PHYSICAL REVIEW C VOLUME 56, NUMBER 1 JULY 1997

Δ excitation in inelastic scattering of nucleons on nuclei

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(Received 17 March 1997)

We outline an elegant way of deducing the spin structure of any reaction $A(a,b)B$ with arbitrary spins s_A, s_a, s_b, s_B and apply the same to $NN\to N\Delta$, taking into consideration the Pauli exclusion principle. This method, based on irreducible tensor techniques is then extended to Δ excitation in $A(N,N'\pi)B$ by considering the target excitation process (TDP) as $A(N,N')B^*_\Delta$ followed by $B^*_\Delta\to B+\pi$ and the projectile excitation process (PDP) as $A(N,\Delta)B$ followed by $\Delta\to N'\pi$. Expressions for the double differential cross section and inelastic nucleon spin observables are given, which are of current experimental interest.
[S0556-2813(97)50206-8]

PACS number(s): 25.40.Ep, 25.40.Fq, 24.70.+s, 14.20.Gk

The inelastic scattering of nucleons on nuclei at intermediate energies is currently a subject of considerable interest both experimentally and theoretically. Apart from measurements of differential cross section and spin observables like asymmetries and spin transfers, simultaneous measurements [1] of $(\mathbf{p},\mathbf{p}')$ and $(\mathbf{p},\mathbf{p}'\gamma)$ observables have been reported recently on ^{12}C. The motivation for such measurements is provided by theoretical work [2–4] based purely on invariance considerations wherein, the T matrix is expressed in terms of invariants constructed out of the initial and final c.m. momenta $\mathbf{p}_i, \mathbf{p}_f$ and the relevant operators in the spin space. Of particular interest in the context of inelastic nucleon scattering is the Δ excitation which has often been discussed [5–8] using models inspired by π and ρ exchange mechanisms and DWBA formalisms. Using invariance arguments Silbar, Lombard, and Kloet [9] have shown that the $NN\to N\Delta$ transition matrix contains as many as 16 amplitudes of which 10 are associated with second rank spin tensors. Ray [10] has drawn attention to the importance of the tensor amplitudes based on a partial wave expansion model wherein he says, ''The total and differential cross sections were reduced by about one-half, the structure in the analyzing powers increased dramatically, the predictions for D_{NN} became much too negative, while that for D_{LL} became much too positive, and the spin correlation predictions were much too small'' when all ''ten of the rank 2 tensor amplitudes were set to zero, while the remaining six amplitudes were unchanged.'' The 16 amplitudes of Silbar *et al.* have been employed in several recent studies [11] on Δ excitation. Discussing inelastic nucleon scattering via Δ excitation in nuclei, Jain and Kundu [12] have emphasized that it is necessary to develop a formalism which incorporates the unstable nature of Δ, since such a formalism is relevant to discuss the $(N,N'\pi)$ experiments. Jo and Lee [13] have recently drawn attention to the need for taking into consideration the projectile Δ excitation process which is usually ignored since the differential cross section at forward angles is dominated by contributions coming from target Δ excitation process.

The purpose of this paper is to develop a general formalism based on invariance considerations to discuss reactions involving particles/nuclei with arbitrary spins and to apply the same to the case of inelastic nucleon scattering on nuclei involving Δ excitation and decay wherein the PDP is fully taken into consideration along with the TDP.

First of all we note that the matrix $\mathcal{M}$ in the spin space for any reaction $A(a,b)B$ with arbitrary spins s_A, s_a, s_b, s_B and isospins I_A, I_a, I_b, I_B may be defined in terms of its elements

$$\mathcal{M}_{\mu_b\mu_B;\mu_a\mu_A}=\sqrt{\frac{2\pi D_2}{v}}\langle f|T|i\rangle, \tag{1}$$

where the initial and final states, with momenta $\mathbf{p}_i, \mathbf{p}_f$ in the c.m. system are given by

$$|i\rangle=|\mathbf{p}_i;s_a\mu_a;I_a\nu_a;s_A\mu_A;I_A\nu_A\rangle, \tag{2}$$

$$|f\rangle=|\mathbf{p}_f;s_b\mu_b;I_b\nu_b;s_B\mu_B;I_B\nu_B\rangle. \tag{3}$$

The magnitude of the relative velocity in the initial state is denoted by v, the two particle density of final states is denoted by D_2, and the on-energy shell T matrix may be expressed in the form

$$T=\sum_I C(I_bI_BI;\nu_b\nu_B\nu)T^IC(I_aI_AI;\nu_a\nu_A\nu), \tag{4}$$

if isospin I is conserved. Each T^I, characterized by channel isospin I, has matrix elements in terms of initial and final channel spin states $|s_i\mu_i\rangle,|s_f\mu_f\rangle$, and the matrix elements have the form [14]

$$\begin{aligned}\langle s_f\mu_f;\mathbf{p}_f|T^I|\mathbf{p}_i;s_i\mu_i\rangle &=\sum_{l_f,l_i,j,\lambda}(-1)^{l_i+s_i+l_f-j}(-1)^{\mu}T^{Ij}_{l_fs_f;l_is_i}(E)[j]^2[\lambda]\\ &\times[s_f]^{-1}W(s_il_is_fl_f;j\lambda)C(s_i\lambda s_f;\mu_i\mu\mu_f)\\ &\times(Y_{l_f}(\hat{p}_f)\otimes Y_{l_i}(\hat{p}_i))^{\lambda}_{-\mu},\end{aligned} \tag{5}$$

where $T^{Ij}_{l_fs_f;l_is_i}(E)$ denote partial wave amplitudes [15] at c.m. energy E. In the particular case of elastic scattering of spin s particles on spin 0 targets, i.e., $s_i=s_f=s$, the Clebsh-Gordan coefficient $C(s\lambda s;\mu_i\mu\mu_f)$ may be replaced [16] by an irreducible tensor operator $[\lambda]^{-1}\tau^{\lambda}_{\mu}(\mathbf{S})$ constructed out of spin operators $\mathbf{S}$. We use the shorthand $[k]=\sqrt{2k+1}$ and

$$(A^{k_1}\otimes B^{k_2})^k_q=\sum_{q_1} C(k_1k_2k,q_1q_2q)A^{k_1}_{q_1}B^{k_2}_{q_2} \tag{6}$$

to denote an irreducible tensor of rank k constructed out of a pair of irreducible tensors $A^{k_1}_{q_1}$ of rank k_1 and $B^{k_2}_{q_2}$ of rank k_2. The rest of the notations follow Rose [17]. We may then express

$$\mathcal{M}=\sum_{s_f,\mu_f,s_i,\mu_i} |s_f\mu_f\rangle\langle s_f\mu_f;\mathbf{p}_f|\mathcal{M}|\mathbf{p}_i;s_i\mu_i\rangle\langle s_i\mu_i| \tag{7}$$

and note [18] that

$$K^{s_f}_{\mu_f}=|s_f\mu_f\rangle,\quad B^{s_i}_{-\mu_i}=(-i)^{2\mu_i}\langle s_i\mu_i| \tag{8}$$

are irreducible tensors of rank s_f, s_i, respectively, so that

$$|s_f\mu_f\rangle\langle s_i\mu_i|=\sum_{\lambda} C(s_fs_i\lambda;\mu_f-\mu_i\mu)(i)^{2\mu_i}(K^{s_f}\otimes B^{s_i})^{\lambda}_{\mu}. \tag{9}$$

Introducing irreducible spin tensors $S^{\lambda}_{\mu}(s_2,s_1)$ of rank λ connecting the spin spaces of s_1 and s_2 through

$$S^{\lambda}_{\mu}(s_2,s_1)=(i)^{2s_1}[s_2](K^{s_2}\otimes B^{s_1})^{\lambda}_{\mu}, \tag{10}$$

we may note that

$$(S^{\lambda''}(s_3,s_2)\otimes S^{\lambda'}(s_2,s_1))^{\lambda}_{\mu}=(-1)^{\lambda'+\lambda''-\lambda}[\lambda'][\lambda''][s_2]W(s_1\lambda's_3\lambda'';s_2\lambda)S^{\lambda}_{\mu}(s_3,s_1). \tag{11}$$

Using Eq. (10) in Eq. (9) and expressing $S^{\lambda}_{\mu}(s_f,s_i)$ in terms of $S^{\lambda_1}_{\mu_1}(s_b,s_a)$ and $S^{\lambda_2}_{\mu_2}(s_B,s_A)$, we obtain the general spin structure for $A(a,b)B$ in the elegant form

$$\mathcal{M}=\sum_{\lambda_1}\sum_{\lambda_2}\sum_{\lambda=|\lambda_1-\lambda_2|}^{\lambda_1+\lambda_2}[(S^{\lambda_1}(s_b,s_a)\otimes S^{\lambda_2}(s_B,s_A))^{\lambda}\cdot\mathcal{T}^{\lambda}(\lambda_1,\lambda_2)], \tag{12}$$

where the amplitudes $\mathcal{T}^{\lambda}_{\mu}(\lambda_1,\lambda_2)$ are given explicitly by

$$\mathcal{T}^{\lambda}_{\mu}(\lambda_1,\lambda_2)=\sum_{l_i,l_f,s_i,s_f,j} G\mathcal{M}^{j}_{l_fs_f;l_is_i}(E)(Y_{l_f}(\hat{\mathbf{p}}_f)\otimes Y_{l_i}(\hat{\mathbf{p}}_i))^{\lambda}_{\mu} \tag{13}$$

in terms of

$$\mathcal{M}^{j}_{l_fs_f;l_is_i}(E)=\sqrt{\frac{2\pi D_2}{v}}\sum_{I} T^{Ij}_{l_fs_f;l_is_i}(E)C(I_bI_BI;\nu_b\nu_B\nu)C(I_aI_AI;\nu_a\nu_A\nu), \tag{14}$$

which completely determine the energy dependence, while the angular dependence is contained in $(Y_{l_f}(\hat{\mathbf{p}}_f)\otimes Y_{l_i}(\hat{\mathbf{p}}_i))^{\lambda}_{\mu}$. The geometrical factors are

$$G=(-1)^{l_i+l_f-j+s_a+s_A}\begin{Bmatrix} s_bs_Bs_f\\ s_as_As_i\\ \lambda_1\lambda_2\lambda\end{Bmatrix}\frac{[j]^2[\lambda][s_f][s_i][\lambda_1][\lambda_2]}{[s_b][s_B]}W(s_il_is_fl_f;j\lambda), \tag{15}$$

where $\{\}$ denotes Wigner 9-j symbol. If $A(a,b)B$ is parity conserving, the summations over l_i,l_f in Eq. (13) are restricted to

$$(-1)^{l_f}\pi_b\pi_B=(-1)^{l_i}\pi_a\pi_A, \tag{16}$$

where $\pi_{i=a,A,b,B}$ denote the respective intrinsic parities.

Considering in particular $NN\rightarrow N\Delta$, we note that the channel isospin can take only one value $I=1$ and the Pauli principle can be taken into consideration by writing the initial state $|i\rangle$ as

$$|i\rangle=\frac{1}{\sqrt{2}}[|\mathbf{p}_i;\tfrac{1}{2}\mu_P;\tfrac{1}{2}\mu_T\rangle-|-\mathbf{p}_i;\tfrac{1}{2}\mu_T;\tfrac{1}{2}\mu_P\rangle], \tag{17}$$

where the suffixes P and T refer to the projectile and target nucleon, respectively. Noting that the Pauli principle implies also

$$-1=(-1)^{l_i+s_i+I}, \tag{18}$$

the matrix $\mathcal{M}$ for $NN\rightarrow N\Delta$ takes the form

$$\mathcal{M}=\frac{1}{\sqrt{2}}\sum_{\lambda_1=0}^{1}\sum_{\lambda_2=1}^{2}\sum_{\lambda=|\lambda_1-\lambda_2|}^{\lambda_1+\lambda_2}(\Sigma^{\lambda}(\lambda_1,\lambda_2)\cdot\mathcal{T}^{\lambda}(\lambda_1,\lambda_2)), \tag{19}$$

where

$$\Sigma^{\lambda}_{\mu}(\lambda_1,\lambda_2)=(\sigma^{\lambda_1}_{P}\otimes S^{\lambda_2}_{T}(\tfrac{3}{2},\tfrac{1}{2}))^{\lambda}_{\mu}+(\sigma^{\lambda_1}_{T}\otimes S^{\lambda_2}_{P}(\tfrac{3}{2},\tfrac{1}{2}))^{\lambda}_{\mu} \tag{20}$$

in terms of the unit matrix σ^0_0 and the spherical components,

$$\sigma^1_0=\sigma_z\,,\quad \sigma^1_{\pm1}=\mp\frac{1}{\sqrt{2}}(\sigma_x\pm i\sigma_y) \tag{21}$$

of Pauli spin matrices $\boldsymbol{\sigma}$ of the projectile/target nucleon. It may be noted that the first and second terms in Eq. (20) correspond, respectively, to TDP and PDP, while the amplitudes $\mathcal{T}^{\lambda}_{\mu}(\lambda_1,\lambda_2)$ have the same form Eq. (13) with the additional constraint Eq. (18) on the summation, apart from the parity constraint Eq. (16).

If we choose a right-handed transverse frame with the z axis along $\mathbf{p}_i\times\mathbf{p}_f$ and x axis along $\mathbf{p}_i$, it is clear that

$$(Y_{l_f}(\hat{\mathbf{p}}_f)\otimes Y_{l_i}(\hat{\mathbf{p}}_i))^{\lambda}_{\mu}=\sum_{m_f}C(l_fl_i\lambda;m_fm_i\mu)Y_{l_fm_f}\left(\frac{\pi}{2},\theta\right)Y_{l_im_i}\left(\frac{\pi}{2},0\right), \tag{22}$$

where θ denotes the scattering angle $\cos\theta=\hat{\mathbf{p}}_i\cdot\hat{\mathbf{p}}_f$. The property [19] that, $Y_{lm}(\pi/2,\phi)=0$ if $l-m$ is odd together with Eq. (16) shows that

$$\mathcal{T}^{\lambda}_{\mu}(\lambda_1,\lambda_2)_{\mathrm{TF}}=0, \tag{23}$$

if μ is odd and TF stands for transverse frame. On the other hand, if we choose the right-handed frame recommended by the Madison convention [20] viz., z axis along $\mathbf{p}_i$ and y axis along $\mathbf{p}_i\times\mathbf{p}_f$, we have

$$(Y_{l_f}(\hat{\mathbf{p}}_f)\otimes Y_{l_i}(\hat{\mathbf{p}}_i))^{\lambda}_{\mu}=C(l_fl_i\lambda,\mu0\mu)[l_i](4\pi)^{-1/2}Y_{l_f\mu}(\theta,0). \tag{24}$$

The property [19] that

$$Y_{l-m}(\theta,\phi)=(-1)^{-m}e^{-2im\phi}Y_{lm}(\theta,\phi) \tag{25}$$

then leads to

TABLE I. The irreducible tensor amplitude $\mathcal{T}^{\lambda}_{\mu}(\lambda_1,\lambda_2)$ for $NN\to N\Delta$. The μ of nonzero amplitudes in transverse and Madison frames together with the number n of independent amplitudes are given.

			TF		MF	
λ_1	λ_2	λ	μ	n	μ	n
0	1	1	0	1	±1	1
0	2	2	$0,\pm2$	3	$0,\pm1,\pm2$	3
1	1	0	0	1	0	1
1	1	1	0	1	±1	1
1	1	2	$0,\pm2$	3	$0,\pm1,\pm2$	3
1	2	1	0	1	±1	1
1	2	2	$0,\pm2$	3	$0,\pm1,\pm2$	3
1	2	3	$0,\pm2$	3	$\pm1,\pm2,\pm3$	3
Total number of independent amplitudes				16		16

$$\mathcal{T}^{\lambda}_{-\mu}(\lambda_1,\lambda_2)_{\mathrm{MF}}=(-1)^{\lambda-\mu}\mathcal{T}^{\lambda}_{\mu}(\lambda_1,\lambda_2)_{\mathrm{MF}} \tag{26}$$

in the Madison Frame (MF). The nonzero amplitudes for $NN\to N\Delta$ in TF and MF are as shown in Table I, from which it is clear that there are 16 independent amplitudes, which may be related to those in [9,10]. Moreover, the amplitudes $\mathcal{T}^{\lambda}_{\mu}$ in any frame may readily be expressed in terms of either of these two sets since $\mathcal{T}^{\lambda}_{\mu}$ transform under rotations as elements of an irreducible tensor of rank λ.

The formalism outlined above can next be used to discuss Δ excitation in $A(N,N'\pi)B$ if we picture the TDP as $A(N,N')B^*_{\Delta}$ followed by $B^*_{\Delta}\to B+\pi$, where B^*_{Δ} denotes a Δ excited nuclear state with spin parity and isospin quantum numbers $s^{*\pi^*_B}_{B}I^*_B\nu^*_B$, while the PDP is $A(N,\Delta)B$ followed by $\Delta\to N'\pi$. The matrix T^{π} in spin space for the decay of an excited state with spin parity and isospin quantum numbers $s^{*\pi^*}I^*\nu^*$ into a state characterized by $s^{\pi}I\nu$ after pion emission may be written, using invariance considerations as before, as

$$T^{\pi}=\sum_{l=|s^*-s|}^{s^*+s}\delta_{\pi^*,\pi(-1)^{l+1}}C(I\;1I^*;\nu\nu_{\pi}\nu^*)T^{I^*,s^*}_{l}(Y_l(\hat{\mathbf{q}}^*)\cdot S^l(s,s^*)), \tag{27}$$

where $\nu_{\pi}=+1,0,-1$ denote the z component of isospin (or charge) of the pion, whose momentum in the rest frame of the excited system is denoted by $\mathbf{q}^*$ and the T^{I^*,s^*}_{l} denote the decay amplitudes into a state of orbital angular momentum l restricted by the δ function expressing conservation of parity to those values satisfying $\pi^*=\pi(-1)^{l+1}$. The matrix $\mathcal{M}$ for $A(N,N'\pi)B$ may then be written as

$$\mathcal{M}=T^{\pi}(B^*_{\Delta})\mathcal{M}[A(N,N')B^*_{\Delta}][D_3/D_2(\mathrm{TDP})]^{1/2} \tag{28}$$

in case of TDP, whereas the same in the case of PDP is

$$\mathcal{M}=T^{\pi}(\Delta)\mathcal{M}[A(N,\Delta)B][D_3/D_2(\mathrm{PDP})]^{1/2}, \tag{29}$$

where D_3 denotes the final 3 particle density of states. In either case of Eq. (28) or Eq. (29), $\mathcal{M}$ may be expressed, using Eq. (11), in the form

$$\mathcal{M}=\sum_{k_1=0}^{1}\sum_{k_2=|s_B-s_A|}^{s_B+s_A}\sum_{k=|k_2-k_1|}^{k_2+k_1}[(\sigma^{k_1}\otimes S^{k_2}(s_B,s_A))^k\cdot\mathcal{A}^k(k_1,k_2)]. \tag{30}$$

Taking both TDP and PDP into account, the $A(N,N'\pi)B$ amplitudes are given by

$$\begin{aligned}\mathcal{A}_q^k(k_1,k_2)=&\sum_{l,\lambda_2,\lambda}\delta_{\pi_B^*,\pi_B(-1)^{l+1}}C(I_B1I_B^*;\nu_B\nu_\pi\nu_B^*)T_l^{I_B^*,s_B^*}[s_B^*][l][k_2][\lambda_2][\lambda]W(s_A\lambda_2s_Bl;s_B^*k_2)W(l\lambda_2kk_1;k_2\lambda)\\&\times(Y_l(\hat{\mathbf{q}}_{B_\Delta^*})\otimes\mathcal{T}^\lambda_{\mathrm{TDP}}(k_1,\lambda_2))^k_q+2\sqrt{3}\sum_{\lambda_1,\lambda}C(\tfrac{1}{2}1\tfrac{3}{2};\nu_{N'}\nu_\pi\nu_\Delta)T_1^{3/2,3/2}(-1)^{k-\lambda+\lambda_1-k_1}[\lambda_1][k_1][\lambda]\\&\times W(\tfrac{1}{2}\lambda_1\tfrac{1}{2}1;\tfrac{3}{2}k_1)W(1\lambda_1kk_2;k_1\lambda)(Y_1(\hat{\mathbf{q}}_\Delta)\otimes\mathcal{T}^\lambda_{\mathrm{PDP}}(\lambda_1,k_2))^k_q.\end{aligned} \tag{31}$$

The unpolarized double differential cross section and inelastic nucleon spin observables are given by

$$\begin{aligned}D_{\alpha,\beta}=\mathrm{Tr}(\sigma_\alpha\mathcal{M}\sigma_\beta\mathcal{M}^\dagger)=&\sum_{n=0}^{1}\sum_{n'=0}^{1}\\&\times\sum_{\kappa=|n-n'|}^{(n+n')}((P^n(\alpha)\otimes P^{n'}(\beta))^\kappa\cdot B^\kappa(n,n')),\end{aligned} \tag{32}$$

where $\alpha,\beta=0,x,y,z$

$$\begin{aligned}B_Q^\kappa(n,n')=&[s_B]^2[s_A]^{-2}\sum_{k_1,k_1',k_2,k,k'}[k][k'][k_1][k_1']\\&\times[n][n']W(k_1k_2\kappa k';kk_1')\begin{Bmatrix}\frac{1}{2}&\frac{1}{2}&k_1\\\frac{1}{2}&\frac{1}{2}&k_1'\\n&n'&\kappa\end{Bmatrix}\\&\times(-1)^{k+n'-\kappa}(\mathcal{A}^k(k_1,k_2)\otimes\mathcal{A}^{\dagger k'}(k_1',k_2))^\kappa_Q,\end{aligned} \tag{33}$$

$$[\mathcal{A}_q^k(k_1,k_2)]^*=(-1)^q\mathcal{A}^{\dagger k}_{-q}(k_1,k_2) \tag{34}$$

and

$$P^n_\mu(\alpha)=\mathrm{Tr}(\sigma_\alpha\sigma^n_\mu), \tag{35}$$

the spherical components σ^1_μ of $\boldsymbol{\sigma}$ being given by Eq. (21) and $\sigma^0_0=\sigma_0$ denotes the unit matrix.

The formalism outlined above makes no assumptions like DWBA or DWIA. Based as it is purely on invariance considerations, it may be used with advantage to analyze experimental data on inelastic nucleon scattering on nuclei via Δ excitation.

One of us (G.R.) acknowledges with thanks the support of the CSIR (India).

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