximation is rank-one separable near this pole [20]. Thus, at low energies we can write

$$V(k,k')=\lambda v(k)v(k'), \tag{2}$$

where λ is a constant. Then it is easily seen from Eq. (1) that the gap function can be written as $\Delta_F v(k)$, where Δ_F is the energy gap. Inserting this form of $\Delta(k)$ into Eq. (1) one obtains

$$1=-\frac{1}{\pi}\int_0^\infty dk' k'^2 \frac{\lambda v^2(k')}{E(k')}, \tag{3}$$

which shows that the energy gap Δ_F is determined by the diagonal elements $\lambda v^2(k)$ of the *NN* interaction. The crucial point is that in scattering theory it can be shown that the inverse scattering problem, that is, the determination of a two-particle potential from the knowledge of the phase shifts at all energies, is exactly, and uniquely, solvable for rank-one separable potentials [20,21]. Following the notation of Ref. [20] we have

$$\lambda v^2(k)=-\frac{k^2+\kappa_B^2}{k^2}\frac{\sin\delta(k)}{k}e^{-\alpha(k)}, \tag{4}$$

for an attractive potential with a bound state at energy $E=-\kappa_B^2$. In our case we take $\kappa_B\approx 0$. Here $\delta(k)$ is the $^1S_0$ phase shift as a function of momentum k, while $\alpha(k)$ is given by a principle value integral:

$$\alpha(k)=\frac{1}{\pi}\mathrm{P}\int_{-\infty}^{+\infty} dk' \frac{\delta(k')}{k'-k}, \tag{5}$$

where the phase shifts are extended to negative momenta through $\delta(-k)=-\delta(k)$. Equations (4) and (5) can also be rewritten in terms of the Jost function [21] as done in Ref. [22].

From this discussion we see that $\lambda v^2(k)$, and therefore also the energy gap Δ_F, is completely determined by the $^1S_0$ phase shifts. However, there are two obvious limitations on the practical validity of this statement. First of all, the separable approximation can only be expected to be good at low energies, near the pole in the *T* matrix. Secondly, we see from Eq. (5) that knowledge of the phase shifts $\delta(k)$ at all energies is required. This is, of course, impossible, and most phase shift analyses stop at a laboratory energy $E_{\mathrm{lab}}=350$ MeV. Strictly speaking, the rank-one separable approximation to the $^1S_0$ interaction breaks down already where the $^1S_0$ phase shift changes sign from positive to negative at

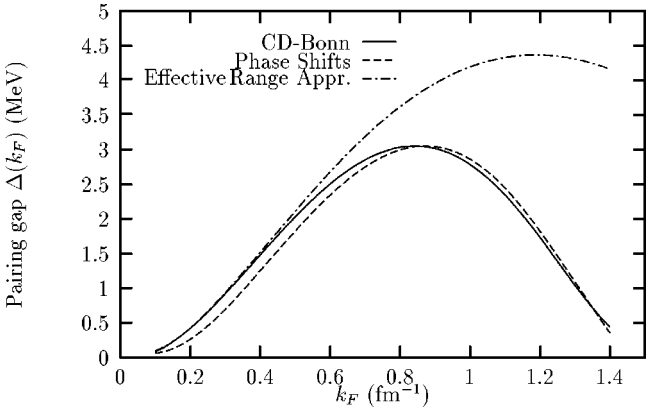

FIG. 2. $^1S_0$ energy gap in neutron matter calculated with the CD-Bonn potential compared with the direct calculation from $^1S_0$ phase shifts.

$E_{\mathrm{lab}}\approx 248$ MeV, corresponding to a single-particle momentum of $k\approx 1.73$ fm^{-1}. However, at low values of k_F, knowledge of $v(k)$ up to this value of k may actually be enough to determine the value of Δ_F, as the integrand in Eq. (3) is strongly peaked around k_F. We therefore found it worthwhile to try to calculate the energy gap directly from the $^1S_0$ phase shifts using Eqs. (3)–(5). A possible improvement to the rank-one separable approach for potentials which change sign is discussed by Kwong and Köhler [22].

The input in our calculation is the $^1S_0$ phase shifts taken from the recent Nijmegen phase shift analysis [23]. We then evaluated $\lambda v^2(k)$ from Eqs. (4) and (5), using methods described in Ref. [24] to evaluate the principle value integral in Eq. (5). Finally, we evaluated the energy gap Δ_F for various values of k_F by solving Eq. (3). Numerically the integral on the right-hand side of this equation depended very weakly on the momentum structure of $\Delta(k)$, so in our calculations we could take $\Delta(k)\approx\Delta_F$ in Eq. (3), and thus it became an algebraic equation for the energy gap Δ_F. The resulting energy gap is plotted in Fig. 2 (dashed line) together with the gap obtained with the CD-Bonn potential (full line). As the reader can see, the agreement between the direct calculation from the phase shifts and the CD-Bonn calculation of Δ_F is very good, even at densities as high as $k_F=1.4$ fm^{-1}. The energy gap is to a great extent determined by the available $^1S_0$ phase shifts. This can also be understood from the fact that for a rank-one separable potential, the equations for the scattering state and the pair state become identical, as also pointed out by Carlson *et al.* [17]. In the same figure we also report the results (dot-dashed line) obtained using the effective range approximation to the phase shifts:

$$k\cot\delta(k)=-\frac{1}{a_0}+\frac{1}{2}r_0k^2, \tag{6}$$

where $a_0=-18.8\pm 0.3$ fm and $r_0=2.75\pm 0.11$ fm are the singlet neutron-neutron scattering length and effective range, respectively. In this case an analytic expression can be obtained for $\lambda v^2(k)$, as shown in Ref. [21]:

$$\lambda v^2(k) = -\frac{1}{\sqrt{k^2 + r_0^2/4(k^2+\alpha^2)^2}} \sqrt{\frac{k^2+\beta_2^2}{k^2-\beta_1^2}}, \tag{7}$$

with $\alpha^2 = -2/a_0 r_0$, and where $\beta_1 \approx -0.0498$ fm^{-1} and $\beta_2 \approx 0.777$ fm^{-1} are the two roots of the quadratic equation

$$\beta^2 - \frac{2}{r_0}\beta - \alpha^2 = 0. \tag{8}$$

The phase shifts using this approximation are positive at all energies, and this is reflected in Eq. (7) where $\lambda v^2(k)$ is attractive for all k. From Fig. 1 we see that below $k_F = 0.5$ fm^{-1} the energy gap can with reasonable accuracy be calculated with the interaction obtained directly from the effective range approximation. One can therefore say that at densities below $k_F = 0.5$ fm^{-1}, and at the crudest level of sophistication in many-body theory, the superfluid properties of neutron matter are determined by just two parameters, namely the free-space scattering length and effective range. At such densities, more complicated many-body terms are also less important. Also interesting is the fact that the phase shifts predict the position of the first zero of $\Delta(k)$ in momentum space, since we see from Eq. (4) that $\Delta(k) = \Delta_F v(k) = 0$ first for $\delta(k) = 0$, which occurs at $E_{\rm lab} \approx 248$ MeV (pp scattering) corresponding to $k \approx 1.73$ fm^{-1}. This is in good agreement with the results of Khodel *et al.* [8]. In Ref. [8] it is also shown that this first zero of the gap function determines the Fermi momentum at which $\Delta_F = 0$. Our results therefore indicate that this Fermi momentum is in fact given by the energy at which the $^1S_0$ phase shifts become negative. This can also be seen from the weak coupling approximation to the gap at the Fermi momentum $\Delta_F = 2\epsilon_F \exp[-1/N(0)\lambda v^2(k_F)]$, when $v(k_F) = 0$, $\Delta_F = 0$.

The calculation of the $^1S_0$ gap in symmetric nuclear matter is closely related to the one for neutron matter. In fact, with charge-independent forces, such as the older Bonn potentials, and free single-particle energies one would, of course, obtain exactly the same results. However, the new potentials on the market are charge dependent, in order to achieve high quality fits to both np and pp scattering data, and therefore we should in principle solve three coupled gap equations for neutron-neutron (nn), proton-proton (pp), and neutron-proton (np) pairing [25]:

$$\Delta_i(k) = -\frac{1}{\pi}\int_0^\infty dk' k'^2 V_i(k,k') \frac{\Delta_i(k')}{E(k')}, \tag{9}$$

where $i = nn$, pp, and np, and the quasiparticle energy is still given by $E(k) = \sqrt{[\epsilon(k) - \epsilon(k_F)]^2 + \Delta(k)^2}$, but the energy gap is now given by

$$\Delta(k)^2 = \Delta_{nn}(k)^2 + \Delta_{pp}(k)^2 + \Delta_{np}(k)^2. \tag{10}$$

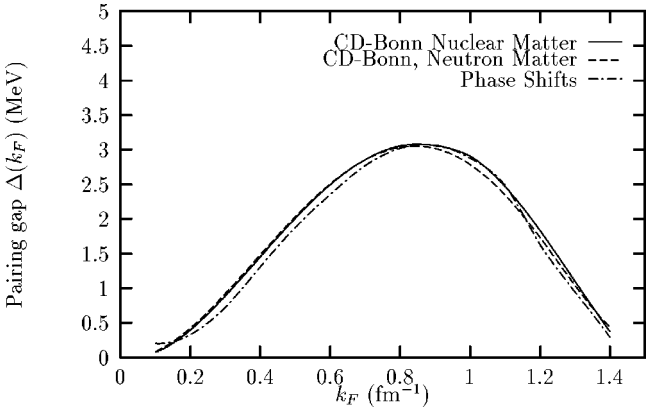

FIG. 3. $^1S_0$ energy gap in nuclear matter calculated with the CD-Bonn potential compared with the direct calculation from the $^1S_0$ np and pp phase shifts. Also shown are the results for neutron matter with the CD-Bonn potential.

Solving these equations, both with the CD-Bonn potential and with the phase shift approximation we get the results shown in Fig. 3. For comparison we have in the same figure plotted the results for pure neutron matter with the CD-Bonn potential (dashed line). From the figure it is clear that the phase shift approximation works well also in this case. As could be expected, the results are very close to those obtained earlier with charge-independent interactions [4–7].

In summary, we have shown that in infinite neutron and nuclear matter, owing to the near rank-one separability of the NN interaction in the $^1S_0$ partial wave, we are able to compute the $^1S_0$ pairing gap directly from the NN phase shifts. This explains why all NN potentials which fit the scattering data result in almost identical $^1S_0$ pairing gaps. Our findings conform with the conclusions of Khodel *et al.* [8] and Carlson *et al.* [17]: The virtual bound state in $^1S_0$ NN scattering determines the features of nucleon pairing in that partial wave. Even though this result is not likely to survive in a more refined calculation, for instance, if one includes polarization effects in the effective pairing interaction as in, e.g., Refs. [13,14], one can argue that our results demonstrate that upper limits for the value of the energy gap and for the density where a $^1S_0$ neutron/nucleon superfluid can exist, can be set directly from the $^1S_0$ phase shifts, since the polarization term serves to cut down the value of the gap, and leave the upper density for this superfluid more or less unchanged. These are the main results of this paper.

The fact that a bound state or a virtual bound state can be used to determine the properties of pairing in a physical system, may be of use in studies of superfluidity and superconductivity in atomic gases, such as a spin-polarized ^{6}Li gas, recently studied by Stoof *et al.* in [26]. The scattering length of lithium is large and negative, as is the case for the $^1S_0$ state discussed here. Since this is a very dilute system one can then even use an effective range approach to the interparticle interaction and determine the gap uniquely for such dilute systems, by simply employing a separable interaction of the form shown in Eq. (7) and discussed in Fig. 2.

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