

Nucleon strange magnetic moment and relativistic covariance

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(Received 1 May 1997)

We calculate the corrections to the strange matrix elements of the nucleon resulting from the breaking of rotational invariance on the light-cone. In the meson cloud model, the strange magnetic moment μ_S is seen to change sign once the spurious form factors arising from this violation are subtracted. The resulting μ_S is small and slightly positive, in agreement with the trend of the recent data from the SAMPLE experiment. The value of the strange magnetic form factor is predicted to be largely Q^2 independent over the range accessible in upcoming experiments. [S0556-2813(97)50311-6]

PACS number(s): 14.20.Dh, 13.40.Em, 13.60.Fz, 24.85.+p

Unraveling the strangeness content of the nucleon is one of the more intriguing aspects of recent nucleon structure studies [1,2]. From the first polarized deep-inelastic muon-proton scattering experiments at large Q^2 [3], which suggested a negatively polarized strange quark sea in the nucleon, and the studies of elastic neutrino-proton scattering cross sections at small Q^2 [4], evidence has been accumulating for the nontrivial presence of strange quarks in the nucleon. Although the total number of strange and anti-strange quarks in the nucleon must of course be the same, their distributions in coordinate or momentum space need not be equivalent [5]. While perturbative QCD predicts equal s and \bar{s} distributions in the nucleon, there is no fundamental symmetry which imposes this restriction outside of perturbation theory. Understanding the strangeness properties of the nucleon should therefore lead to a better appreciation of the role of nonperturbative QCD in nucleon structure.

In response to the interest created by the original measurements [3,4] in the strangeness distributions in the nucleon, several experiments involving parity-violating electron scattering from protons and deuterons were proposed at MIT/Bates and Jefferson Lab to measure neutral current form factors, from which various strange quark matrix elements could be extracted. Recently the SAMPLE Collaboration at MIT/Bates [6] reported the results of the first measurement of the strange magnetic form factor of the proton, G_M^S , in the elastic scattering of 200 MeV electrons from protons at backward angles with an average $Q^2=0.1$ GeV². The trend of the data indicate a positive magnetic moment to the proton, albeit with large errors [6]: $G_M^S(Q^2=0.1 \text{ GeV}^2)=+0.23\pm 0.44$ nucleon magnetons (n.m.).

A nonzero strangeness content of the nucleon can be naturally accommodated within a number of models of nucleon structure. One of the simplest and most popular of these is the kaon cloud model. Through quantum fluctuations, the nucleon here is viewed as having some probability of dissociating into a virtual kaon and hyperon, both of which carry strangeness quantum numbers. Because of the very different masses and momentum distributions of the kaon and hyperon, the overall strange and anti-strange quark distributions are therefore expected to be quite different, leading, for ex-

ample, to a nonzero strangeness radius, as well as a strange magnetic moment $\mu_S \equiv G_M^S(Q^2=0)$.

A common assumption in the application of the kaon cloud model is the impulse approximation, in which one truncates the hadronic Fock space at the one-meson level, and omits contributions arising from many-body currents. It is known, however, that the use of one-body currents alone for composite systems leads to a violation of Lorentz covariance [7], which would not be the case if one were to include the complete Fock space in the calculation [8]. In this note we will investigate in more detail the consequences of the impulse approximation assumption, and in particular show that the magnitude and even the sign of the strange magnetic moment are very sensitive to the corrections which arise from the Lorentz covariance breaking.

One should note that the issue of relativistic covariance is relevant both for light-front [8] as well as instant-form [9,10] quantization. While on the light-front it is closely connected with the well-known problem of violation of rotational invariance, in the instant-form approach the restriction to one-body currents also leads to a violation of Lorentz covariance, as discussed in Ref. [10]. Irrespective of the orientation of the quantization surface, the problem exists because one-body currents, which do not include interactions, do not commute with the interaction-dependent generators of the Poincaré group. Consequently, an incorrect four-vector structure will appear in the matrix elements of the current operator, resulting in the appearance in the full electroweak current of additional unphysical, or spurious, form factors, which would not be present if the symmetry were preserved. Nevertheless, it is still possible to estimate quantitatively the extent of the covariance violation within a specific model.

In the explicitly covariant formulation of light-front dynamics developed in Refs. [8,11], a specific method was proposed to extract the nucleon's physical form factors, excluding the spurious contributions. It turns out that the unphysical form factors are most evident for quantities that are small in magnitude, such as the neutron's electromagnetic form factors, or the strangeness form factors. The former was investigated in Ref. [8] in a simple constituent quark model of the nucleon, where it was found that the difference between the physical form factors and those obtained from the electromagnetic current without subtracting

the spurious contributions can be as large as 50% at high momentum transfers. In view of the importance of determining accurately the strange properties of the nucleon in current and upcoming experiments, one should obviously reexamine the effect of the impulse approximation in the calculation of strange form factors of the nucleon. As discussed in Ref. [12], there are a number of reasons why a light-cone description of the dynamics is more attractive for this application. We shall, therefore, restrict ourselves to the calculation of the form factors within light-front quantization, and adopt the approach developed in Ref. [8] to expunge the unphysical contributions.

In the covariant light-front approach [8], the state vector is defined on a light-front given by the invariant equation $n \cdot x = 0$, where n is an arbitrary light-like four vector, $n^2 = 0$. In constructing the most general form of the electroweak current on the light-cone one has, in addition to the nucleon p^μ and current q^μ four-momenta, the vector n^μ , specifying the orientation of the light-cone plane. In principle, no physical quantity can depend on n^μ . Following Refs. [8,10], one can show that the most general covariant form of the strange current operator J_μ^S can be written in terms of 5 form factors—the usual F_1^S and F_2^S , plus an additional three, whose coefficients depend on n^μ [8]:

$$J_\mu^S = \bar{u}(p') \left[F_1^S(Q^2) \gamma_\mu + F_2^S(Q^2) \frac{i\sigma_{\mu\nu} q^\nu}{2M} + B_1^S(Q^2) \left(\frac{\not{n}}{n \cdot p} - \frac{1}{(1+\eta)M} \right) P_\mu + B_2^S(Q^2) \frac{M}{n \cdot p} n_\mu + B_3^S(Q^2) \frac{M^2}{(n \cdot p)^2} \not{n} n_\mu \right] u(p), \quad (1)$$

where $\eta = Q^2/4M^2$, $Q^2 = -q^2 = -(p' - p)^2$ and $P = p' + p$, with the auxiliary condition $n \cdot q = 0$. The new n -dependent structures appearing in the decomposition of the amplitude for a spin 1/2 particle in Ref. [8] are essentially the same as those in Ref. [10], differing only in the normalization integral, which in [8] coincides with the electric form factor at $Q^2 = 0$. In principle, the form factors themselves can also depend on the vector n^μ , however, the n -dependence can only enter in the form of the ratio $n \cdot p / n \cdot p'$ [8], which is unity from the condition $n \cdot q = 0$.

One can compare the covariant light-front formulation with the standard approach of calculating matrix elements on the light-cone from the “+” component of the current [13], $J_+^S = n \cdot J^S$, where a particular choice is made for n : $n^\mu = (1; 0, 0, -1)$, i.e., $t + z = 0$. Taking the $\mu = +$ component of the current in Eq. (1) eliminates two of the unphysical contributions, although one is still left with the B_1^S form factor. In fact, for the $\mu = +$ component Eq. (1) can be written

$$J_+^S = \bar{u}(p') \left[F_1^S(Q^2) \gamma_+ + F_2^S(Q^2) \frac{i\sigma_{+\nu} q^\nu}{2M} + 2B_1^S(Q^2) \left(\gamma_+ - \frac{P_+}{(1+\eta)M} \right) \right] u(p) \quad (2a)$$

$$\equiv \bar{u}(p') \left[\bar{F}_1^S(Q^2) \gamma_+ + \bar{F}_2^S(Q^2) \frac{i\sigma_{+\nu} q^\nu}{2M} \right] u(p), \quad (2b)$$

where $\bar{F}_1^S = F_1^S + 2\eta B_1^S / (1 + \eta)$ and $\bar{F}_2^S = F_2^S + 2B_1^S / (1 + \eta)$ (henceforth the symbol \sim refers to form factors obtained from the “+” component of the current). Note that even though Eq. (2b) takes a form just like that allowed by Lorentz covariance constraints, the physical form factors are $F_{1,2}^S$, and not $\bar{F}_{1,2}^S$. The dependence of $\bar{F}_{1,2}^S$ on the spurious B_1^S contribution is a manifestation of the violation of rotational invariance inherent in the “+” prescription, which introduces an asymmetry between the z and (x, y) axes.

An important observation from the above expression for \bar{F}_1^S is that $\bar{F}_1^S(0) = F_1^S(0)$, which is essentially the splitting function from which the deep-inelastic strange quark distribution in the meson cloud model is calculated [12,14]. Consequently, within the light-cone approach the structure functions can be calculated unambiguously within the impulse approximation, without any unphysical n -dependent contributions. Furthermore, for the measured Sachs electric and magnetic form factors one can write the physical form factors as:

$$G_E^S = F_1^S - \eta F_2^S = \bar{G}_E^S, \quad (3a)$$

$$G_M^S = F_1^S + F_2^S = \bar{G}_M^S - 2B_1^S. \quad (3b)$$

Therefore, it is only the magnetic form factor which suffers contamination from the unphysical contributions due to Lorentz covariance breaking, while the electric form factor remains unchanged. This can be understood from the fact that it is only the magnetic form factor which is related to the matrix element of the spatial components of the current, so that it receives contributions from the spurious form factors associated with the rotational invariance breaking.

One should also note that, unlike for spin-1/2 and 1 particles, the matrix elements of the J_+ component for spin-0 particles such as the pion do not involve unphysical form factors [8,10]. For spin-1 systems, for example, the deuteron, analogous expressions for the form factors were obtained in Ref. [11]. In fact, the problem of Lorentz covariance was explored for the deuteron axial current using light-front dynamics in Ref. [15]. There the axial form factor was explicitly shown to be sensitive to the choice of the matrix element of the axial current, which reflects the fact that this operator must contain contributions from two-body currents if rotational covariance is to be maintained [16]. This problem was recently reanalyzed by Keister [17] in the manifestly covariant scheme of Gross [18], and compared with the light-front approach in an attempt to study the sensitivity to the form of relativistic dynamics.

Having outlined the general approach to obtaining the physical form factors of the nucleon consistently within the impulse approximation, we can now proceed to investigate the effect on the strange magnetic form factor within the kaon cloud model. It is straightforward to evaluate the contributions to B_1^S from the interaction of the current with the kaon and hyperon components of the nucleon. (In practice, although the Σ , Σ^* , ... hyperons can be included, the Λ is by far the most dominant contribution to the strange nucleon form factors.) One can define a generic strange form factor G^S as a sum of Λ and K contributions:

$$\mathcal{G}^S(Q^2) = \mathcal{G}^{(\Lambda)}(Q^2) + \mathcal{G}^{(K)}(Q^2), \quad (4)$$

where $\mathcal{G}^S = G_M^S$ or B_1^S . The results for the contributions to \tilde{G}_M^S were given in Ref. [12]. For the interaction with the Λ , the contribution to \mathcal{G}^S can be written

$$\mathcal{G}^{(\Lambda)}(Q^2) = Q_\Lambda \frac{g_{KN\Lambda}^2}{16\pi^3} \int_0^1 dy \int \frac{d^2\mathbf{k}_\perp}{y^2(1-y)} \frac{\mathcal{F}(\mathcal{M}_{\Lambda K,i}^2) \mathcal{F}(\mathcal{M}_{\Lambda K,f}^2)}{(\mathcal{M}_{\Lambda K,i}^2 - M^2)(\mathcal{M}_{\Lambda K,f}^2 - M^2)} \mathcal{T}^{(\Lambda)}, \quad (5)$$

where

$$\mathcal{T}^{(\Lambda)} = \begin{cases} (k_\perp^2(1-2y) - 2(\mathbf{k}_\perp \cdot \hat{\mathbf{q}}_\perp)^2(1-y) + (M_\Lambda^2 - y^2 M^2) + (1-y)^2 q_\perp^2/4)/y & \text{for } G_M^{(\Lambda)}, \\ (-k_\perp^2(1-3y) + 2(\mathbf{k}_\perp \cdot \hat{\mathbf{q}}_\perp)^2(1-y) - (1-y)(M_\Lambda^2 - y^2 M^2) - (1+y)(1-y)^2 q_\perp^2/4)/2y & \text{for } B_1^{(\Lambda)}, \end{cases} \quad (6)$$

where $\hat{\mathbf{q}}_\perp^2 = \mathbf{q}_\perp^2/q_\perp^2$ is a unit vector.

For the $KN\Lambda$ vertex we assume a pseudoscalar $i\gamma_5$ interaction (the same results are obtained with a pseudovector coupling), with $g_{KN\Lambda}$ the coupling constant, and \mathcal{F} the hadronic vertex function, which we parametrize in terms of a cutoff mass $\Lambda_{K\Lambda}$ [12]: $\mathcal{F}(\mathcal{M}_{\Lambda K,i(f)}^2) = (\Lambda_{K\Lambda}^2 + M^2)/(\Lambda_{K\Lambda}^2 + \mathcal{M}_{\Lambda K,i(f)}^2)$. The squared center of mass energies $\mathcal{M}_{\Lambda K,i(f)}^2$ in Eq. (5) depend on the invariant mass squared $\mathcal{M}^2 = (p_K + p_\Lambda)^2 = (k_\perp^2 + M_K^2)/y + (k_\perp^2 + m_K^2)/(1-y)$ of the $K\Lambda$ system, where m_K and M_Λ are the masses of kaon and hyperon, respectively, and p_K and p_Λ are their four-momenta. For the sign of the strangeness we adopt the convention of Ref. [19], in which $Q_\Lambda = +1$ is the strangeness charge of the Λ . One can check the consistency of Eqs. (5) and (6) using Eq. (3b) to obtain the $\tilde{G}_M^{(\Lambda)}$ as calculated in Ref. [12].

The kaon contribution to \mathcal{G}^S can be written

$$\mathcal{G}^{(K)}(Q^2) = Q_K \frac{g_{KN\Lambda}^2}{16\pi^3} \int_0^1 dy \int \frac{d^2\mathbf{k}_\perp}{y(1-y)^2} \frac{\mathcal{F}(\mathcal{M}_{K\Lambda,i}^2) \mathcal{F}(\mathcal{M}_{K\Lambda,f}^2)}{(\mathcal{M}_{K\Lambda,i}^2 - M^2)(\mathcal{M}_{K\Lambda,f}^2 - M^2)} \mathcal{T}^{(K)}, \quad (7)$$

where

$$\mathcal{T}^{(K)} = \begin{cases} 2k_\perp^2 - 2(\mathbf{k}_\perp \cdot \hat{\mathbf{q}}_\perp)^2 & \text{for } G_M^{(K)}, \\ (k_\perp^2(1-y)/y - 2k_\perp^2 + 2(\mathbf{k}_\perp \cdot \hat{\mathbf{q}}_\perp)^2 + (M_\Lambda^2 - y^2 M^2)(1-y)/y - y(1-y)q_\perp^2/4)/2 & \text{for } B_1^{(K)}, \end{cases} \quad (8)$$

and $Q_K = -1$ is the strangeness charge of the kaon. The expression for $G_M^{(K)}$ especially is now very simple, depending only on the relative orientation of the kaon and photon momenta. In particular, there is no contribution to $G_M^{(K)}$ from the configuration where \mathbf{k}_\perp and \mathbf{q}_\perp are parallel, for which it would be impossible to flip the nucleon spin through the interaction with the spin-0 kaon. As we shall see below, the contribution from the K -interaction diagram to the magnetic moment will be instrumental to its change of sign.

The results for the B_1^S form factor turn out to be quite large in magnitude, but opposite in sign compared with \tilde{G}_M^S , for the same value of the cutoff mass parameter $\Lambda_{K\Lambda}$. The combined effect, illustrated in Fig. 1 (solid curve), is a value for μ_S which is almost independent of $\Lambda_{K\Lambda}$. Also shown is the uncorrected value $\tilde{\mu}_S \equiv \tilde{G}_M^S(0)$ (dotted curve), and $-2B_1^S(0)$ (dashed curve). In fact, the $B_1^{(\Lambda)}$ form factor is negative, making the contribution to the magnetic moment from the Λ interaction more positive than for the uncorrected $\tilde{G}_M^{(\Lambda)}$. The same is true for the K interaction contribution, where $B_1^{(K)}$, which is also negative, gives a less negative contribution to $G_M^{(K)}$ compared with $\tilde{G}_M^{(K)}$. The final effect is that both the K interaction and Λ interaction diagrams now

contribute with almost the same magnitude and opposite sign, making the total magnetic moment very small and positive: for $\Lambda_{K\Lambda} = 1$ GeV, one has $G_M^{(\Lambda)}(0) = 0.044$ n.m. and $G_M^{(K)}(0) = -0.034$ n.m., for a total strange magnetic moment $\mu_S = +0.010$ n.m. This is to be compared with $\tilde{G}_M^{(\Lambda)}(0)$

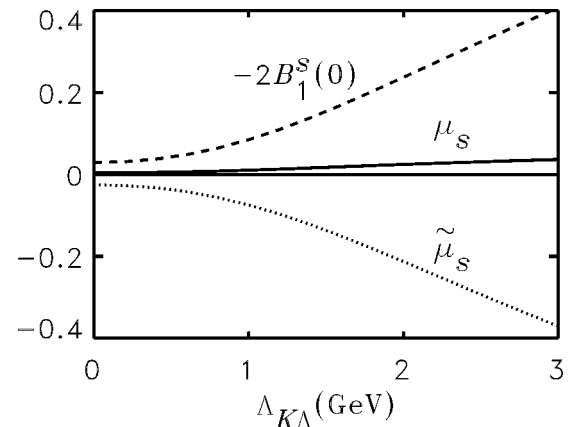


FIG. 1. Strange magnetic moment μ_S (solid) as a function of meson-nucleon vertex function cutoff mass $\Lambda_{K\Lambda}$. Also shown are the unphysical $\tilde{\mu}_S$ (dotted) and $-2B_1^S(0)$ contributions (dashed).

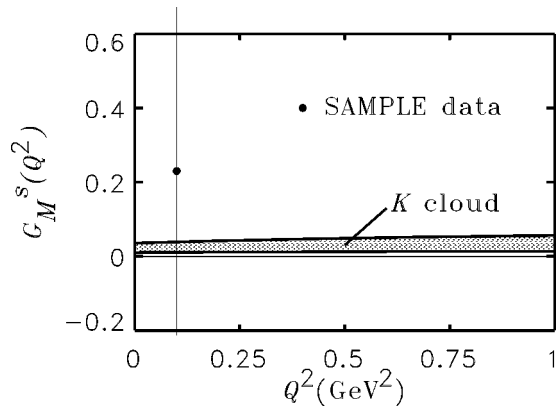


FIG. 2. Strange magnetic form factor of the proton $G_M^S(Q^2)$ as a function of Q^2 . The shaded region is the kaon cloud prediction, for $\Lambda_{K\Lambda}=1$ (lower curve) and 3 (upper curve) GeV. The data point is from the SAMPLE experiment [6].

$=0.011$ n.m. and $\tilde{G}_M^{(K)}(0)=-0.085$ n.m., with $\tilde{\mu}_S=-0.074$ n.m. In fact, all previous estimates of the strange magnetic moment in meson cloud models have obtained negative values [9,12]. The above results clearly illustrate the point that calculations of form factors in the impulse approximation, which utilize only single-particle operators, need to be treated with considerable care. Even with the inclusion of so-called seagull terms [20], which are examples of many-body contributions, the effects from the Lorentz covariance breaking will still remain dominant.

In Fig. 2 we show the total strange magnetic form factor $G_M^S(Q^2)$ as a function of Q^2 , for values of $\Lambda_{K\Lambda}$ between 1 and 3 GeV (shaded region). Surprisingly, the result seems almost entirely independent of Q^2 , which is a consequence of the cancellation between the B_1^S and \tilde{G}_M^S terms above.

Also shown is the recent data point from the SAMPLE Collaboration, which gave a value of $G_M^S(0.1 \text{ GeV}^2)=+0.23\pm 0.37\pm 0.15\pm 0.19$ n.m., where the errors are, respectively, statistical, systematic, and that coming from the uncertainty due to the axial radiative corrections and uncertainties in extracting G_M^S from the measured asymmetry [6].

In summary, the breaking of Lorentz covariance due to the use of the impulse approximation in the calculation of form factors of composite systems manifests itself in the appearance of unphysical form factors in the definition of the current. We have investigated the consequences of this breaking within the meson cloud model of the nucleon, by removing the spurious contributions according to the prescription outlined in Ref. [8]. For the strange magnetic form factor these contributions are significant, and give rise to large cancellations, leading to an overall change of sign for the physical G_M^S form factor compared with earlier estimates [9,12]. The small (positive) value of the strange magnetic moment μ_S is found to be largely independent of the details of the $KN\Lambda$ vertex function, which is the main parameter in the model. It will be of considerable interest to see to what extent the results in Figs. 1 and 2 are supported in upcoming parity-violating experiments at Jefferson Lab [21], which will map out the Q^2 dependence of the strange form factors over a larger range of kinematics.

ACKNOWLEDGMENTS

We would like to thank X. Ji and J.-F. Mathiot for helpful discussions. M.M. wishes to thank the TQHN group at the University of Maryland for their hospitality during his extended visit, and the Brazilian agency CAPES Grant No. BEX1278/95-2 for the financial support which made this visit possible. This work was supported by the DOE Grant No. DE-FG02-93ER-40762.

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