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Neutrino scattering from ^{12}C and ^{16}O

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Exclusive and inclusive (ν_μ,μ^-), (ν_e,e^-) cross sections and μ^--capture rates are calculated for ^{12}C and ^{16}O using the consistent random phase approximation (RPA) and pairing model. After a pairing correction is introduced to the RPA results, the flux-averaged theoretical (ν_μ,μ^-), (ν_e,e^-) cross sections and μ^--capture rates in ^{12}C are in good agreement with experiment. Predictions of (ν_μ,μ^-) and (ν_e,e^-) cross sections in ^{16}O are also presented. [S0556-2813(97)50111-7]

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Investigations of neutrino-nucleus interactions are presently in the forefront of particle and nuclear astrophysics research. The study of basic properties of the neutrino such as its mass, possibility of decay or oscillations, is probed using the interaction of neutrinos with nuclei. The neutrino interaction with matter is very weak and therefore neutrinos that are emitted from the inner parts of a star are ''messengers'' that bring information about the processes occurring inside the stars [1]. These extraterrestrial neutrinos are detected in reactions involving nuclear targets. It is of considerable importance to provide a theoretical framework that takes into account the various aspects of nuclear structure and is able to describe well these reactions. Attempts of this kind were made in the past [2] and in the last few years such attempts have been revived [3,4] in view of the availability of new experimental results in this field of neutrino-nucleus interaction. In particular we will refer here to the KARMEN results and the results obtained at the Liquid Scintillator Neutrino Detector (LSND) in Los Alamos. The ^{12}C$(\nu_\mu,\mu^-)X$ measurements at the LSND were made with the same experimental setup as the one used in the neutrino oscillation experiment—the results of which were recently published [5]. A study of the ^{12}C$(\nu_\mu,\mu^-)X$ reactions will shed indirectly some light on the question of validity of the neutrino oscillation experiments. For example a strong disagreement between theory and the experimental results for the ^{12}C$(\nu_\mu,\mu^-)X$ could probably lead to some skepticism concerning the experimental setup of the neutrino experiments in general. In fact, the recently published theoretical results [4] for the inclusive ^{12}C$(\nu_\mu,\mu^-)X$ are in disagreement, by a factor of two, with the LSND result [6].

In the present paper we examine a range of neutrino-nucleus cross sections as well as μ^--capture rates with a special emphasis on the ^{12}C case. Our calculations are performed first in the Hartree-Fock (HF) approximation and then in the consistent Hartree-Fock- Random Phase Approximation (HF-RPA). The consistency refers here to the fact that the HF mean field and the particle-hole (p-h) interaction result from the same effective nucleon-nucleon, two-body force. Such RPA calculation preserves the energy-weighted sum rule of the strength distribution of one-body operators [7] and therefore it is a favorable scheme when it comes to calculating the distribution of total strength. One is led to expect that, for closed shell nuclei inclusive cross sections of processes governed by one-body transition operators should be well reproduced in an RPA-type calculation. Indeed, the RPA had much success in providing an adequate description of giant resonances [8,9] and of a variety of inclusive processes in nuclei with good closed shells. An example relevant to our subject of neutrino-nucleus interactions is the inclusive μ^- capture on nuclei. The HF-RPA was able to reproduce [9] the inclusive capture rates in a number of closed shell nuclei. It was found that the collectivity and RPA ground state correlations are very important in reaching good agreement with experiment. This suggests that other inclusive neutrino processes such as for example the (ν_μ,μ^-) reaction (which is the inverse to μ capture) will be well described by the HF-RPA. It is known, however, that the ground state of ^{12}C is not a good closed shell. Admixtures of the $(p_{1/2})^2(p_{3/2})^2$ configuration into the $(p_{3/2})^4$ configuration are large and one should expect substantial corrections to the matrix elements obtained in the RPA. As we will see these corrections play a very important role when one calculates the (ν_l,l^-) cross sections, in particular the exclusive ones to the ground state of the daughter nucleus. The inclusive cross sections to the excited states are less affected but, in order to achieve agreement with experiment one must, nevertheless, include these corrections also for the excited states.

The numerical applications are performed with the Skyrme forces SGII and SIII [10,11]. The interaction SGII was adjusted so as to give the correct value of the Landau parameter F_0' [11] in the spin-isospin particle-hole channel which should be of particular importance in charged current neutrino reactions on nuclei. First, the Hartree-Fock equations are solved in coordinate space to obtain the self-consistent mean field. This mean field determines the single-particle spectrum. For the present problem it is not necessary to treat exactly the single-particle continuum since we are not studying specific (exclusive) channels where a nucleon would be emitted following the (ν_μ,μ^-) or (ν_e,e^-) reaction. Therefore, it is convenient to discretize the single-particle spectrum by diagonalizing the Hartree-Fock mean field on a harmonic oscillator basis. The reference Hartree-Fock state $|HF\rangle$ corresponds to the target nucleus, e.g., ^{12}C. Let us denote by $i,j,\dots$ $(a,b,\dots)$ the proton (neutron) occupied states and by $I,J,\dots$ $(A,B,\dots)$ the proton (neutron) unoccupied states. The proton (neutron) creation and annihilation operators are, respectively, p_i^+ and p_i (n_i^+ and n_i). In reactions of the (ν_μ,μ^-) or (ν_e,e^-) type the final states $|\lambda\rangle$ belong to the $\Delta T_Z=-1$ daughter nucleus (e.g., ^{12}N) and they can be described by the charge-exchange RPA [12,13] model:

$$|\lambda\rangle=\left(\sum_{I,a} X_{Ia}^{(\lambda)}\;\; p_I^+ n_a+\sum_{i,A} Y_{iA}^{(\lambda)}\;\; p_i^+ n_A\right)|\tilde{0}\rangle, \tag{1}$$

where $|\tilde{0}\rangle$ is the correlated RPA ground state. The $X^{(\lambda)}$ and $Y^{(\lambda)}$ are solutions of the charge-exchange RPA equations [12,13]. For a one-body charge-exchange operator of the general form

$$O=\sum_{\alpha,\beta} O_{\alpha\beta} p_\alpha^+ n_\beta, \tag{2}$$

the transition amplitude $\langle\lambda|F|\tilde{0}\rangle$ can be expressed simply as

$$\langle\lambda|O|\tilde{0}\rangle=\sum_{Ia} X_{Ia}^{(\lambda)*}O_{Ia}-\sum_{iA} Y_{iA}^{(\lambda)*}O_{iA}. \tag{3}$$

In the case of a parent nucleus with zero angular momentum in the ground state the cross section is [2,3]

$$\sigma=\frac{G^2}{2\pi}\cos^2(\theta_C)\sum_\lambda p_l E_l \mathcal{F}(Z,E_l)\int_{-1}^{1} d[\cos(\theta)]\mathcal{M}_{\lambda 0}, \tag{4}$$

where G and θ_C are the Fermi constant and the Cabibbo angle, p_l and E_l are the momenta and energies of outgoing leptons (muon or electron), θ is the angle between the momenta of the lepton and the incoming neutrino. The factor $\mathcal{F}$ accounts for the effects of the final state interaction (FSI) of the outgoing lepton with the daughter nucleus of charge Z [2,3]. For the case of (ν_e,e^-) reactions the mass of the outgoing lepton is small and the effect is not so important. The effect of the FSI for the negatively charged muon is more significant, increasing the cross section approximately by 15–20%. In Eq. (4), the sum goes over the available nuclear excitations, denoted by λ. The nuclear structure effects are incorporated into $\mathcal{M}_{\lambda 0}$, the bilinear combination of the nuclear matrix elements between the ground state $|\tilde{0}\rangle$ and the excited states $|\lambda\rangle$ of the daughter nucleus. These are given by [3]

$$\mathcal{M}_{\lambda 0}=M_F|\langle\lambda|F|\tilde{0}\rangle|^2+\tfrac{1}{3}M_{GT}|\langle\lambda|GT|\tilde{0}\rangle|^2+M'_{GT}\Lambda. \tag{5}$$

The coefficients M_i are obtained by the Foldy-Wouthuysen transformation of the weak Hamiltonian where the terms up to third order in the momentum transfer q/M are kept (M is the nucleon mass)[3]. The first matrix element squared of Eq. (5) is

$$|\langle\lambda|F|\tilde{0}\rangle|^2=4\pi\sum_J |\langle\lambda,J||t_- j_J(qr)Y_J||\tilde{0}\rangle|^2. \tag{6}$$

Here, $\vec{q}$ is the momentum transfer, $q=|\vec{q}|$, $\vec{\sigma}$ and t_- refer to the nucleon spin Pauli matrices and isospin-lowering operator, respectively, $||\ldots||$ stands for the standard definition of the reduced matrix elements, j_L are the spherical Bessel functions and Y_J are the spherical harmonics. The remaining combinations of the matrix elements are

$$|\langle\lambda|GT|\tilde{0}\rangle|^2=4\pi\sum_{l,J} |\langle\lambda,J||t_- j_l(qr)[Y_l\times\vec{\sigma}]_J||\tilde{0}\rangle|^2, \tag{7}$$

$$\Lambda=4\pi\left(\frac{5}{6}\right)^{1/2}\sum_{l,l',J}(-1)^{(l-l')/2+J}[(2l+1)(2l'+1)]^{1/2} \times\begin{pmatrix} l & l' & 2\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} 1 & 1 & 2\\ l' & l & J\end{Bmatrix}\langle\lambda,J||t_- j_l(qr) \times[Y_l\times\vec{\sigma}]_J||\tilde{0}\rangle\langle\lambda,J||t_- j_{l'}(qr)[Y_{l'}\times\vec{\sigma}]_J||\tilde{0}\rangle^*. \tag{8}$$

Here, $[\times]_J$ means the coupling to the total angular momentum J. The quantity Λ in Eq. (8) is an interference term which appears when the square of the weak interaction matrix element is calculated [see Eq. (12) of Ref. [3]]. In the limit of zero momentum transfer it vanishes. In our calculation we take into account states with $J\leqslant 3$ with positive and negative parity.

We perform the integration over angle θ in Eq. (4) with the step size $\Delta\theta=2\pi/30$. The single-particle matrix elements of the operators O are calculated using the Hartree-Fock wave functions in steps of $\Delta r=0.1$ fm in the radial coordinate.

The ^{12}C nucleus is not described well by a closed $p_{3/2}$ subshell and configuration mixing is important in the ground state. Besides the RPA correlations, one of the important correlations is introduced by the pairing force. Here, we estimate the effect of pairing on the *exclusive and inclusive* neutrino cross sections. In the expressions (4)–(8) for the cross section, two types of single-particle matrix elements enter: (a) those which do not contain the spin operator $\vec{\sigma}$ and (b) those which contain $\vec{\sigma}$. The two kinds have different symmetry properties under time reversal. Consequently, the corresponding expressions for these single-particle operators in terms of quasiparticles are [14]

$$\langle i^*|O\sigma|k^*\rangle=(u_iv_k-u_kv_i)\langle i|O\sigma|k\rangle,$$

$$\langle i^*|O|k^*\rangle=(u_iv_k+u_kv_i)\langle i|O|k\rangle, \tag{9}$$

where the asterisks mean that the pairing has been taken into account, O is an operator that depends on $\vec{r}$ only, and the u and v are the coefficients of the Bogoliubov-Valatin transformation. In the case of ^{12}C, the u and v factors differ significantly from their values without pairing only for the $p_{3/2}$ and $p_{1/2}$ levels. This results in reduction factors

$$\zeta^2=(u_{p_{1/2}}v_{p_{3/2}}-v_{p_{1/2}}u_{p_{3/2}})^2, \tag{10}$$

which multiply the single-particle matrix elements of the operators containing σ in the expressions for the RPA matrix elements $\langle\lambda|O|\tilde{0}\rangle$. The evaluation of pairing effects in the cross sections is done by introducing the factors $u_iv_k\pm u_kv_i$

in the matrix elements obtained from the RPA solutions without pairing. For orbits different from the $p_{3/2}$ and $p_{1/2}$ we take the v_k to be zero or one.

To obtain the cross sections that allow for comparison with the experimental data, one has to fold the energy-dependent cross section of Eq. (4) with a corresponding neutrino flux $f(E)$

$$\langle \sigma \rangle_f = \int dE \sigma(E) \tilde{f}(E), \tag{11}$$

where $\tilde{f}(E)$ is a properly normalized neutrino flux from an available neutrino source,

$$\tilde{f}(E) = \frac{f(E)}{\int_{E_0}^{\infty} dE' f(E')}, \tag{12}$$

and $f(E)$ is the initial (unnormalized) flux from the source. Here, the value of E_0 depends on the neutrino source used in each experiment. It is taken to be zero for the case of the electron neutrino [15], while for the case of the muon neutrino source $E_0 = E_{\mathrm{thr}}$ [6], where E_{thr} is the threshold energy for the $^{A}Z(\nu_\mu, \mu^-)$ reaction. We calculate the cross sections with the neutrino energy steps $\Delta E = 1$ MeV and $\Delta E = 5$ MeV for the electron and muon neutrino cases, respectively. The spectra for the muon and electron neutrinos were taken from Refs. [6,15]. The endpoint for the electron neutrino flux is 52.7 MeV, thus the cross section for the $^{A}Z(\nu_e, e^-)$ reaction is sensitive to the low-energy excitations in the daughter nucleus, mainly the giant resonances. For the case of the muon neutrino, the flux is cut at $E =$ 260 MeV.

The results of the calculations are shown in Table I. We note first that the results obtained do not differ very much for the two interactions used and the cross sections or capture rates agree typically within 10% for the SGII and SIII interactions. In order to assess the influence of collective effects and of the RPA ground state correlations we have calculated each cross section or capture rate first in the Hartree-Fock approximation and then in the HF-RPA. The comparison shows that in the RPA the μ^--capture rates are reduced by 30% and 50% in ^{12}C and ^{16}O, respectively. This is in agreement with the results in Ref. [9]. Note that in the present calculation of the μ^--capture rates the SU4 assumption is not used and the vector, axial vector and induced pseudoscalar contributions are directly calculated. From Table I we see that the RPA effects are also very important in the (ν_e, e^-) process. In ^{12}C and ^{16}O the flux averaged (ν_e, e^-) cross section is reduced by nearly a factor two when one goes from HF to HF-RPA.

One of the manifestations of the fact that ^{12}C is not a closed shell is the need for a large suppression factor in order to reproduce the experimental exclusive cross sections to the ground states of the $A = 12$ nuclei [2,4]. In Ref. [4] a reduction factor $\zeta^2 = 3.88$–4.13 was introduced in the computation of exclusive processes, however, these authors have not considered the influence of configuration mixing in the ^{12}C ground state on the inclusive cross sections to the excited states. In the present work, by including the pairing correlations we are able to treat the effects of such configuration mixing in both exclusive and inclusive cross sections. The pairing calculations are done employing Eq. (9) and a $v_{p1/2} = 0.65$, a choice in accordance with the value of the gap parameter $\Delta \simeq 3$–4 MeV [14]. The pairing correction reduces the flux-averaged exclusive cross sections and μ^--capture rates to the ground state of the daughter nucleus by factors 4–7 compared to the RPA results, depending on the type of reaction one calculates. The inclusive cross sections to the excited states (denoted as $\langle \sigma^* \rangle$) are affected less by the pairing correction but still the effect is sizable, reducing the $\langle \sigma^* \rangle$ cross sections by 25%. The reduction due to pairing of the total inclusive processes is about a factor of 5 for the (ν_e, e^-) cross section (because it is dominated by the transition to the $J^\pi = 1^+$ ground state) and about 33% reduction for the (ν_μ, μ^-) cross section with respect to the RPA result. Note that the RPA and pairing correction decrease the cross sections calculated in the HF approximation by a factor of 6–7 for (ν_e, e^-) and a factor of 2 for the (ν_μ, μ^-) and μ^--capture processes. We emphasize that the agreement is achieved in all the quantities calculated by using the same

TABLE I. Flux-averaged cross sections and μ^--capture rates in ^{12}C. σ_{exc} stands for exclusive cross sections to the ^{12}N ground state, σ^* is the inclusive cross section to excited states. The capture rates $\omega_{\mathrm{g.s.}}$ and ω_{tot} correspond to the partial rate to the ground state and to the total rate, respectively. The (ν_μ, μ^-) cross sections are in 10^{-40} cm^2, (ν_e, e^-) cross sections are in 10^{-42} cm^2, capture rates are in 10^4 s^{-1}. For each quantity, values calculated with SGII and SIII are in the upper and lower row, respectively. The results in brackets correspond to the choice $v_{p1/2} = 0.60$.

Channel	HF	HF-RPA	HF-RPA +pair	Exp.
(ν_μ, μ^-)				
σ_{exc}	4.28	3.35	0.39 (0.64)	0.66±0.1±0.1 [16]
	4.70	3.80	0.50 (0.77)	
σ^*	22.7	17.7	13.1 (13.7)	
	24.1	18.6	14.0 (14.4)	
σ_{inc}	27.0	21.1	13.5 (14.3)	11.2±0.3±1.8 [16]
	28.8	22.4	14.5 (15.2)	
(ν_e, e^-)				
σ_{exc}	78.1	54.8	7.1 (11.4)	10.5±1.±1. [19,20]
	100.4	68.2	10.1 (16.0)	8.2±0.65±0.75 [15]
σ^*	8.6	8.3	5.6 (6.2)	6.4±1.45±1.4 [21]
	14.0	8.1	6.4 (6.7)	3.6±2.7[20]
				5.7±0.6±0.6 [22]
				8.6±1.2±1.5 [15]
σ_{inc}	90.6	63.2	12.9 (17.6)	14.1±2.3 [20]
	114.4	76.3	16.5 (22.7)	16.8±1.7 [15]
μ^--capt.				
$\omega_{\mathrm{g.s.}}$	3.61	3.24	0.41 (0.67)	0.60±0.04 [23,24]
	3.70	3.48	0.45 (0.73)	
ω_{tot}	8.0	6.87	3.09 (3.48)	3.7±0.1 [18]
	8.4	7.22	3.23 (3.64)	

value of the parameter $v_{p1/2}$. We did not attempt to find the best value for this parameter. In parentheses of column 4 we show the results obtained when we use $v_{p1/2}$=0.60. We should remark that the results in Table I corresponding to interaction SGII are obtained with a slightly changed spin-orbit parameter (compared to the original SGII force) so as to reproduce in the RPA the experimental threshold energy. Our results for the (ν_μ,μ^-) inclusive cross section of 13.5 $\times 10^{-40}$ cm^2 (for SGII) and 14.5$\times 10^{-40}$ cm^2 (for SIII) should be compared with the recently revised experimental value of (11.2$\pm$0.3$\pm$1.8)$\times 10^{-40}$ cm^2 from the LSND experiment [16].

At this point several remarks should be added.

(a) Our procedure of introducing the effects of pairing on the cross sections is approximate and several corrections must be considered. We have estimated the correction that stems from the transitions from the $p_{1/2}$ to higher orbits (that is transitions proportional to $v^2_{p1/2}$) in the inclusive cross-section as well as the correction that comes from the energy shift of the spectrum caused by the pairing force. The two types of corrections cancel each other so that the results of Table I are affected only by a few percent by these additional terms.

(b) It is well known that there is a 30–40% universal quenching of GT strength in nuclei (see for example Ref. [17]). The origin of the observed quenching is still disputed. One suggestion is that the missing strength is shifted to energies as high as 300 MeV (the Δ_{33} region). The other suggestion is that the strength is actually spread out into many states each carrying a small fraction of the missing strength. If the second suggestion is the correct one, then we do not need to introduce additional quenching because we sum over all the RPA strength. It is not clear if the above quenching applies uniformly to all states, including the ones that carry little GT strength. It is possible that the parameter $v_{p1/2}$ we introduce might effectively take into account some of this quenching. The fact that we reproduce all the exclusive cross sections suggests that we do not need to introduce additional quenching.

(c) We should remark that the strong quenching of GT strength (in particular in the β^+ decay) due to pairing effects was discussed extensively in the past (see for example, Ref. [17] and references therein).

In ^{16}O the nucleons form a good closed shell and pairing effects are not important. Except for the μ^- capture there are no experimental neutrino data. Our HF-RPA predictions in ^{16}O for the flux-averaged inclusive (ν_μ,μ^-) cross sections are 27.8×10^{-40} cm^2 (SGII) and 27.1×10^{-40} cm^2 (SIII) whereas for (ν_e,e^-) they are 17.2×10^{-42} cm^2 (SGII) and 16.9×10^{-42} cm^2 (SIII). The calculated HF-RPA μ^--capture rates are 1.05×10^5 s^{-1} (SGII) and 1.00×10^5 s^{-1} (SIII) to be compared with the experimental value of 0.98×10^5 s^{-1} [18].

In summary, we have shown that the use of the consistent HF-RPA scheme and the introduction of pairing corrections can successfully reproduce the experimental μ^--capture rates and neutrino cross sections in ^{12}C. The main limitation and also the main source of uncertainties in ^{12}C is that the effect of pairing correlations on the various cross sections is evaluated in several steps, each step involving approximations. An improved treatment should attempt to introduce the pairing correlations in the ground state from the start and to perform RPA calculations on the correlated ground state. Such a computational scheme is known as the quasiparticle RPA (QRPA). It is of considerable importance to calculate the neutrino cross sections in the QRPA. Such calculations are presently in progress. One should keep in mind, however, that although the QRPA will be an improvement over the present scheme, one will still face uncertainties in the calculation. In order to reduce uncertainties one should be able to constrain the wave functions by examining related data, such as for example the existing (e,e') cross sections. The pairing correlations in the ground state of ^{12}C are not the only possible ones. Other types of nucleon-nucleon correlations might contribute to the cross sections. A larger space shell-model calculation should be able to determine better the ground state wave function. However, the calculation of highly excited states (excitations across several major shells) will be quite difficult and will involve necessary approximations. Although the present approach is not as elaborate as an extended shell-model calculation or a QRPA, it does contain some of the basic features that will emerge from such more extensive methods and it goes beyond the work that was published on this subject in the past.

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