Parity conserving γ asymmetry in n-p radiative capture

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The importance of n-p radiative capture, utilizing polarized cold neutrons, as a means of measuring the weak pion coupling constant is reviewed. Parity conserving processes of the form $\mathbf{k}_\gamma \cdot (\mathbf{s}_n \times \mathbf{k}_n)$ can contribute to the $\mathbf{s}_n \cdot \mathbf{k}_\gamma$ photon asymmetry in any such experiment, if the apparatus is not perfectly symmetric. For an incident laboratory neutron energy of 0.003 eV a value of $A_\gamma^{\mathrm{PC}} = 0.67 \times 10^{-8}$ is obtained for two different potential models (Argonne AV14 and Nijmegen Reid93). Serving as an extreme test case, the Reid soft core potential yields 0.61×10^{-8}, close to the result of the contemporary forces. Implications for extracting the weak pion coupling constant and for monitoring the beam polarization are discussed. [S0556-2813(97)04908-X]

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I. INTRODUCTION

It was more than 60 years ago that the first photonuclear experiment was performed [1]: $^2\mathrm{H} + \gamma \rightarrow n + p$. Over the following 15 years, it came to be realized [2] that the approximately 330 mb cross section for the inverse reaction (thermal neutron capture by hydrogen) [3,4] was some 10% larger than that for which theoretical models could account. This discrepancy between experiment and theory produced the first incontrovertible evidence for the importance of meson-exchange currents in nuclear reactions [5,6]. Some 20 years ago it was observed that deuteron photodisintegration yielded forward going (0°) protons [7] in greater numbers than nonrelativistic theory predicted. This was subsequently confirmed by two independent measurements [8,9]. The observation was demonstrated to provide evidence for the contribution of the relativistic spin-orbit dipole operator even at photon energies well below 50 MeV [10,11]. These are but two examples that illustrate the significant role played by low-energy neutron-proton radiative capture and threshold deuteron photodisintegration experiments in developing our understanding of nuclear physics in terms of the physically observable hadrons, the baryons and mesons, and their interactions.

The deuteron has also played an important role in efforts to understand the weak interaction in nuclei. In the same year that parity nonconservation (PNC) was observed in β and μ decay, a first search for parity violation in the NN interaction was reported [12]. The first evidence for such parity nonconservation was found in the radiative capture of neutrons by $^{181}\mathrm{Ta}$ [13]. The observed signal was of the expected size:

$$V_{NN}^{\mathrm{PNC}} / V_{NN}^{\mathrm{PC}} \sim G m_\pi^2 \sim 10^{-7}, \tag{1}$$

where $G = 1.01 \times 10^{-5} / M_N^2$ is the weak coupling constant. The neutron capture technique was later extended in an attempt to detect the parity nonconserving circular polarization signal in n-p radiative capture. Unfortunately, the initial measurement was contaminated by circularly polarized photons from bremsstrahlung, but an improved measurement [14] yielded a final upper limit of $P_\gamma = (1.8 \pm 1.8) \times 10^{-7}$. The nominal value was consistent with theoretical expectations of 0.6×10^{-7}. However, to obtain an accuracy sufficient to extract more than an upper limit on the weak coupling constants via this method would be difficult, because of the small analyzing power (0.045) of the γ-ray polarimeter.

A summary of efforts over the years to understand the coupling constants defining the weak Hamiltonian H_W can be found in the reviews by Adelberger and Haxton [15] and by Haeberli and Holstein [16]. Various combinations of the weak meson-exchange coupling constants [17] $(f_\pi, h_\rho^0, h_\rho^1, h_\rho^2, h_\omega^0, \ldots)$ contribute to a number of scattering and reaction processes. However, most PNC measurements are sensitive to a linear combination of these parameters or to only the $\Delta I = 0,2$ components. In particular, the n-p radiative capture P_γ measurements and possible helicity measurements are sensitive to $\Delta I = 0,2$ mixing effects in the $^1S_0$-$^3P_0$ and $^3S_1$-$^3P_1$ amplitudes. Similarly, the PNC effects in low-energy pp scattering (at LAMPF [18] and PSI [19]) explored primarily $\Delta I = 0,2$ but failed to investigate π-exchange effects. (The f_π coupling does not enter the picture in the case that projectile and target are identical.) Alternatively, a measurement of the photon emission asymmetry A_γ^{PNC} when polarized neutrons are captured by protons is sensitive to $\Delta I = 1$ mixing effects in the $^3S_1$-$^3P_1$ amplitude and, thus, to the weak pion coupling constant f_π. (P_γ and A_γ are essentially independent.) The dependence of this photon asymmetry on the weak coupling constants is calculated to be [16,20]

$$A_\gamma^{\mathrm{PNC}} = -0.107 f_\pi + 0.003 h_\omega^1 - 0.001 h_\rho^1 \simeq -0.11 f_\pi + (\text{negligible } \rho/\omega \text{ contributions}). \tag{2}$$

A definitive measurement of the parity violating component of A_γ would test whether f_π agrees with the the neutral-current-enhanced weak current prediction of Desplanques, Donoghue, and Holstein [17] (-0.5×10^{-7}) or is, in fact, significantly smaller. Studies of a parity-mixed doublet in $^{18}\mathrm{F}$ indicate a strong suppression of f_π relative to the DDH best values [16,21]. A comparison of f_π extracted from A_γ in the n-p radiative capture reaction with that coming from measurements in ^{18}F and in other light and heavy nuclei would provide a first insight regarding possible modification of f_π when embedded in the nuclear medium.

The measurement of A_γ in n-p radiative capture reached feasibility with the development of intense cold, polarized neutron beams from the high flux reactor at ILL (Institut Laue-Langevin). The initial result was reported [22] to be $A_\gamma=(-6\pm21)\times10^{-8}$. The final analysis provided only a refined upper limit [23] of $A_\gamma=(-1.5\pm4.7)\times10^{-8}$. Because the experiment was limited by statistical uncertainties, it is anticipated that a longer running might achieve an order of magnitude improvement. Such a measurement could answer the question of whether there exists a significant neutral current enhancement of f_π in the NN weak interaction.

The experimental situation which we study in this paper is the following. A transversely polarized neutron beam is incident on an unpolarized proton target. The z axis of our coordinate system is defined by the neutron momentum $\mathbf{k}_n$, while the direction of the neutron polarization, $\mathbf{s}_n$, is parallel to the x axis. The momentum of the outgoing photon, $\mathbf{k}_\gamma$, makes an angle θ with $\mathbf{k}_n$, while ϕ is the angle between the plane of $\mathbf{k}_n$ and $\mathbf{k}_\gamma$ and the plane defined by $\mathbf{k}_n$ and $\mathbf{s}_n$. All angles are given in the center-of-mass frame, where the total momentum is zero; all energies and momenta are specified in the laboratory frame, where the proton is at rest.

The observable in which we are interested is A_γ, the asymmetry of the photon distribution with respect to the polarization direction. The only parity conserving (PC) scalar, built from $\mathbf{k}_n$, $\mathbf{s}_n$, and $\mathbf{k}_\gamma$, describing this situation, is $\mathbf{s}_n\cdot[\mathbf{k}_n\times\mathbf{k}_\gamma]$, while the only parity nonconserving (PNC) pseudoscalar is $\mathbf{s}_n\cdot\mathbf{k}_\gamma$. The former leads to a left-right ($\sin\theta\sin\phi$) asymmetry, while the latter leads to an up-down ($\sin\theta\cos\phi$) asymmetry. Because of these different symmetry properties, the two can be separated exactly in a perfect detector. However, if there is a small asymmetry in the left-right spatial acceptance of the detector, then A_γ^{PC} leads to an A_γ^{PNC}-like background. The asymmetry in the up-down or left-right acceptance is typically less than 1% in a carefully constructed detector. The question is whether this is adequate to avoid any contamination of A_γ^{PNC} from A_γ^{PC}. In the ILL experiment it was *assumed* that contamination of A_γ^{PNC} due to a parity conserving asymmetry A_γ^{PC} was negligible. Whether such an assumption was at that time warranted, any effort to push such a measurement to obtain more than an upper limit should take into account A_γ^{PC}.

Our goal here is to calculate A_γ^{PC}, using contemporary nucleon-nucleon potential models. We seek to provide an estimate upon which to base any new measurement of the parity nonconserving photon angular asymmetry in n-p radiative capture in order to determine a value for the pion weak coupling constant. We want to understand how robust is the estimate of A_γ^{PC} in order to shed light on two important questions: (1) If the measured asymmetry that determines A_γ^{PNC} is sufficiently small that one must separate the PNC and PC asymmetries, then how large might the PC contamination be? (2) If the separation can be achieved experimentally, then can A_γ^{PC} be used as a polarization monitor in the measurement? An *in situ* polarization monitor would considerably enhance the reliability of any measurement.

This short paper is structured as follows: In the next section, we outline the expressions required to calculate the parity conserving photon asymmetry. In the following section we discuss briefly our method for obtaining the nucleon-nucleon bound state and continuum wave functions. In the last section we summarize our results, compare them with the known parity violating asymmetry estimates, and discuss the implications of our calculations for future measurements of f_π.

II. ANGULAR DISTRIBUTION

For the cold neutron energies of interest, only the lowest multipoles in the radiative capture process are important. The angular distribution has the general $\sin^2\theta$ form characteristic of an $E1$-dominated transition. The angular asymmetry arises from an $E1\times M1$ interference, so that the photon asymmetry exhibits an expected $\sin\phi$ dependence. These properties have been thoroughly discussed in the literature.

We adopt the nonrelativistic phenomenological treatment of n-p radiative capture published by Partovi [25] (and use $\hbar=c=1$ units). The primary approximation is the neglect of any nucleon structure and meson-exchange currents. The parity conserving n-p capture cross section can be expressed as

$$\frac{d\sigma}{d\Omega}=I_0(\theta)[1+P_tB(\theta)\sin\phi], \tag{3}$$

where P_t is the transverse polarization of the incident nucleon, and θ and ϕ are defined above.

The function $I_0(\theta)$ appearing which determines the unpolarized differential cross section can be written as

$$\begin{aligned}I_0(\theta)=\frac{\omega^2}{4k^2}\sum_{M=0}^{2}\sum_{L'=L}\sum_{L=1}(2-\delta_{M0})(2-\delta_{LL'})\Bigg\{&[d_{1M}^{(L)}(\theta)d_{1M}^{(L')}(\theta)+d_{1-M}^{(L)}(\theta)d_{1-M}^{(L')}(\theta)]\\&\times\mathrm{Re}\sum_{s=0}^{1}\sum_{m^d=-1}^{1}[(sM+m^d|E^{(L)}|m^d)(sM+m^d|E^{(L')}|m^d)^*+(sM+m^d|M^{(L)}|m^d)(sM+m^d|M^{(L')}|m^d)^*]\\&+[d_{1M}^{(L)}(\theta)d_{1M}^{(L')}(\theta)-d_{1-M}^{(L)}(\theta)d_{1-M}^{(L')}(\theta)]\mathrm{Re}\sum_{s=0}^{1}\sum_{m^d=-1}^{1}[(sM+m^d|E^{(L)}|m^d)(sM+m^d|M^{(L')}|m^d)^*\\&+(sM+m^d|M^{(L)}|m^d)(sM+m^d|E^{(L')}|m^d)^*]\Bigg\}.\end{aligned} \tag{4}$$

Here k is the relative n-p momentum, $d^{(J)}_{mm'}$ is the reduced rotation matrix, and the electric and magnetic multipoles are obtained by the usual expansion of $\boldsymbol{\epsilon}\exp(i\mathbf{k}_\gamma\cdot\boldsymbol{\xi})$ in terms of the photon polarization $\boldsymbol{\epsilon}$, the nucleon coordinate $\boldsymbol{\xi}$, and the photon momentum (energy) $\mathbf{k}_\gamma(\omega)$:

$$\boldsymbol{\epsilon}e^{(i\mathbf{k}_\gamma\cdot\boldsymbol{\xi})}=\sum_{LM}D^{(L)}_{M\mu}(0,-\theta,-\phi)\left(\frac{2\pi(2L+1)}{L(L+1)}\right)^{1/2}\left\{-\frac{i^{L+1}}{\omega}\nabla\left[\left(1+\xi\frac{d}{d\xi}\right)j_L(\omega\xi)Y^{(L)}_M(\theta,\phi)\right]\right.$$
$$\left.-i^{L+1}\omega\boldsymbol{\xi}j_L(\omega\xi)Y^{(L)}_M(\theta,\phi)-\mu i^L j_L(\omega\xi)[\boldsymbol{L}Y^{(L)}_M(\theta,\phi)]\right\}. \tag{5}$$

The $D^{(L)}_{M\mu}$ function appearing above is the standard rotation matrix. The first two terms in the summation constitute the electric multipoles; the last term is the magnetic multipole. The first term gives rise to usual Sigert theorem electric multipole operators; the second term generates the electric spin-dependent operator as well as a retardation correction to the electric multipole transitions.

The term giving rise to the photon asymmetry in Eq. (5) can be expressed as follows:

$$I_0(\theta)B(\theta)=-\frac{\omega^2}{\sqrt{2}k^2}\mathrm{Im}\sum_{LL''m^d}\{[d^{(L)}_{1-m^d}(\theta)d^{(L')}_{11-m^d}(\theta)-d^{(L)}_{1m^d}(\theta)d^{(L')}_{1m^d-1}(\theta)][(10|E^{(L)}|m^d)(11|E^{(L')}|m^d)^*$$
$$+(10|M^{(L)}|m^d)(11|M^{(L')}|m^d)^*]-(00|E^{(L)}|m^d)(11|E^{(L')}|m^d)^*-(00|M^{(L)}|m^d)(11|M^{(L')}|m^d)^*]$$
$$+[d^{(L)}_{1-m^d}(\theta)d^{(L')}_{11-m^d}(\theta)+d^{(L)}_{1m^d}(\theta)d^{(L')}_{1m^d-1}(\theta)][(10|E^{(L)}|m^d)(11|M^{(L')}|m^d)^*+(10|M^{(L)}|m^d)(11|E^{(L')}|m^d)^*]$$
$$-(00|E^{(L)}|m^d)(11|M^{(L')}|m^d)^*-(10|M^{(L)}|m^d)(11|E^{(L')}|m^d)^*]\}. \tag{6}$$

The relative n-p momentum k is related to the photon energy by

$$k^2=M\left[\omega\left(1-\frac{E_d}{2M}\right)-E_d\left(1-\frac{E_d}{4M}\right)\right], \tag{7}$$

where E_d is the deuteron binding energy and M is the nucleon mass.

At very low energies, using the properties of the d functions and the electromagnetic matrix elements, $B(\theta)$ in Eq. (3) reduces to $B(\theta)=A^{\mathrm{PC}}_\gamma\sin\theta$, where A^{PC}_γ is the parity conserving γ asymmetry in which we are interested. Thus we recover the $\mathbf{s}_n\cdot\mathbf{k}_\gamma$ angular distribution, discussed above.

III. WAVE FUNCTION SOLUTION

The outgoing scattering wave functions are evaluated by numerically solving the two-body Schrödinger equation for each partial wave. For each value of the total angular momentum j, there are four equations corresponding to the four possible combinations of orbital angular momentum l and spin s. Two of these equations, $(l=j,s=0)$ and $(l=j,s=1)$, are uncoupled. Following Partovi [25], we label them by $\lambda=2$ and $\lambda=4$. The other two sets of equations, $(l=j-1,s=1)$ and $(l=j+1,s=1)$, are coupled and have two independent sets of regular solutions. These are labeled by $\lambda=1$ and $\lambda=3$, where $\lambda=1$ is the solution that reduces to the $l=j-1$ function when there is no coupling. The other solution $\lambda=3$ reduces to the $l=j+1$ function in this limit. The boundary conditions for the reduced partial wave functions are such that the functions are zero at the origin and have the asymptotic form

$$v^j_{ls\lambda}(kr)\rightarrow\sin\left(kr-\frac{1}{2}l\pi+\delta^j_\lambda\right). \tag{8}$$

The numerical solutions were obtained by expanding each function in a complete set of cubic splines [24] and then using the collocation method to generate a matrix equation for the coefficients of the spline expansion. The eigenvalue coupled equations for the bound-state wave functions are solved in an analogous manner, except that the boundary condition in the asymptotic region requires that the functions go to zero.

The wave functions were first tested for accuracy by comparing with the numerical results reported by Partovi. Agreement of better than 1% was obtained with his matrix elements for various partial waves. In addition, we reproduced his published polarization functions. Finally, we obtained excellent agreement with the total capture cross section for thermal neutrons reported by Arenhövel and Sanzone [26] using two different potential models [27,28].

IV. NUMERICAL RESULTS

We considered the Reid soft core (RSC) [27], the Argonne V_{14} (AV14) [28], and the Nijmegen Reid93 [29] potential models to represent nucleon-nucleon scattering and the deuteron bound state. The RSC model was constructed to fit p-p scattering data in the $^1S_0$ channel, whereas the AV14 and Reid93 models were fitted to n-p scattering data in that channel. The simple form of the RSC model makes it a good test case for numerical checks. Moreover, by using the RSC p-p-based force, we also obtain the widest possible range of values for A^{PC}_γ. That is, the Coulomb-corrected p-p scattering length is $a^C_{pp}\simeq-17$ fm in contrast to the n-p spin-singlet scattering length which is $a^s_{np} \simeq -23.7$ fm. However, because the photon asymmetry is a ratio of matrix elements, the dependence on the $^1S_0$ channel interaction largely cancels. The RSC, AV14, and Reid93 forces yield $A_\gamma^{\rm PC} = 0.607 \times 10^{-8}$, $A_\gamma^{\rm PC} = 0.668 \times 10^{-8}$, and $A_\gamma^{\rm PC} = 0.665 \times 10^{-8}$, respectively.

Taking into account the spatial resolution of a typical detector, this means that $A_\gamma^{\rm PC}$ should not significantly contaminate a measurement of $A_\gamma^{\rm PNC}$ unless the latter should prove to be an order of magnitude smaller than expected, that is, unless $A_\gamma^{\rm PNC}$ is found to be of the order of 10^{-9}.

Likewise, the theoretical estimate of $A_\gamma^{\rm PC}$ appears to be sufficiently model independent that measurement of this quantity can serve as a valid polarization monitor in experiments involving polarized cold neutrons, if an experimental sensitivity of a few times 10^{-9} can be obtained.

Finally, the dependence of $A_\gamma^{\rm PC}$ upon the neutron energy was found to be linear, within the region where $B(\theta) = A_\gamma^{\rm PC} \sin\theta$ holds, for several orders of magnitude above threshold. Thus, one can easily explore the photon asymmetry at energies convenient to a given experiment.

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