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Unitarity constraint for threshold coherent pion photoproduction on the deuteron

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The contribution of the two-step process $\gamma d \to pn \to \pi^0 d$ to the imaginary part of the amplitude for coherent pion production on the deuteron is calculated at threshold exploiting unitarity constraints. The result shows that this absorptive process is not negligible and has to be considered in an extraction of the elementary neutron production amplitude from the $\gamma d \to \pi^0 d$ cross section at threshold. [S0556-2813(97)50109-9]

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Recently there has been considerable interest in the coherent electromagnetic production of pions from the deuteron near threshold. The main motivation thereby is to gain information on the elementary neutron amplitude $\gamma n \to \pi^0 n$ which is experimentally not directly accessible. A first measurement of the $ed \to e\pi^0 d$ reaction near $q_\mu^2 = -0.075$ GeV$^2/c^2$ will be performed soon at MAMI [1]. However, it is already known for a long time that at threshold the $\gamma d \to \pi^0 d$ process is dominated by two-nucleon production mechanisms [2]. Therefore, a careful theoretical analysis which allows the separation of the one-nucleon process is essential. Recently, chiral perturbation theory (χPT) in the heavy baryon formulation has been applied to this reaction by Beane *et al.* [3] predicting a real threshold amplitude. Very recently, this work has been improved and extended beyond next-to-leading order in the chiral power counting scheme [4].

It is the purpose of the present paper to point out that an additional contribution of an absorptive two-step process, $\gamma d \to pn \to \pi^0 d$, has to be included, which leads to a complex amplitude even at threshold. The presence of such a competing deuteron disintegration channel has an analogue in the case of πd elastic scattering, where the contribution of the absorptive process $\pi d \to NN \to \pi d$ is known to be of the order of 10% of the total amplitude (see, e.g., [5]). An effect of this size would not be negligible for the electromagnetic reaction due to the relative smallness of the single-nucleon amplitude, one is mainly interested in. In the case of $\pi d \to \pi d$, the imaginary part of the scattering length $a_{\pi d}$ is related to the total absorption cross section through

$$\mathrm{Im}\, a_{\pi d} = \frac{1}{4\pi} \lim_{p_{\pi d} \to 0} p_{\pi d}\, \sigma(\pi d \to X), \tag{1}$$

where $p_{\pi d}$ is the pion momentum in the c.m. system. This relation follows directly from the optical theorem. An analogous unitarity constraint on the $\gamma d \to \pi^0 d$ amplitude near threshold is present and will be treated below.

As is well known, the unitarity of the S-matrix leads to constraints for the corresponding reaction amplitudes. For our purposes, it is sufficient to consider coupled two-particle channels where the particles are subject to interactions which are invariant under time reversal. In this case, following the conventions of Ref. [6], the imaginary part of the partial wave T-matrix element of total angular momentum J for the reaction $a = (a_1 a_2) \to b = (b_1 b_2)$ fulfills the following unitarity constraint:

$$\mathrm{Im} T^J_{\lambda_{a_1}\lambda_{a_2} \to \lambda_{b_1}\lambda_{b_2}}(W) = \sum_c p_c \sum_{\lambda_{c_1}\lambda_{c_2}} T^J_{\lambda_{a_1}\lambda_{a_2} \to \lambda_{c_1}\lambda_{c_2}}(W) \left(T^J_{\lambda_{b_1}\lambda_{b_2} \to \lambda_{c_1}\lambda_{c_2}}(W)\right)^*. \tag{2}$$

Here, λ_{c_i} is the helicity of particle i in the channel c, and p_c denotes the c.m. momentum of the two particles in the channel c. The first sum on the right-hand side (RHS) of Eq. (2) runs over all open channels c for a given total c.m. energy W. In terms of the T^J the total helicity amplitude for $a \to b$ is given by

$$T_{\lambda_{a_1}\lambda_{a_2} \to \lambda_{b_1}\lambda_{b_2}}(W,\phi,\theta) = 8\pi W \sum_J (2J+1) e^{i(\lambda_a - \lambda_b)\phi} d^J_{\lambda_a \lambda_b}(\theta) \times T^J_{\lambda_{a_1}\lambda_{a_2} \to \lambda_{b_1}\lambda_{b_2}}(W), \tag{3}$$

with $\lambda_a = \lambda_{a_1} - \lambda_{a_2}$, $\lambda_b = \lambda_{b_1} - \lambda_{b_2}$, and θ, ϕ are the spherical coordinates of the outgoing particle b_1.

We apply relation (2) to the three coupled channels $\pi^0 d$, pn, and γd at the π^0 production threshold, i.e., in the limit $W \to W_0 \equiv m_{\pi^0} + m_d$, which, of course, implies $p_{\pi^0 d} \to 0$. In this limit, therefore, no contribution from $c = (\pi^0 d)$ to the sum occurs due to the vanishing phase space factor. Moreover, at threshold we can restrict ourselves to partial waves of angular momentum and parity $J^\pi = 1^-$. Thus $\pi^0 d \to pn$ is described by a single matrix element A, with

$$T^1_{\lambda_d \to \lambda_p \lambda_n} = \tfrac{1}{3}(101\lambda_p - \lambda_n | 1\lambda_p - \lambda_n) A. \tag{4}$$

For $\gamma d \to \pi^0 d$ at threshold, two independent matrix elements remain, $E_{\pi^0 d}$ and $M_{\pi^0 d}$. They correspond to the electric dipole ($E1$) and magnetic quadrupole ($M2$) radiation allowed for the $1^+ \to 1^-$ transition of the hadronic system. One finds

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$$T^1_{\lambda_\gamma\lambda_d\to\lambda'_d}=\frac{1}{3\sqrt{6}}\sum_{L=1,2}\sqrt{2L+1}(1-\lambda_d L\lambda_\gamma|1\lambda_\gamma-\lambda_d)\times(\delta_{L,1}E_{\pi^0 d}+\lambda_\gamma\delta_{L,2}M_{\pi^0 d}). \tag{5}$$

Finally, for $\gamma d\to{}^3P_1(pn)$ one obtains

$$T^1_{\lambda_\gamma\lambda_d\to\lambda_p\lambda_n}=\frac{1}{3\sqrt{2}}(101\lambda_p-\lambda_n|1\lambda_p-\lambda_n)\times\sum_{L=1,2}\sqrt{2L+1}(1-\lambda_d L\lambda_\gamma|1\lambda_\gamma-\lambda_d)\times(\delta_{L,1}E_{pn}+\lambda_\gamma\delta_{L,2}M_{pn}), \tag{6}$$

where E_{pn} and M_{pn} are the corresponding $E1$ and $M2$ matrix elements of the disintegration process. The various total cross sections in terms of these matrix elements are

$$\sigma(\pi^0 d\to pn)=\frac{4\pi}{3}\frac{p_{pn}}{p_{\pi^0 d}}|A|^2, \tag{7}$$

$$\sigma(\gamma d\to\pi^0 d)=\frac{4\pi}{6}\frac{p_{\pi^0 d}}{p_{\gamma d}}(|E_{\pi^0 d}|^2+|M_{\pi^0 d}|^2), \tag{8}$$

$$\sigma(\gamma d\to{}^3P_1(pn);E1+M2)=\frac{4\pi}{6}\frac{p_{pn}}{p_{\gamma d}}(|E_{pn}|^2+|M_{pn}|^2). \tag{9}$$

Taking now $a=(\gamma d)$ and $b=(\pi^0 d)$, relation (2) leads to

$$\mathrm{Im}\ E_{\pi^0 d}(W_0)=\frac{1}{\sqrt{3}}p_{pn}E_{pn}(W_0)A^*(W_0), \tag{10}$$

and an analogous relation for $M_{\pi^0 d}$ and M_{pn}, respectively. Since the left-hand side (LHS) of Eq. (10) is real, relation (10) implies that the phases of the complex matrix elements A and E_{pn} are equal. Indeed, evaluating Eq. (2) with $a=(\pi^0 d)$, $b=(pn)$, and $a=(\gamma d)$, $b=(pn)$ provides us with

$$A(W_0)=|A(W_0)|\exp[i\delta_{^3P_1}(W_0)+in\pi], \tag{11}$$

and

$$E_{pn}(W)=|E_{pn}(W)|\exp[i\delta_{^3P_1}(W)+ik\pi],\quad W\leqslant W_0, \tag{12}$$

respectively, where $\delta_{^3P_1}$ is the nucleon-nucleon scattering phase shift in the $^3P_1$ channel. Equation (12) is simply Watson's theorem applied to deuteron photodisintegration and is valid for all energies below the pion production threshold, whereas Eq. (11) is valid for $W=W_0$ only. Finally, we mention that taking $a=b=(\pi d)$ in Eq. (2), leads to the constraint for the imaginary part of the πd scattering length in Eq. (1).

Using Eqs. (7) and (9), and the detailed balance relation

$$3p^2_{\pi^0 d}\sigma(\pi^0 d\to pn)=4p^2_{pn}\sigma(pn\to\pi^0 d), \tag{13}$$

our main result (10) can be rewritten as

TABLE I. Relative contributions of various electromagnetic currents to the $E1(\gamma d\to{}^3P_1(pn))$ matrix element at $E_\gamma=140$ MeV.

Siegert operator	122%
spin current	7%
spin-orbit current	−23%
meson-exchange currents (beyond the Siegert operator) and Δ current	−6%

$$|\mathrm{Im}E_{\pi^0 d}|=\frac{1}{\sqrt{2}\pi}p_{pn}\sqrt{\frac{p_{\gamma d}}{p_{\pi^0 d}}\sigma(\gamma d\to{}^3P_1(pn);E1)\sigma(pn\to\pi^0 d)}. \tag{14}$$

At this level, there is, however, no way to fix the sign. In order to get a numerical value, we take for the hadronic cross section the experimental result given by Hutcheon *et al.* [7],

$$\lim_{p_{\pi^0 d}\to 0}2\frac{m_\pi}{p_{\pi^0 d}}\sigma(pn\to\pi^0 d)=184\pm5\pm13\mu b. \tag{15}$$

The partial cross section $\gamma d\to{}^3P_1(pn)$ is at present not available although it could in principle be obtained from a multipole analysis. There are, however, reliable theoretical models available which reproduce all deuteron photodisintegration data in this energy region [8]. At this energy, the $E1$ matrix element is mainly given by π-exchange current contributions, which can be calculated in a largely model-independent way by taking advantage of gauge-invariance constraints (Siegert's theorem). The underlying nucleon-nucleon interaction and also all model-dependent transverse electromagnetic currents, like the Δ(1232) excitation current, have little effect on this matrix element. We take the value

$$\sigma(\gamma d\to{}^3P_1(pn);E1)=10.5\mu\ \mathrm{b} \tag{16}$$

from an updated version of the model of Ref. [9]. Table I gives some details of how this value arises. It is interesting to note that the relativistic spin-orbit current gives a sizable contribution.

In order to relate our result to the χPT calculations of Beane *et al.* [4], we switch to their normalization (and notation) of the electric dipole amplitude, $E_d\equiv E_{\pi^0 d}/4$, and obtain

$$|\mathrm{Im}E_d|=0.22\times10^{-3}/m_{\pi^+}. \tag{17}$$

The cross section $\sigma(\gamma d\to{}^3P_1(pn);M2)$ is more than two orders of magnitudes smaller than Eq. (16) and leads to $|\mathrm{Im}M_d|=0.018\times10^{-3}/m_{\pi^+}$, where $M_d\equiv M_{\pi^0 d}/4$. Nevertheless, the role of the $M2$ transition in the coherent production at threshold deserves a more detailed investigation which will be presented elsewhere. The main reasons are: (i) it provides an additional possibility to test theoretical predictions (an experimental separation of $E1$ and $M2$ requires a polarized deuteron target), and (ii) the relative importance of the $M2$ transition grows with the momentum transfer in the electroproduction process.

The result (17) has to be compared with the value calculated in [4],

$$E_d^{\chi PT} = 0.38 E_{0+}^{\pi^0 n} - 2.6\times 10^{-3}/m_{\pi^+} = -1.8\times 10^{-3}/m_{\pi^+}, \tag{18}$$

where the latter value is obtained taking the χPT prediction of [10] for the neutron electric dipole amplitude, $E_{0+}^{\pi^0 n} = 2.13\times 10^{-3}/m_{\pi^+}$. Thus the threshold cross section itself is affected by less than 2% only by this imaginary part (17). However, there is no reason at all to assume that the contribution of the absorptive process to the real part of the amplitude is much smaller in magnitude than the imaginary part. Unfortunately, unitarity does not allow to estimate it. It can only be calculated within a model which to our knowledge has not yet been done. For the moment, in order to get a rough idea, one may look into the in many respects analogous situation for πd elastic scattering. There, three-body calculations suggest for the absorptive contribution to the scattering length, $\mathrm{Re}\, a_{\pi d}^{\mathrm{abs}} \approx -\mathrm{Im}\, a_{\pi d}$ (see [5] for an overview). Assuming, therefore $|\mathrm{Re} E_d^{\mathrm{abs}}| = 0.22\times 10^{-3}/m_{\pi^+}$, one has to conclude that the neglect of the absorption process in an analysis of the $\gamma d \rightarrow \pi^0 d$ cross section based on Eq. (18) would lead to a systematic error of the order of $\delta E_{0+}^{\pi^0 n} = \pm 0.6\times 10^{-3}/m_{\pi^+}$ for the neutron amplitude. This rough argument at least demonstrates that a calculation of the absorptive contribution is necessary before definite conclusions on the neutron amplitude can be drawn. One way to do this is to combine conventional models for deuteron photodisintegration [8] with those for the pionic disintegration of the deuteron [11].

Finally, one may wonder what is the role of this absorptive process in the framework of heavy baryon χPT. The process $\gamma d \rightarrow \pi^0 d$ has been calculated in [4] up to and including all terms of order $\nu \leqslant 0$ of the expansion in terms of powers of small momenta $(Q/\Lambda)^\nu$ where typically $Q \sim m_\pi$, $\Lambda \sim m_N$. A real amplitude has been obtained since the absorptive contribution (defined as the sum of all time-ordered diagrams which contain at least one pure two-nucleon intermediate state) is of higher order in the power counting of [4]. Now, the fact that $\mathrm{Im}\, E_d$ in Eq. (17) turns out to be of the same order of magnitude as the three-body contribution of order $\nu = 0$ calculated by Beane *et al.*, $E_d^{tb,4} = -0.25 \times 10^{-3}/m_{\pi^+}$ in their notation, leads to the conclusion that either the convergence of the expansion is simply bad, or eventually a modified power counting should be applied. The necessity for a modified power counting when typical momenta are $\sim (m_\pi m_N)^{1/2}$ rather than $\sim m_\pi$ due to the kinematics of the reaction was first noted by Cohen *et al.* [12], studying the reaction $pp \rightarrow \pi^0 pp$ near threshold. Indeed, the on-shell momentum of the intermediate np pair within the absorptive process is of order $(m_\pi m_N)^{1/2}$. Therefore, the second point needs to be clarified in the future.

In summary, the contribution of the two-step process $\gamma d \rightarrow pn \rightarrow \pi^0 d$ to the imaginary part of the electric dipole (and magnetic quadrupole) amplitude for coherent pion photoproduction on the deuteron has been calculated near threshold utilizing unitarity constraints. The result shows that this absorptive process cannot be neglected in the extraction of the elementary neutron amplitude.

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