Bound states of ΔΔ and ΔΔΔ systems

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We study possible bound states of the ΔΔ and ΔΔΔ systems by using a two-body interaction derived from the chiral quark cluster model. The systems of two and three deltas, which will appear in nature as dibaryon and tribaryon resonances with zero strangeness, have large similarities with the corresponding two- and three-nucleon systems. The two deepest bound ΔΔ states are those with angular momentum and isospin $(j,i)=(1,0)$ and $(j,i)=(0,1)$ which have the same quantum numbers as the $^3S_1$-$^3D_1$ (deuteron) and $^1S_0$ (virtual) NN states. Similarly, the more strongly bound ΔΔΔ state is that with angular momentum and isospin $(J,I)=(\frac{1}{2},\frac{1}{2})$ which has precisely the same quantum numbers as the triton. [S0556-2813(97)02806-9]

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I. INTRODUCTION

The possible existence of nonstrange dibaryon resonances has been studied in the past by considering the bound-state problem for the $N\Delta$ and $\Delta\Delta$ systems [1–5]. Since the Δ is an unstable particle, bound-state solutions of the two-body system will actually appear in nature as resonances that decay mainly into two nucleons and one pion or into two nucleons and two pions, respectively. If we now make a straightforward generalization of this concept, we can investigate the possible existence of tribaryon resonances that decay mainly into three nucleons and either one, two, or three pions, by looking into the bound-state problem of the $NN\Delta$, $N\Delta\Delta$, and $\Delta\Delta\Delta$ systems, respectively.

In this first paper we will consider the simplest case where we have only identical particles, i.e., we will discuss here the bound-state problems of the ΔΔ and ΔΔΔ systems. The cases of combined systems of nucleons and deltas will be studied in future works.

In order to perform the ΔΔΔ calculations we will take advantage of the experience gained in the three-nucleon bound-state problem. In that case one knows that the dominant configuration of the system is that in which all particles are in S-wave states. However, in order to get reasonable results for the binding energy, the S-wave two-body amplitudes used as input in the Faddeev equations must already contain the effect of the tensor force. Thus, for example, in the case of the Reid soft-core potential if one considers only the S-wave configurations but neglects the tensor force in the two-body subsystems the triton is unbound. However, if one includes the effect of the tensor force in the nucleon-nucleon $^3S_1$-$^3D_1$ channel but uses only the $^1S_0$ and $^3S_1$ components of the two-body amplitudes in the three-body equations (two-channel calculation) one gets a triton binding energy of 6.6 MeV. Notice that including the remaining configurations (34-channel calculation) leads to a triton binding energy of 7.35 MeV [6]. This means that the S-wave truncated T-matrix approximation leads to a binding energy which differs from the exact value by less than 1 MeV. Therefore by means of our approach we will not study exact binding energies but which are the best candidates for bound states and the ordering of the different ΔΔ and ΔΔΔ states.

We will use delta-delta interactions derived from the chiral quark cluster model that reproduces the nucleon-nucleon data [7–9]. This model contains π and σ exchange in addition to the quarks and gluons [7–9]. The main advantage of the model comes from the fact that it works with a single qq-meson vertex. Therefore its parameters (coupling constants, cutoff masses, etc.) are independent of the baryon to which the quarks are coupled, the difference among them being generated by SU(2) scaling.

The lifetime of the bound ΔΔ and ΔΔΔ systems should be similar to that of the Δ in the case of very weakly bound systems and larger if the system is very strongly bound. Therefore, these dibaryon and tribaryon resonances will have widths similar or smaller than the width of the Δ so that, in principle, they are experimentally observable.

In order to perform our calculations we will assume that the Δ is a stable particle, that is, we will neglect the width of the Δ and the effects of the retardation in the one-pion-exchange interaction of the ΔΔ subsystem. These two effects have been estimated recently in the case of the simpler $N\Delta$ system [5]. There, it was found that the assumption of a stable Δ leads to very reliable predictions for the mass of $N\Delta$ resonances since the effects of retardation and width of the Δ are responsible for producing the width of the $N\Delta$ resonance but have almost no effect over its mass. Thus, this gives us confidence that our predictions for the masses of the ΔΔ and ΔΔΔ states will not change very much when the unstable nature of the Δ is explicitly taken into account.

Finally, we want to emphasize that the possible detection of dibaryon and tribaryon resonances does not constitute an exotic subject since, in principle, any nucleus with at least three nucleons can serve as test system that may be excited by forming a tribaryon.

II. THE TWO-BODY SYSTEM

We consider two deltas in a relative S-state interacting through a potential that contains a tensor force. Thus, there is a coupling to the ΔΔ D wave so that the Lippmann-Schwinger equation of the system is of the form

TABLE I. Coupled channels (l,s) that contribute to a given $\Delta\Delta$ state with total angular momentum j and isospin i.

j	i	(l,s)
0	1	(0,0),(2,2)
0	3	(0,0),(2,2)
1	0	(0,1),(2,1),(2,3)
1	2	(0,1),(2,1),(2,3)
2	1	(0,2),(2,0),(2,2)
2	3	(0,2),(2,0),(2,2)
3	0	(0,3),(2,1),(2,3)
3	2	(0,3),(2,1),(2,3)

$$t_{ji}^{ls,l''s''}(p,p'';E)$$
$$=V_{ji}^{ls,l''s''}(p,p'')+\sum_{l's'}\int_0^\infty p'^2dp'V_{ji}^{ls,l's'}(p,p')\times\frac{1}{E-p'^2/M_\Delta+i\epsilon}t_{ji}^{l's',l''s''}(p',p'';E), \tag{1}$$

where j and i are the angular momentum and isospin of the system, while ls, $l's'$, and $l''s''$ are the initial, intermediate, and final orbital angular momentum and spin of the system, respectively. We give in Table I the two-body channels that are coupled together for the eight possible values of j and i that involve two deltas in a relative S state. Notice that the Pauli principle requires that $(-)^{l+s+i}=-1$. For bound-state problems $E<0$ so that the singularity of the propagator is never touched and we can forget the $i\epsilon$ in the denominator. If we make the change of variables

$$p'=b\frac{1+x'}{1-x'}, \tag{2}$$

where b is a scale parameter and similarly for p and p'', we can write Eq. (1) as

$$t_{ji}^{ls,l''s''}(x,x'';E)=V_{ji}^{ls,l''s''}(x,x'')+\sum_{l's'}\int_{-1}^{1}b^2\left(\frac{1+x'}{1-x'}\right)^2\times\frac{2b}{(1-x')^2}dx'V_{ji}^{ls,l's'}(x,x')\times\frac{1}{E-p'^2/M_\Delta}t_{ji}^{l's',l''s''}(x',x'';E). \tag{3}$$

We solve this equation by replacing the integral from -1 to 1 by a Gauss-Legendre quadrature [10] which results in the set of linear equations

$$\sum_{l's'}\sum_{m=1}^{N}M_{ji}^{nls,ml's'}(E)t_{ji}^{l's',l''s''}(x_m,x_k;E)=V_{ji}^{ls,l''s''}(x_n,x_k), \tag{4}$$

with

TABLE II. Quark model parameters.

m_q (MeV)	313
b (fm)	0.518
α_s	0.485
a_c (MeV fm^{-2})	46.938
$\alpha_{\rm ch}$	0.027
m_σ (fm^{-1})	3.421
m_π (fm^{-1})	0.70
Λ (fm^{-1})	4.2

$$M_{ji}^{nls,ml's'}(E)=\delta_{nm}\delta_{ll'}\delta_{ss'}-w_mb^2\left(\frac{1+x_m}{1-x_m}\right)^2\frac{2b}{(1-x_m)^2}\times V_{ji}^{ls,l's'}(x_n,x_m)\frac{1}{E-p_m'^2/M_\Delta}, \tag{5}$$

and w_m and x_m are the weights and abscissas of the Gauss-Legendre quadrature [10] while p'_m is obtained by putting $x'=x_m$ in Eq. (2).

For the solution of the three-body system we will use only the component of the T matrix with $l=l''=0$, so that for that purpose we define the S-wave amplitude

$$t^{si}(p,p'';E)\equiv t_{si}^{0s,0s}(p,p'';E). \tag{6}$$

If a bound state exists at an energy E_B, the determinant of the matrix $M_{ji}^{nls,ml's'}(E_B)$ vanishes, i.e.,

$$|M_{ji}(E_B)|=0. \tag{7}$$

We took the scale parameter b of Eq. (2) as b = 3 fm^{-1} and used a Gauss-Legendre quadrature with N = 20 points [10].

The interaction between two deltas was obtained from the chiral quark cluster model developed elsewhere [8]. In this model baryons are described as clusters of three interacting massive (constituent) quarks, the mass coming from the breaking of the chiral symmetry. The ingredients of the quark-quark interaction are confinement, one-gluon (OGE), one-pion (OPE), and one-sigma (OSE) exchange terms, and whose parameters are fixed from the NN data. Explicitly, the quark-quark (qq) interaction is

$$V_{qq}(\vec{r}_{ij})=V_{\rm con}(\vec{r}_{ij})+V_{\rm OGE}(\vec{r}_{ij})+V_{\rm OPE}(\vec{r}_{ij})+V_{\rm OSE}(\vec{r}_{ij}), \tag{8}$$

where $\vec{r}_{ij}$ is the ij interquark distance and

$$V_{\rm con}(\vec{r}_{ij})=-a_c\vec{\lambda}_i\cdot\vec{\lambda}_jr_{ij}^2, \tag{9}$$

$$V_{\rm OGE}(\vec{r}_{ij})=\frac{1}{4}\alpha_s\vec{\lambda}_i\cdot\vec{\lambda}_j\left\{\frac{1}{r_{ij}}-\frac{\pi}{m_q^2}\left[1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right]\times\delta(\vec{r}_{ij})-\frac{3}{4m_q^2r_{ij}^3}S_{ij}\right\}, \tag{10}$$

$$V_{\mathrm{OPE}}(\vec{r}_{ij})=\frac{1}{3}\alpha_{ch}\frac{\Lambda^2}{\Lambda^2-m_\pi^2}m_\pi\Bigg\{\Bigg[Y(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}Y(\Lambda r_{ij})\Bigg]\times\vec{\sigma}_i\cdot\vec{\sigma}_j+\Bigg[H(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}H(\Lambda r_{ij})\Bigg]S_{ij}\Bigg\}\times\vec{\tau}_i\cdot\vec{\tau}_j, \tag{11}$$

$$V_{\mathrm{OSE}}(\vec{r}_{ij})=-\alpha_{\mathrm{ch}}\frac{4m_q^2}{m_\pi^2}\frac{\Lambda^2}{\Lambda^2-m_\sigma^2}m_\sigma\Bigg[Y(m_\sigma r_{ij})-\frac{\Lambda}{m_\sigma}Y(\Lambda r_{ij})\Bigg], \tag{12}$$

where

$$Y(x)=\frac{e^{-x}}{x},\quad H(x)=\left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y(x). \tag{13}$$

a_c is the confinement strength, the $\vec{\lambda}$'s are the SU(3) color matrices, the $\vec{\sigma}$'s ($\vec{\tau}$'s) are the spin (isospin) Pauli matrices, S_{ij} is the usual tensor operator, m_q (m_π, m_σ) is the quark (pion, sigma) mass, α_s is the qq-gluon coupling constant, α_{ch} is the qq-meson coupling constant, and Λ a cutoff parameter. In order to derive a $\Delta\Delta$ potential from the basic qq interaction defined above we use a Born-Oppenheimer approximation

$$V_{\Delta\Delta(LST)\rightarrow\Delta\Delta(L'S'T)}(R)=\xi_{LST}^{L'S'T}(R)-\xi_{LST}^{L'S'T}(\infty), \tag{14}$$

where

$$\xi_{LST}^{L'S'T}(R)=\frac{\langle\Psi_{\Delta\Delta}^{L'S'T}(\vec{R})|\Sigma_{i<j=1}^{6}V_{qq}(\vec{r}_{ij})|\Psi_{\Delta\Delta}^{LST}(\vec{R})\rangle}{\sqrt{\langle\Psi_{\Delta\Delta}^{L'S'T}(\vec{R})|\Psi_{\Delta\Delta}^{L'S'T}(\vec{R})\rangle}\sqrt{\langle\Psi_{\Delta\Delta}^{LST}(\vec{R})|\Psi_{\Delta\Delta}^{LST}(\vec{R})\rangle}}. \tag{15}$$

In the last expression the quark coordinates are integrated out keeping R fixed, the resulting interaction being a function of the Δ-Δ distance. The parameters of the model are summarized in Table II. From this table an $f_{\pi\Delta\Delta}$ baryonic coupling constant can be consistently derived by quark scaling [11]. The value is the one inconsistently used by most of the mesonic $\Delta\Delta$ potentials [12], in the sense that they only consider the quark model to obtain the vertex coupling constant.

III. THE THREE-BODY SYSTEM

If we restrict ourselves to the configurations where all three particles are in S-wave states, the Faddeev equations for the bound-state problem in the case of three identical particles with total spin S and total isospin I are [13]

$$T_{\mathrm{SI}}^{si}(p_i,q_i;E)=2\sum_{s'i'}h_{\mathrm{SI}}^{si,s'i'}\int_0^\infty dq_j\times\int_{|q_i-q_j/2|}^{q_i+q_j/2}dp_j t^{si}(p_i,p_i'';E-3q_i^2/4M_\Delta)\times\frac{p_j}{E-p_j^2/M_\Delta-3q_j^2/4M_\Delta}\frac{q_j}{q_i}\times T_{\mathrm{SI}}^{s'i'}(p_j,q_j;E), \tag{16}$$

where t^{si} are the two-body amplitudes defined by Eq. (6),

$$p_i''=\left(p_j^2+\frac{3}{4}q_j^2-\frac{3}{4}q_i^2\right)^{1/2}, \tag{17}$$

and

$$h_{\mathrm{SI}}^{si,s'i'}=(-)^{s'+3/2-S}\sqrt{(2s+1)(2s'+1)}W\left(\frac{3}{2}\frac{3}{2}S\frac{3}{2};ss'\right)\times(-)^{i'+3/2-I}\sqrt{(2i+1)(2i'+1)}W\left(\frac{3}{2}\frac{3}{2}I\frac{3}{2};ii'\right), \tag{18}$$

are the spin-isospin coefficients for the case of three deltas with W the Racah coefficient. We give in Table III the contributing two-body channels for all the possible values of total spin and isospin of the three-delta system.

If we now apply the transformations

$$p_i=b\frac{1+x}{1-x},\quad p_j=b\frac{1+x'}{1-x'}, \tag{19}$$

$$q_i=b\frac{1+y}{1-y},\quad q_j=b\frac{1+y'}{1-y'}, \tag{20}$$

then Eq. (16) becomes

$$T_{\mathrm{SI}}^{si}(x,y;E)=2\sum_{s'i'}h_{\mathrm{SI}}^{si,s'i'}\int_{-1}^{1}\frac{2b}{(1-y')^2}dy'\times\int_{x'_-}^{x'_+}\frac{2b}{(1-x')^2}dx't^{si}(x,x'';E-3q_i^2/4M_\Delta)\times\frac{p_j}{E-p_j^2/M_\Delta-3q_j^2/4M_\Delta}\frac{q_j}{q_i}T_{\mathrm{SI}}^{s'i'}(x',y';E). \tag{21}$$

TABLE III. Two-body channels (s,i) that contribute to a given ΔΔΔ state with total spin S and isospin I.

S	I	(s,i)
1/2	1/2	(1,2),(2,1)
1/2	3/2	(1,0),(1,2),(2,1),(2,3)
1/2	5/2	(1,2),(2,1),(2,3)
1/2	7/2	(1,2),(2,3)
1/2	9/2	(2,3)
3/2	1/2	(0,1),(1,2),(2,1),(3,2)
3/2	3/2	(0,1),(0,3),(1,0),(1,2), (2,1),(2,3),(3,0),(3,2)
3/2	5/2	(0,1),(0,3),(1,2),(2,1), (2,3),(3,2)
3/2	7/2	(0,3),(1,2),(2,3),(3,2)
3/2	9/2	(0,3),(2,3)
5/2	1/2	(1,2),(2,1),(3,2)
5/2	3/2	(1,0),(1,2),(2,1),(2,3), (3,0),(3,2)
5/2	5/2	(1,2),(2,1),(2,3),(3,2)
5/2	7/2	(1,2),(2,3),(3,2)
5/2	9/2	(2,3)
7/2	1/2	(2,1),(3,2)
7/2	3/2	(2,1),(2,3),(3,0),(3,2)
7/2	5/2	(2,1),(2,3),(3,2)
7/2	7/2	(2,3),(3,2)
7/2	9/2	(2,3)
9/2	1/2	(3,2)
9/2	3/2	(3,0),(3,2)
9/2	5/2	(3,2)
9/2	7/2	(3,2)
9/2	9/2	

Since the variable x runs from -1 to 1, we can make the expansion in terms of Legendre polynomials

$$q_i T_{\mathrm{SI}}^{si}(x,y;E)=\sum_{n=1}^{L} T_{\mathrm{SI}}^{\sin}(y;E)P_n(x), \tag{22}$$

so that the integral equation in two variables (21) becomes an integral equation in one variable

$$T_{\mathrm{SI}}^{\sin}(y;E)=\sum_{s'i'}\sum_{m=1}^{L}\int_{-1}^{1}dy' B_{\mathrm{SI}}^{\sin,s'i'm}(y,y';E) \times T_{\mathrm{SI}}^{s'i'm}(y';E), \tag{23}$$

where

$$B_{\mathrm{SI}}^{\sin,s'i'm}(y,y';E) =(2n+1)h_{\mathrm{SI}}^{si,s'i'}\frac{2b}{(1-y')^2}\int_{-1}^{1}dx \times\int_{x'_-}^{x'_+}\frac{2b}{(1-x')^2}dx' P_n(x)t^{si}(x,x'';E-3q_i^2/4M_\Delta) \times\frac{p_j}{E-p_j^2/M_\Delta-3q_j^2/4M_\Delta}P_m(x'). \tag{24}$$

TABLE IV. Binding energies B_2 of the ΔΔ states with total angular momentum j and isospin i. We also give between parentheses the energies obtained by neglecting the tensor force.

j	i	B_2 (MeV)
0	1	108.4(100.3)
0	3	0.4(unbound)
1	0	138.5(122.7)
1	2	5.7(4.8)
2	1	30.5(21.8)
2	3	Unbound(unbound)
3	0	29.9(9.3)
3	2	Unbound(unbound)

We now approximate the integral in Eq. (23) by a Gauss-Legendre quadrature [10] in order to obtain the system of homogeneous linear equations

$$\sum_{s'i'}\sum_{m=1}^{L}\sum_{k=1}^{N} M_{\mathrm{SI}}^{\sin j,s'i'mk}(E)T_{\mathrm{SI}}^{s'i'm}(y_k)=0, \tag{25}$$

with

$$M_{\mathrm{SI}}^{\sin j,s'i'mk}(E)=\delta_{ss'}\delta_{ii'}\delta_{nm}\delta_{jk}-w_k B_{\mathrm{SI}}^{\sin,s'i'm}(y_j,y_k;E), \tag{26}$$

where w_k and y_k are, respectively, the weights and abscissas of the Gauss-Legendre quadrature [10].

In order for Eq. (25) to have a solution, the Fredholm determinant of the system must vanish, so that a bound state exists at the energy E_B if

$$|M_{\mathrm{SI}}(E_B)|=0. \tag{27}$$

Thus, our method of solution consists simply in searching for the zeros of the Fredholm determinant as a function of energy.

We checked our program by comparing with known results for the three-nucleon bound-state problem with the Reid soft-core potential [14]. We found very stable results taking for the scale parameter $b=3$ fm^{-1}, a number of Legendre polynomials $L=10$, and a number of Gauss-Legendre points $N=12$.

IV. RESULTS

We give in Table IV our results for the ΔΔ system. Out of the eight possible ΔΔ channels six have a bound state (there are no excited states in any of the channels). It is interesting that the deepest bound state is the $(j,i)=(1,0)$ which has precisely the same quantum numbers as the deuteron, while the next deepest bound state is the $(j,i)=(0,1)$ which has precisely the quantum numbers of the NN $^1S_0$ virtual state. This clearly shows that there is a qualitative similarity between the ΔΔ and NN systems. In order

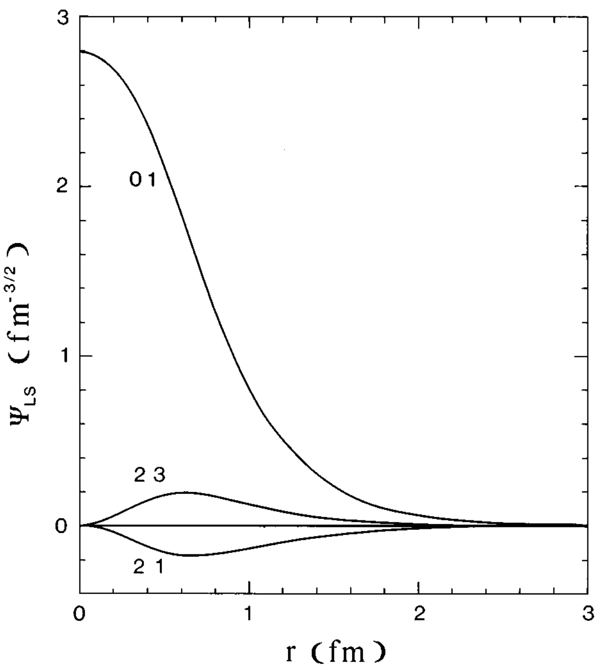

FIG. 1. Wave function of the $\Delta\Delta$ bound state in the $(j,i)=(1,0)$ channel. We show the three components corresponding to the states $(l,s)=(0,1)$, $(l,s)=(2,1)$, and $(l,s)=(2,3)$.

to show the effect of the tensor force we also give in parentheses the corresponding binding energies that are obtained if one neglects the tensor force. As one can see, the effect of the tensor force is to add somewhat to the attraction but without changing the qualitative behavior of the spectrum.

We show in Fig. 1 the bound-state wave function of the $(j,i)=(1,0)$ channel which has the same quantum numbers as the deuteron. This channel has three components, namely, $(l,s)=(0,1)$, $(l,s)=(2,1)$, and $(l,s)=(2,3)$. The probability of the $(l,s)=(0,1)$ state is 97% while the $(l,s)=(2,1)$ and (2,3) states have each one a probability of 1.5%. This wave function may be useful in calculations of the $\Delta\Delta$ components of the deuteron.

The channels $(j,i)=(2,3)$ and (3,2) are unbound because they have a strong repulsive barrier at short distances in the S-wave central interaction. This strong repulsion originates from the quark Pauli blocking produced by the saturation of states that occurs when the total spin and isospin are near their maximum values [15]. As we will see next in the discussion of the $\Delta\Delta\Delta$ results, these repulsive cores in the (3,2) and (2,3) channels largely determine the three-body spectrum.

TABLE V. Binding energies B_3 and separation energies B_3-B_2 of the $\Delta\Delta\Delta$ states with total spin S and isospin I. We also give between parentheses the corresponding energies obtained by neglecting the tensor force.

S	I	B_3 (MeV)	B_3-B_2 (MeV)
1/2	1/2	84.0(73.0)	53.5(51.2)
1/2	3/2	139.2(unbound)	0.7
1/2	7/2	6.3(unbound)	0.6
3/2	1/2	109.5(100.6)	1.1(0.3)
5/2	1/2	39.1(28.9)	8.6(7.1)
7/2	1/2	31.7(unbound)	1.2
7/2	3/2	35.1(unbound)	4.6

We show in Table V the results for the $\Delta\Delta\Delta$ system with and without including the tensor force in the two-body interaction. We give both B_3 the binding energy of the system and B_3-B_2 the separation energy, where B_2 is the binding energy of the deepest bound two-body channel that contributes to the three-body state (see Tables III and IV). The full model has seven bound states while the model without tensor force has only three. Notice, however, that the extra states produced by including the tensor force are barely bound, i.e., they have very small separation energies, so that there is really not much difference by including the tensor force. The more strongly bound three-body state (that is, the one with the largest separation energy) is the $(S,I)=(\frac{1}{2},\frac{1}{2})$ which has precisely the quantum numbers of the triton. This shows again like in the two-body case the similarity between the $\Delta\Delta\Delta$ and NNN systems. As stated in the Introduction, our procedure only provides an approximation to the binding energies, therefore it is meaningless to go further in the comparison with the NNN system, the main objective being the comparison in the ordering of the different states. In fact, the most interesting comparison would be to calculate the $\Delta\Delta$ and $\Delta\Delta\Delta$ bound states by means of a baryonic interaction, where the strong repulsion, that is a consequence of the quark substructure is absent. We are actually working along this line [16].

The reason why the $(S,I)=(\frac{1}{2},\frac{1}{2})$ state is the more strongly bound is very simple. As shown in Table III, this is the only state where none of the two-body channels with a strong repulsive core $(s,i)=(2,3)$ or (3,2) contribute. In all the other three-body states the strong repulsion of the $(s,i)=(2,3)$ and (3,2) channels either completely destroys the bound state or allows just a barely bound one.

The state $(S,I)=(\frac{5}{2},\frac{1}{2})$ comes next with respect to separation energy. Notice that like in the $(S,I)=(\frac{1}{2},\frac{1}{2})$ state, the separation energy comes out very similar whether we include or not the tensor force. This means that the tensor force essentially shifts both the two-body and three-body binding energies by roughly a constant amount.

The third state that is bound in both the scheme with tensor force and the one without it is the $(S,I)=(\frac{3}{2},\frac{1}{2})$ and again like in the two previous cases the separation energy is not so different between the two schemes.

The state $(S,I)=(\frac{7}{2},\frac{3}{2})$ has a somewhat anomalous behavior since it has a relatively large separation energy in the scheme with tensor force (B_3-B_2=4.6 MeV) while it is unbound in the scheme without tensor force. This behavior is sort of accidental and it can be understood as follows. As seen in Table III, there are four two-body channels contributing to the $(S,I)=(\frac{7}{2},\frac{3}{2})$ state, the two attractive ones $(s,i)=(2,1)$ and (3,0) and the two repulsive ones $(s,i)=(2,3)$ and (3,2). In the case without tensor force, the channels (2,1) and (3,0) have bound states at $E=-21.8$ and $E=-9.3$ MeV, respectively, so that the poles in the scattering amplitudes of these two-body channels are very far apart and only the deepest one contributes effectively to the three-body bound-state problem. In the case with tensor force, on the other hand, channels (2,1) and (3,0) have bound states at $E=-30.5$ and $E=-29.9$ MeV, respectively, so that the poles in the scattering amplitudes of these two channels are very close together and both of them contribute effectively to the three-body bound state.

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