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Efimov effect in the nuclear halo ^{14}Be nucleus

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The Efimov effect in the halo nucleus ^{14}Be, considered as a three-body (n-n-^{12}Be) system, is studied employing separable potential for the binary systems. Assuming a possibility for the existence of a low lying s-orbital state for the halo neutrons with ^{12}Be as core, we investigate its effect on the possibility of occurrence of the Efimov states by computing the three-body integral equations. A virtual state of 2 to 4 keV for the n-^{12}Be binary system predicts not only the ground state energy for ^{14}Be, which is in reasonable agreement with the experimental value, but also shows excited states close to about zero binding energy for the halo neutrons. [S0556-2813(97)50107-5]

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The discovery of neutron rich isotopes of the lightest elements on the neutron dripline exhibiting a halo structure has opened up new vistas in research activities. The novel structural features associated with the halo phenomena have been the subject for extensive theoretical and experimental investigations in recent times [1]. From a purely phenomenological standpoint, it is now generally recognized that simple few body cluster models provide a natural premises to account largely for the structural properties of nuclear halos. In the case of two neutron halos, these nuclei, characterized by large spatial extension and very low separation energy of the neutrons, may well be regarded as three-body systems which are ideally suited for studying the Efimov effect [2]. The conditions for the occurrence of Efimov states in two neutron halos have recently been investigated [3,4] employing the Faddeev equations in coordinate space. It has thus been pointed out that the obvious place to look for the Efimov states is among the halo nuclei with the outer neutrons in the sd shells. One of the promising candidates to investigate for the possible occurrence of Efimov states is the halo ^{14}Be nucleus.

Some time ago we proposed a three-body model employing two-body separable potential to study the structural properties of ^{11}Li nucleus [5] (henceforth referred to as [A]). The results on two neutron separation energy, the momentum distributions of the cluster ^{9}Li and of the halo neutrons as well as the root-mean-square separations of n-n and n-^{9}Li pairs within the nucleus were quite satisfactory.

In the present Rapid Communication, we extend this three-body model to investigate the Efimov effect in the nucleus ^{14}Be. For the neutron-neutron and neutron-^{12}Be binary subsystems, we assume the s-state interaction. As far the n-n pair, the separable potential

$$v_{nn}=-\frac{\lambda_n}{2\mu_{nn}}g(p_{nn})g(p'_{nn}),\quad g(p)=\frac{1}{(p^2\ +\ \beta^2)}\quad (1)$$

is known to reproduce the low-energy scattering data reasonably well. Thus for instance, the strength parameter $\lambda_n=18.6\alpha^3$, $\beta=5.8\alpha$ (where α is the deuteron binding energy parameter; $\alpha^2/m=2.225$ MeV) yields the value of the $^1s_0$ scattering length $a_{nn}=-23.69$ fm and that of the effective range $r_{nn}=2.32$ fm. In the present analysis, we keep the parameters of the n-n potential fixed. However, for the n-^{12}Be potential

$$v_{nc}=-\frac{\lambda_c}{2\mu_{nc}}f(p_{nc})f(p'_{nc}),\quad f(p)=\frac{1}{(p^2\ +\ \beta_1^2)}\quad (2)$$

we allow the parameters λ_c and β_1 to vary so as to obtain different sets, producing virtual and bound two-body systems near zero energy. Here it is worth pointing out that while there is an experimental evidence for a narrow peak in ^{13}Be corresponding to a $\frac{5}{2}^+$ ($l=2$) resonance unbound by more than 2 MeV [6], a strong possibility for the existence of a low lying s-orbital state which could have been difficult to identify experimentally cannot be ruled out. To explore the consequences of considering such an intruder state in the context of studying ^{14}Be as a three-body system, particularly, looking for its effect on the possibility of Efimov states, is the main motivation of the present work.

The basic structure of the three-body equation in terms of the spectator functions, $F(p)$ and $G(p)$, of the ^{12}Be core and of the halo neutron satisfying the coupled integral equations is essentially the same as given in [A] [cf. Eqs. (13) and (14) and the details given therein]. However, for the purpose of studying the sensitive computational details of the Efimov effect, we here recast these equations involving only dimensionless quantities. Thus, by defining

$$\tau_n^{-1}(p)F(p)\equiv\phi(p)\quad\text{and}\quad\tau_c^{-1}(p)G(p)\equiv\chi(p)\,,\quad (3)$$

$$\text{where}\quad \tau_n^{-1}(p)=\mu_n^{-1}-\left[\beta_r\left(\beta_r+\sqrt{\frac{p^2}{2a}+\epsilon_3}\right)^2\right]^{-1},\quad (4)$$

$$\text{and}\quad \tau_c^{-1}(p)=\mu_c^{-1}-2a\left[1+\sqrt{2a\left(\frac{p^2}{4c}+\epsilon_3\right)}\right]^{-2},\quad (5)$$

and where $\mu_n=\pi^2\lambda_n/\beta_1^3$ and $\mu_c=\pi^2\lambda_c/2a\beta_1^3$ are now the dimensionless strength parameters. We reduce the two coupled equations into one integral equation for $\chi(\vec{p})$ by

TABLE I. ^{14}Be ground and excited states three-body energy for different two-body input parameters.

n-^{12}Be Energy keV	$\lambda 1$	a_s fm	ϵ_0 keV	ϵ_1 keV	ϵ_2 keV
50	11.71	-21	1350		
5.8	12.32	-61.6	1408	.053	
2	12.46	-105	1450	2.56	0.061
1	12.52	-149	1456	3.8	0.22
0.1	12.62	-483	1488	6.1	0.62
0.05	12.63	-658	1490	6.4	0.68
0.01	12.65	-1491	1490	6.9	0.72

substituting Eq. (13) into Eq. (14) (of Ref. [A]) and using the definition (3) for $\phi(p)$ and $\chi(p)$ given above. Thus

$$\lambda_i \chi(\vec{p}) = \int d\vec{q} k_3(\vec{p},\vec{q},\epsilon)\tau_c(\vec{q})\chi(\vec{q}) + 2\int\int d\vec{q} d\vec{q}\,' k_2 \times(\vec{p},\vec{q},\epsilon)\tau_n(\vec{q})k_1(\vec{q},\vec{q}\,',\epsilon)\tau_c(\vec{q}\,')\chi(\vec{q}\,'), \qquad (6)$$

where the kernels k_1, k_2, and k_3 are the same as given in [A] [of Eq. (15)] except that the variables p, q, etc., are now dimensionless quantities; $p/\beta_1 \to p$ and $q/\beta_1 \to q$, and

$$-\frac{mE}{\beta_1^2} \equiv \epsilon_3, \quad \beta_r = \frac{\beta}{\beta_1}. \qquad (7)$$

The integral Eq. (6) is the eigenvalue equation in λ_i. After having performed the angular integration over the vectors $\vec{q}$ and $\vec{q}\,'$ and having properly symmetrized the final kernel, we compute the above integral equation as an eigenvalue equation in λ_i. In fact by feeding the parameters of the binary systems, i.e., for n-n and n-c potentials in the right-hand side of Eq. (6), we seek the solution of the above equation for the three-body energy parameter ϵ_3 when the eigenvalue λ_i approaches to 1, accurate to at least four decimal places. Here it may be pointed out that the factors τ_n and τ_c defined above through Eq. (4) and Eq. (5) are quite sensitive particularly when the scattering lengths of the binary systems get infinitely large values. In fact, these factors blow up as the variable $p \to 0$ and the three-body energy parameter ϵ_3 approaches extremely small values. This necessitates, from the computational point of view, a rather large size of the three-body matrix with double precision so as to minimize the possible truncation errors.

Table I summarizes the results for the ^{14}Be ground and excited states three-body energy as a result of different two-body input parameters. Keeping the range parameter $\beta_1 = 5.0\alpha$ as fixed, we vary the strength parameter λ_1 to produce virtual n-^{12}Be states at energies varying from 50 keV to 0.01 keV. The corresponding values of the scattering length range from -21.0 fm to -1491.0 fm. We observe that at 50 keV virtual state [7], the three-body binding energy for ^{14}Be, as obtained from Eq. (6), is 1350 keV which is exactly the experimental value of the two neutron separation energy. However, this two-body potential does not reproduce any excited state for ^{14}Be. As the virtual state energy of n-^{12}Be system is decreased, we not only get the ground state energy, but also the excited state energy for the ^{14}Be system. In fact the first excited state appears when the n-^{12}Be virtual state is about 4 keV. At 2 keV virtual state, the second excited state at about 0.06 keV appears for ^{14}Be. We have also considered here the possibility of assuming a nearly zero energy n-^{12}Be bound state. This yields a little higher binding energy for the ^{14}Be ground as well as excited states.

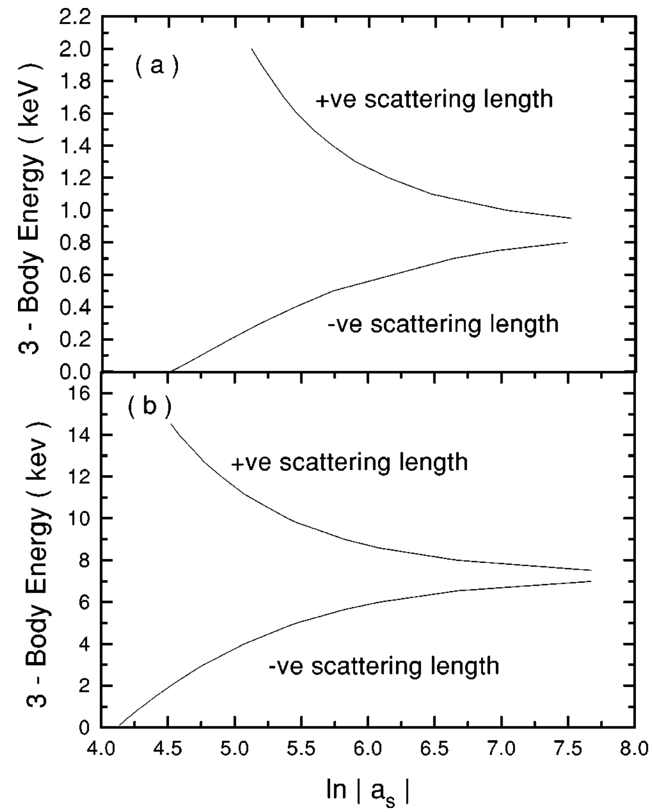

FIG. 1. Plot of three-body energy vs the two-body scattering length (actually, $\ln|a_s|$) for the n-^{12}Be system. (a) shows the behavior of the three-body energy lying in the range from zero to 2 keV and the bottom (b) shows the behavior in higher-energy region (see text).

In Fig. 1 we give a plot of three-body energy vs the two-body scattering length (actually, $\ln|a_s|$) for the n-^{12}Be system. While the lower curve corresponds to the virtual n-^{12}Be states and reproduces the two neutron separation energy on the lower side, the upper curve corresponds to higher separation energy directly proportional to the two-body bound state energy. To depict the behavior of the three-body energy, lying in the range from zero to 2.0 keV, vs the two-body scattering length, we use a different scale for the energy and show the plot in Fig. 1(a).

To summarize, the present analysis makes it amply clear that as the virtual state energy of the binary n-^{12}Be subsystem approaches zero, the resulting three-body system starts swelling and enters into the regime of Efimov effect.

Even a virtual state of a few (2–4 keV) energy corresponding to the scattering length from -50 fm to -100 fm predicts not only the ground state energy, which is in reasonable agreement with the experimental value, but also shows excited states close to about zero binding energy for the halo neutrons. To establish the existence of a low-lying s-orbital n-^{12}Be virtual state in ^{13}Be near zero energy may be a difficult task from the experimental point of view, but, nevertheless, must be challenging from the point of view of seeing the Efimov states as physical reality in halo nuclei.

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[7] The virtual state energy, say, 50 keV for n-^{12}Be system has been defined here by correlating with the value of the scattering length, i.e., $k_s = |1/a_s|$. Thus $E_s = \hbar^2 k_s^2 / 2\mu$, where μ is the reduced mass of the n-^{12}Be system.