Reply to "Comment on 'Quantum-mechanical equivalent-photon spectrum for heavy ion physics'"

C. J. Benesh
Nuclear Theory Center, Indiana University, Bloomington, Indiana 47405

A. C. Hayes
AECL, Chalk River Laboratories, Chalk River, Ontario, Canada K0J 1J0

J. L. Friar
Los Alamos National Laboratory, Los Alamos, New Mexico 87545

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Shortcomings of a momentum-space treatment of strong absorption, as discussed in the preceding Comment, are only of concern at low projectile energies, $\gamma<1.5$. At intermediate and high energies, for which the quantum-mechanical equivalent-photon spectrum is intended, the quantum-mechanical cross sections are reduced relative to the semiclassical results whether one treats strong effects via a momentum-space or an impact-parameter (spatial) cutoff. At these energies the origin of the discrepancy between the predictions of fully quantum-mechanical and the semiclassical calculations cannot be traced to differences in the treatment of strong-interaction effects. Rather, they arise from quantum effects neglected in the semiclassical calculations. [S0556-2813(97)05607-0]

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In the preceding Comment, the authors contend that the discrepancies between the quantum-mechanical calculations [1,2] and more traditional approaches [3] to Coulomb excitation in peripheral heavy-ion collisions are due to an incomplete treatment of strong absorption in [1]. They claim that if strong absorption is treated in terms of a cutoff in the impact parameter, as opposed to the cutoff in momentum space used in Ref. [1], then the quantum-mechanical and semiclassical calculations would yield the same result. For low projectile energies, $\gamma<1.5$, where the cross sections are sensitive to the details of the treatment of diffractive effects, this criticism is well grounded. However, at the intermediate energies of interest ($1.5<\gamma<3$) one is already outside the diffractive limit and strong absorption can be treated adequately in momentum space. In this energy range deviations between semiclassical and quantum calculations arise mostly from quantum effects that are normally neglected in calculations of Coulomb excitation.

To disentangle discrepancies arising from differences in the treatment of strong absorption from discrepancies arising from quantum-mechanical versus semiclassical treatments of the problem we make a comparison between these two approaches and that of an earlier quantum calculation of Jäckle and Pilkuhn [4]. The advantage in doing this is that Jäckle and Pilkuhn calculated the Coulomb excitation cross section in the eikonal approximation, treating strong absorption in exactly the fashion described in the preceding Comment.

The calculations of Jäckle and Pilkuhn find cross sections that are smaller at all energies than the usual semiclassical results. For all but the lowest projectile energies ($\gamma<1.5$), the cross sections are reduced by comparable amounts as we find. In particular, they find a reduced photon flux for the mildly relativistic collisions ($1.5<\gamma<3.0$) of interest, and as shown in Fig 2.2 of [3] this result also holds under the assumption of pointlike projectiles.

In Fig. 1 we show the equivalent photon number per charge for the two quantum predictions (those of Jäckle and Pilkuhn and of [1]), assuming a pointlike projectile, and that for the semiclassical predictions [3] for a 20 MeV dipole transition. It is clear that at low projectile energies the pre-

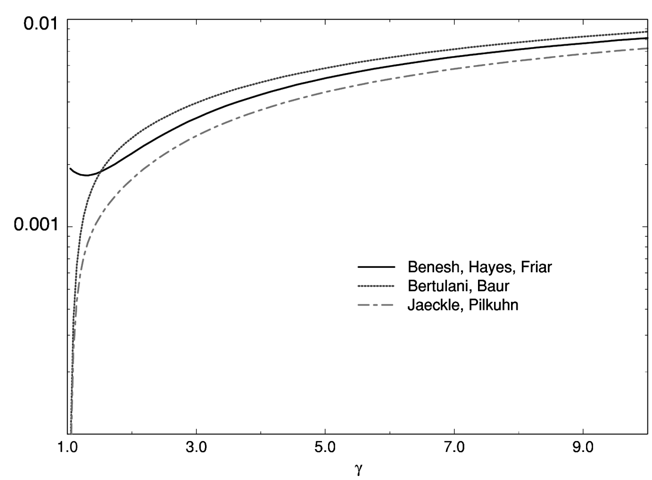

FIG. 1. Predictions from the two quantum-mechanical and one semiclassical calculations for the equivalent photon number corresponding to a 20 MeV dipole transition. The calculation of Jäckle and Pilkuhn follows the prescription deemed necessary by Baur and Bertulani in their Comment. Nonetheless, this quantum calculation also finds the equivalent photon number to be reduced relative to the semiclassical prediction. Thus, at all but low projectile energies ($\gamma<1.5$), differences in the treatment of strong absorption effects cannot explain the discrepancies between the predictions of Refs. [1] and [3]. These differences arise from quantum effects omitted from semiclassical treatments of Coulomb excitation in heavy-ion collisions.

dictions are quite sensitive to the treatment of diffractive effects and, as pointed out in the preceding Comment, the calculation using a momentum-space cutoff deviates considerably from both the semiclassical and the quantum calculations using a cutoff in the impact parameter. However, at intermediate energies ($\gamma > 1.5$) the predictions of the two quantum calculations, using a cutoff in coordinate [4] versus a cutoff in momentum [1] space, are consistently lower than the semiclassical calculation. This discrepancy between the quantum and semiclassical expressions for the equivalent photon spectrum cannot be explained by strong effects. Rather, as noted in [3], the effect arises from quantum and kinematic effects not included in the semiclassical calculations.

On a second issue, Baur and Bertulani suggest in their Comment that the inclusion of the finite size of the projectile does not affect the predicted cross sections. On this point we entirely disagree. The fact that the projectile has a form factor and that there is a kinematic restriction on the magnitude of the three-momentum transfer in the projectile's rest frame, $\sqrt{-q^2} \geqslant \omega_T / \gamma\beta$, necessarily implies small deviations from the predictions for a pointlike projectile.

Finally, Baur and Bertulani's observation that multiphoton effects are a natural consequence of QED is, of course, unassailable. However, they misrepresent our point here. Our claim is that such effects are not required to suppress the single-neutron removal cross sections to the levels observed experimentally. We stand by that conclusion.

In summary, the degree of validity of a momentum-space treatment of strong absorption in heavy-ion collisions depends on the projectile-energy range in question. The concerns expressed in the preceding Comment are justified at low projectile energies ($\gamma < 1.5$), but not at the energies of interest for the experimental data under discussion. At these intermediate energies both momentum-space and coordinate-space treatments of the problem predict that quantum effects omitted from semiclassical treatments lower the predicted cross sections. There appears to be experimental evidence in support of this in single-neutron-removal cross sections (see Tables III and Tables IV of Ref [1]).

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[4] R. Jäckle and H. Pilkuhn, Nucl. Phys. **A247**, 521 (1975).