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Light front treatment of the nucleus: Implications for deep inelastic scattering

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A light front treatment of the nuclear wave function is developed and applied, using the mean field approximation, to infinite nuclear matter. The nuclear mesons are shown to carry about a third of the nuclear plus momentum p^+; but their momentum distribution has support only at $p^+=0$, and the mesons do not contribute to nuclear deep inelastic scattering. This zero mode effect occurs because the meson fields are independent of space-time position. [S0556-2813(97)50207-X]

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The discovery that the deep inelastic scattering structure function of a bound nucleon differs from that of a free one (the EMC effect [1]) changed the way that physicists viewed the nucleus. With a principal effect that the plus momentum (energy plus third component of the momentum, $p^0+p^3 \equiv p^+$) carried by the valence quarks is less for a bound nucleon than for a free one, quark and nuclear physics could not be viewed as being independent. Many different interpretations and related experiments [2] grew out of the desire to better understand the initial experimental observations.

The interpretation of the experiments requires that the role of conventional effects, such as nuclear binding, be assessed and understood [2]. Nuclear binding is supposed to be relevant because the plus momentum of a bound nucleon is reduced by the binding energy, and so is that of its confined quarks. Conservation of momentum implies that if nucleons lose momentum, other constituents such as nuclear pions [3], must gain momentum. This partitioning of the total plus momentum amongst the various constituents is called the momentum sum rule. Pions are quark antiquark pairs so that a specific enhancement of the nuclear antiquark momentum distribution, mandated by momentum conservation, is a testable [4] consequence of this idea. A nuclear Drell Yan experiment [5], in which a quark from a beam proton annihilates with a nuclear antiquark to form a $\mu^+\mu^-$ pair, was performed. No influence of nuclear pion enhancement was seen, leading Bertsch *et al.* [6] to question the idea that the pion is a dominant carrier of the nuclear force.

Here a closer look at the relevant nuclear theory is taken, and the momentum sum rule is studied. The first step is to discuss the appropriate coordinates. The structure function depends on the Bjorken variable $x_{\rm Bj}$ which in the parton model is the ratio of the quark plus momentum to that of the target. Thus $x_{\rm Bj}=p^+/k^+$, where k^+ is the plus momentum of a nucleon bound in the nucleus, so a more direct relationship between the necessary nuclear theory and experiment is obtained by using a theory in which k^+ is one of the canonical variables. Since k^+ is conjugate to a spatial variable $x^- \equiv t - z$, it is natural to quantize the dynamical variables at the light cone time variable of $x^+ \equiv t+z$. To use such a formalism is to use light front quantization, since the other three spatial coordinates $(x^-, \mathbf{x}_\perp)$ are on a hyperplane perpendicular to a light like vector [7]. This light front quantization requires a new derivation of the wave function, because previous work used the equal time formalism.

Such a derivation is provided here, using a simple renormalizable model in which the nuclear constituents are nucleons ψ (or ψ'), scalar mesons ϕ [8], and vector mesons V^μ. The Lagrangian $\mathcal{L}$ is given by

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi\partial^\mu\phi-m_s^2\phi^2)-\frac{1}{4}V^{\mu\nu}V_{\mu\nu}+\frac{m_v^2}{2}V^\mu V_\mu + \overline{\psi'}(\gamma^\mu(i\partial_\mu-g_v V_\mu)-M-g_s\phi)\psi', \tag{1}$$

where the bare masses of the nucleon, scalar, and vector mesons are given by M, m_s, m_v, and $V^{\mu\nu}=\partial^\mu V^\nu-\partial^\nu V^\mu$. This Lagrangian may be thought of as a low-energy effective theory for nuclei under normal conditions. Quarks and gluons would be the appropriate degrees of freedom at higher energies and momentum transfer. Understanding the transition between the two sets of degrees of freedom is of high present interest, and using a relativistic formulation of the hadronic degrees of freedom is necessary to avoid a misinterpretation of a kinematic effect as a signal for the transition.

This hadronic model, when evaluated in mean field approximation, gives [9] at least a qualitatively good description of many (but not all) nuclear properties and reactions. The aim here is to use a simple Lagrangian to study the effects that one might obtain by using a light front formulation. It is useful to simplify this first calculation by studying infinite nuclear matter which has no surface effects.

The light front quantization necessary to treat nucleon interactions with scalar and vector mesons was derived by Yan and collaborators [10,11]. Glazek and Shakin [12] used a Lagrangian containing nucleons and scalar mesons to study infinite nuclear matter. Here vector and scalar mesons are included, and the nuclear plus momentum distribution is obtained.

The next step is to examine the field equations. The nucleons satisfy

$$\gamma(i\partial-g_v V)\psi'=(m+g_s\phi)\psi' . \tag{2}$$

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The independent and dependent Fermion degrees of freedom in light front field theories are constructed using projection operators $\Lambda_\pm \equiv \gamma^0\gamma^\pm/2$, such that $\psi'_\pm \equiv \Lambda_\pm \psi'$ [13]. The relation between the dependent variable ψ'_- and the independent variable ψ'_+ is very complicated unless one may set the plus component of the vector field to zero. This simplification is a matter of a choice of gauge for QED and QCD, but the nonzero mass of the vector meson prevents such a choice here. Instead, one simplifies the equation for ψ'_- by transforming [14] the Fermion field according to $\psi' = e^{-ig_v\Lambda(x)}\psi$ with $\partial^+\Lambda = V^+$. This leads to the result

$$(i\partial^- - g_v\overline{V^-})\psi_+ = [\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\overline{\boldsymbol{V}_\perp}) + \beta(M+g_s\phi)]\psi_-,$$

$$i\partial^+\psi_- = [\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\overline{\boldsymbol{V}_\perp}) + \beta(M+g_s\phi)]\psi_+, \tag{3}$$

where

$$\partial^+\overline{V^\mu} = \partial^+V^\mu - \partial^\mu V^+. \tag{4}$$

The field equations for the mesons are

$$\partial_\mu V^{\mu\nu} + m_v^2 V^\nu = g_v\bar{\psi}\gamma^\nu\psi,$$

$$\partial_\mu\partial^\mu\phi + m_s^2\phi = -g_s\bar{\psi}\psi. \tag{5}$$

The mean field approximation [9] is implemented by assuming that the coupling constants are considered strong and the Fermion density large. Then the meson fields can be approximated as classical—the sources of the meson fields are replaced by their expectation values. In this case, the nucleon mode functions will be plane waves and the nuclear matter ground state can be assumed to be a normal Fermi gas, of Fermi momentum k_F, and of large volume Ω in its rest frame. The number of protons is set equal to the number of neutrons. Then the meson fields are constants given by

$$\phi = -\frac{g_s}{m_s^2}\langle\bar{\psi}\psi\rangle,$$

$$V^\mu = \frac{g_v}{m_v^2}\langle\bar{\psi}\gamma^\mu\psi\rangle = \delta^{0,\mu}\frac{g_v\rho_B}{m_v^2}, \tag{6}$$

where $\rho_B = 2k_F^3/3\pi^2$. The expectation values refer to the nuclear matter ground state.

The quantity $\overline{V^\mu}$ can be obtained from Eqs. (4) and (6). In particular, $\overline{V^+} = 0$ by construction. Furthermore, the conditions that $V^i = 0$ and $\partial^i V^+ = \partial^i V^0 = 0$ tell us that $\overline{V^i} = 0$. Finally $\partial^+\overline{V^-} = \partial^+ V^0$, so the only nonvanishing component of $\overline{V^\mu}$ is $\overline{V^-} = V^0$.

The fields ϕ and $\overline{V^-}$ are constants, within the mean field approximation, so the solutions of Eq. (3) are of the plane wave form $\sim e^{ik\cdot x}$. That equation can then be rewritten as

$$(i\partial^- - g_v\overline{V^-})\psi_+ = \frac{\boldsymbol{k}_\perp^2 + (M+g_s\phi)^2}{k^+}\psi_+. \tag{7}$$

The light front eigenenergy ($i\partial^- \equiv k^-$) is the sum of a kinetic energy term in which the mass is shifted by the presence of the scalar field [recall that for free nucleons $k^- = (\boldsymbol{k}_\perp^2 + M^2)/k^+$)], and an energy arising from the vector field. The nucleon field operator is constructed using the solutions of Eq. (7) as the plane wave basis states [15]. This means that the nuclear matter ground state, defined by operators that create and destroy baryons in eigenstates of Eq. (7), is the correct wave function and that Eqs. (6) and (7) represent the solution of the approximate field equations, and the diagonalization of the Hamiltonian.

The computation of the energy and plus momentum distribution proceeds from taking the appropriate expectation values of the energy momentum tensor $T^{\mu\nu}$ [10,11],

$$P^\mu = \frac{1}{2}\int d^2x_\perp dx^- \langle T^{+\mu}\rangle. \tag{8}$$

We are concerned with the light front energy P^- and momentum P^+. The relevant components of $T^{\mu\nu}$ can be obtained from Refs. [10] and [11] and the field equations. Within the mean field approximation one finds

$$T^{+-} = m_s^2\phi^2 + 2\psi_+^\dagger(i\partial^- - g_v\overline{V^-})\psi_+,$$

$$T^{++} = m_v^2 V_0^2 + 2\psi_+^\dagger i\partial^+\psi_+. \tag{9}$$

Taking the nuclear matter expectation value of T^{+-} and T^{++}, using Eq. (7), and performing the spatial integral of Eq. (8) leads to the result

$$\frac{P^-}{\Omega} = m_s^2\phi^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ \frac{\boldsymbol{k}_\perp^2 + (M+g_s\phi)^2}{k^+}, \tag{10}$$

$$\frac{P^+}{\Omega} = m_v^2V_0^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ k^+. \tag{11}$$

The subscript F denotes that $|\vec{k}| < k_F$ with k^3 defined by the Glazek-Shakin [12] relation

$$k^+ = \sqrt{(M+g_s\phi)^2 + \boldsymbol{k}^2} + k^3. \tag{12}$$

Note that $\overline{V^-}$, associated by Eq. (3) with the variable k^-, does not appear here. Using Eq. (12) allows one to maintain the equivalence between energies computed in the light front and equal time formulations of scalar field theories [16]. A similar equation has been used to restore manifest rotational invariance in light-front QED [17].

Using Eq. (12) allows us to compute the energy of the system as $E = \frac{1}{2}(P^+ + P^-)$. The resulting expression turns out to be identical to that of the equal time treatment [9] as can be seen by summing Eqs. (10) and (11), and changing integration variables from k^+ to k^3 [$(dk^+/k^+ \rightarrow dk^3/\sqrt{(M+g_s\phi)^2 + \boldsymbol{k}^2}$]. The equality of energies is a nice check on the present result, and confirms the use of Eq. (12), because a manifestly covariant solution of the present problem, yielding the standard expression [9] for the energy, has been obtained [18].

Light front field theory is a Hamiltonian theory with states built on a noninteracting vacuum, so the variational principle is expected to apply. In particular, setting $\partial E/\partial\phi$ to zero reproduces the field equation for ϕ, as is also usual [9]. Moreover, the relation $P^+ = P^-$ (which must hold for the

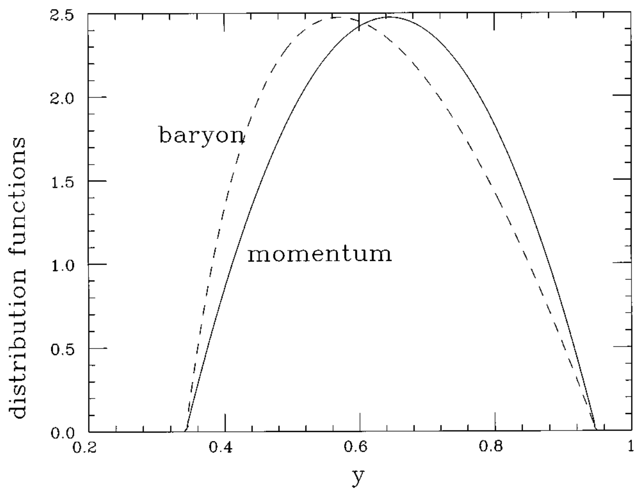

FIG. 1. The momentum distribution, $f(y)$ (solid) and baryon momentum distribution $f_B(y)$ (dashed).

system at rest) is a consequence of minimizing the energy per particle at fixed volume with respect to k_F, or minimizing the energy with respect to the volume [12]. The parameters $g_v^2 M^2/m_v^2 = 195.9$ and $g_s^2 M^2/m_s^2 = 267.1$ have been chosen [19] so as to give the binding energy per particle of nuclear matter as 15.75 MeV with $k_F = 1.42\ \mathrm{fm}^{-1}$. In this case, $g_v \overline{V^-} = 330$ MeV and solving Eq. (6) for ϕ gives $M + g_s\phi = 0.56M$.

The use of Eq. (11) and these parameters leads immediately to the result that only 65% of the nuclear plus momentum is carried by the nucleons; the remainder is carried by the mesons. The nucleonic momentum distribution which is the input to calculations of the nuclear structure function is also of primary interest here. This function can be computed from the integrand of Eq. (11). The probability that a nucleon has plus momentum k^+ is determined from the condition that the plus momentum carried by nucleons, P_N^+, is given by $P_N^+/A = \int dk^+ k^+ f(k^+)$, where $A = \rho_B \Omega$. It is convenient to use the dimensionless variable $y \equiv k^+/\overline{M}$ with $\overline{M} = M - 15.75$ MeV. Then Eq. (11) and simple algebra leads to the equation

$$f(y) = \frac{3}{4}\frac{\overline{M}^3}{k_F^3}\theta(y^+ - y)\theta(y - y^-)\left[\frac{k_F^2}{\overline{M}^2} - \left(\frac{E_F}{\overline{M}} - y\right)^2\right], \tag{13}$$

where $y^\pm \equiv (E_F \pm k_F)/\overline{M}$ and $E_F \equiv \sqrt{k_F^2 + (M + g_s\phi)^2}$. This function is displayed in Fig. 1. Similarly the baryon number distribution $f_B(y)$ (number of baryons per y, normalized to unity) can be determined from the expectation value of $\psi^\dagger\psi$. The result is

$$f_B(y) = \frac{3}{8}\frac{\overline{M}^3}{k_F^3}\theta(y^+ - y)\theta(y - y^-)\left\{\left(1 + \frac{E_F^2}{\overline{M}^2 y^2}\right) \times \left[\frac{k_F^2}{\overline{M}^2} - \left(\frac{E_F}{\overline{M}} - y\right)^2\right] - \frac{1}{2y^2}\left[\frac{k_F^4}{\overline{M}^4} - \left(\frac{E_F}{\overline{M}} - y\right)^4\right]\right\}. \tag{14}$$

Some phenomenological models treat the two distributions $f(y)$ and $f_B(y)$ as identical. The distributions have the same normalization: $\int dy f(y) = 1$, $\int dy f_B(y) = 1$, but they are different.

The nuclear deep inelastic structure function, F_{2A} can be obtained from the light front distribution function $f(y)$ and the nucleon structure function F_{2N} using the relation [20]

$$\frac{F_{2A}(x)}{A} = \int dy f(y) F_{2N}(x/y), \tag{15}$$

where x is the Bjorken variable computed using the nuclear mass divided by $A(\overline{M})$: $x = Q^2/2\overline{M}\nu$. This formula is the expression of the convolution model in which one means to assess, via $f(y)$, only the influence of nuclear binding and Fermi motion. If F_{2N} is obtained from deep inelastic scattering on the free nucleon, other effects such as the nuclear modification of the nucleon structure function (and any influence of the final state interaction between the debris of the struck nucleon and the residual nucleus [21]) are neglected. Consider the present effect of having the average value of y equal to 0.65. Frankfurt and Strikman [2] use Eq. (15) to argue that an average of 0.95 is sufficient to explain the 15% depletion effect observed for the Fe nucleus. The present result then represents a very strong binding effect, even if this infinite nuclear matter result cannot be compared directly with the experiments using Fe targets.

It is interesting to compare the 0.65 fraction with the result of a relativistic calculation using the equal time (et) formalism [22]. In this calculation, which uses Eq. (1) and for which the scalar and vector fields are the same as here, the plus momentum of a nucleon was chosen as the sum of the Dirac eigenenergy and k^3:

$$k_{\mathrm{et}}^+ \equiv \sqrt{(M + g_s\phi)^2 + \boldsymbol{k}^2} + g_v V^0 + k^3. \tag{16}$$

Using this leads to an average nucleon plus momentum fraction $\langle y\rangle_{\mathrm{et}} = (E_F + g_v V^0)/\overline{M}$, which when evaluated with our results for k_F, ϕ, and $\overline{V^-}$, leads to $\langle y\rangle_{\mathrm{et}} = 1.00$. The big difference between our result and the earlier equal time result— compare Eqs. (12) and (16)—arises from our use of the plus momentum as a canonical momentum variable and the consequent use of $T^{+\mu}$ to construct the light front momentum and energy density.

One might think that the mesons, which cause the large binding effect, would also have huge effects on deep inelastic scattering. It is therefore necessary to determine the mesonic momentum distributions. The mesons contribute 35% of the total nuclear plus momentum, but we need to know how this is distributed over different individual values. The paramount feature is that ϕ and V^μ are the same constants for any and all values of the space-time coordinates. This means that the related momentum distribution can only be proportional to a delta function setting both the plus and $\perp$ components of the momentum to zero. This result is attributed to the mean field approximation, in which the meson fields are treated as classical quantitates. Thus the finite plus momentum may be thought of as coming from an infinite number of quanta, each carrying an infinitesimal amount of plus momentum. A plus momentum of 0 can only be accessed experimentally at $x_{Bj} = 0$, which requires an infinite amount of energy. Thus, in the mean field approximation, the scalar and vector mesons cannot contribute to deep inelastic scattering. The usual term for a field that is constant over space is a zero mode, and the present Lagrangian provides a simple example. For finite nuclei, the mesons would carry a very small momentum of scale given by the inverse of the nuclear radius, under the mean field approximation. If fluctuations were to be included, the relevant momentum scale could be of the order of the inverse of the average distance between nucleons (about 2 fm).

The Lagrangian of Eq. (1) and its evaluation in mean field approximation for nuclear matter have been used to provide a simple but semirealistic example. It would be premature to compare the present results with data before obtaining light front dynamics for a model which addresses chiral symmetry, in which the correlational corrections to the mean field approximation are included, and which treats finite nuclei. Thus the specific numerical results of the present work are far less relevant than the central feature that the mesons responsible for nuclear binding need not be accessible in deep inelastic scattering.

More generally, we view the present model as being one of a class of models, such as the conventional shell model and the quark meson coupling model [23] in which the mean field plays an important role. For such models nuclei would have constituents that contribute to the momentum sum rule, but do not contribute to deep inelastic scattering. Thus the predictive and interpretive power of the momentum sum rule is vitiated.

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