Bright interior of nuclei viewed by α particles: High order eikonal expansion

S. M. Eliseev[1] and K. M. Hanna[1,2]
[1]*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia*
[2]*Mathematics and Theoretical Physics Department, Atomic Energy Authority, P.O. Code 13759, Cairo, Egypt*
(Received 12 March 1997)

The possibility of observing a bright interior in the nucleus ''viewed'' by intermediate energy alpha particles (E_α=172.5 MeV), as a probe for the ^{58}Ni nucleus, previously predicted by applying only a first-order non-eikonal correction and one simple optical model potential was confirmed. The first-, second-, and third-order noneikonal corrections to the Glauber model were incorporated in three arbitrary different nominal optical potentials and the effects of switching (off/on) the Coulomb potential were studied. The role of corrections in the effective potentials and also the results of calculations of the imaginary part of the phase shift functions for the different optical potentials used were discussed. [S0556-2813(97)01107-2]

PACS number(s): 25.70.−z, 24.10.Ht

Although quantum chromodynamics (QCD) is considered to be the most probable candidate for building a unified theory of strong interactions, the baryons and their properties are still very useful and remarkably effective in describing the gross properties of nuclei and the mechanisms of nuclear interactions. In addition, recent investigations with different probes have been focused on the study of the fine peculiarities of the nuclear structure and also reaction mechanisms. Indeed, a few papers published a short time ago in this journal were devoted to the investigation of the effect of non-eikonal (NE) corrections to the Glauber approach in nuclear reactions at intermediate energies (see, e.g., [1–7]). This reflects the increasing attention to these NE corrections with the perspective of obtaining some new information on the reaction mechanisms. Moreover, this is consistent with the contemporary trend in nuclear theory to investigate these peculiarities of intermediate energy nuclear reactions, and important signals may come from this energy region.

Further, the understanding of nuclear density distributions within nuclei, using different electromagnetic and hadronic probes, is considered one of the most interesting and fundamental problems in contemporary nuclear physics. The information we can get about the nuclear charge and matter distributions and their radial sensitivity strongly depends on the inherent properties of the used probe. One of the earliest strongly interacting probes was the low energy alpha particle which had been used by Rutherford in his famous experiment in 1911. Some features, besides its strong absorption in the nuclear surface, favor the alpha particle as the only composite-particle probe for investigations of nuclear density distributions. These are its high binding energy (causing it to behave like an inert particle in the considered energy region) and a small size together with its zero spin and isospin.

For medium energy alpha particles, strong absorption in the nuclear surface plays a dominant role, thus leading to a characteristic diffraction pattern for the angular cross section. The large-angle behavior is sensitive to the details of the real optical potential over a wide radial region from the nuclear surface towards the interior (see the review article by Batty *et al.* [8] and references therein).

It seems that this sensitivity has no simple explanation. Recently, Ingemarsson and Fäldt [6] have studied this problem for the scattering of medium energy alpha particles using an optical potential [9] which reproduces the elastic scattering with the ^{58}Ni nucleus. The basic idea was to find an effective potential, corrected by a first-order NE Wallace [10] t-matrix expansion, which is equivalent to the *zeroth-order* potential used in Glauber eikonal model calculations [11]. One of the exciting and interesting results of these authors was the observation of a bright interior ''viewed'' by alpha particles in their interaction with the ^{58}Ni nucleus at 172.5 MeV [6]. However, two questions of principal importance arise from the above result: Is it a consequence of selecting a special type of the optical potential? Moreover, how much will it be affected by the application of more higher-order (higher than the first one) NE corrections? The present work provides a further test of the above interesting result obtained in [6]. We developed higher-order NE corrections in the effective optical potentials in order to investigate the role that they can play in the details of both the real and imaginary parts of the optical potential for a wide radial range from the nuclear surface towards the nuclear interior.

Using intermediate energy alpha particles (E_α=172.5 MeV) as a probe for the ^{58}Ni nucleus, eikonal and first-, second-, and third-order NE corrections were developed to investigate their consequences on the details of both the real and imaginary parts of an effective optical potential and they were applied to a wide radial range from the nuclear surface towards the nuclear interior. Three arbitrary different nominal optical potentials were used and the effect of switching (off/on) the Coulomb potential was studied. Substantial reductions in the absorptive parts of the effective potentials were preserved. The results were confirmed by calculating the imaginary part of the phase shift function $\chi(b)$ in every case.

We chose three different nominal optical potentials, all of them reproducing the elastic scattering data of 172.5 MeV alpha particles from the ^{58}Ni nucleus [9]. These potentials are $(\mathrm{WS})^1(\mathrm{WS})^1$, $(\mathrm{WS})^2(\mathrm{WS})^1$, and $(\mathrm{WS})^2(\mathrm{WS})^2$ where $(\mathrm{WS})^{N_R}$ and $(\mathrm{WS})^{N_I}$ are usual Fermi functions for the real and imaginary parts of the Woods-Saxon potential, respectively, and the powers N_R and N_I take the value 1 or 2. The

 © 1997 The American Physical Society

TABLE I. Optical potential parameters for the $\alpha+{}^{58}$Ni scattering [9].

			$E_\alpha = 172.5$ MeV			
Potential type	W_R (MeV)	r_R (fm)	a_R (fm)	W_I (MeV)	r_I (fm)	a_I (fm)
$(\mathrm{WS})^1$ $(\mathrm{WS})^1$	111.47	1.248	0.792	22.73	1.564	0.580
$(\mathrm{WS})^2$ $(\mathrm{WS})^1$	140.34	1.379	1.266	25.14	1.458	0.766
$(\mathrm{WS})^2$ $(\mathrm{WS})^2$	149.76	1.340	1.336	25.55	1.671	1.123

effect of switching (off/on) the Coulomb potential on the results was studied.

According to Wallace [10], the expansion of the phase shift function $\chi(b)$, as a power series in the strength of any potential possessing spherical symmetry (or at least approaching zero at large distances as rapidly as the Coulomb potential), in the impact parameter representation is given in the following compact form:

$$\chi_j(b) = \sum_{n=0}^{j} \chi^{(n)}(b),$$

$$\chi^{(n)}(b) = -\frac{\mu^{n+1}}{k(n+1)!}\left[\frac{b}{k^2}\frac{\partial}{\partial b} - \frac{\partial}{\partial k}\frac{1}{k}\right]^n \int_{-\infty}^{\infty} V^{n+1}(r)\,dz, \tag{1}$$

where b is the impact parameter, μ is the reduced mass, and k is the momentum in the c.m. system ($\hbar = c = 1$). The zeroth-order term in this expansion is the Glauber eikonal (or a straight-line) phase shift function while the corrections given by higher-order terms correspond to NE effects. The phase shift function $\chi_j(b)$ is given by

$$\chi_j(b) = -\frac{\mu}{k}\int_{-\infty}^{\infty} U_j(r)\,dz, \tag{2}$$

with

$$U_j(r) = \sum_{n=0}^{j} U^{(n)}(r),$$

$$U^{(n)}(r) = \frac{\mu^n}{(n+1)!}\left[\frac{1}{k^2}\left(1 + r\frac{\partial}{\partial r}\right) - \frac{\partial}{\partial k}\frac{1}{k}\right]^n V^{n+1}(r), \tag{3}$$

where $U_j(r)$ is a new "effective" potential of the jth order. As explained by Wallace [10], the calculations including NE corrections up to the jth order are equivalent to those obtained by using simple eikonal calculations with $U_j(r)$ as given by Eq. (3). Further, because of the tedious expansion of Eq. (3) and rather complicated mathematical expressions due to the appearance of higher-order derivatives, it suffices to show in a symbolical form both the third-order terms of the phase shift function $\chi(b)$ and the effective potential $U(r)$ as follows:

$$\chi^{(3)}(b) = -\left(\frac{\mu^4}{2k^7}\right)\left[\frac{5}{4} + \frac{11}{4} b\frac{\partial}{\partial b} + b^2\frac{\partial^2}{\partial b^2} + \frac{1}{12} b^3 \frac{\partial^3}{\partial b^3}\right]\int_{-\infty}^{\infty} V^4(r)\,dz, \tag{4}$$

$$U^{(3)}(r) = \left(\frac{\mu^3}{2k^6}\right)\left[4 + \frac{19}{4} r\frac{d}{dr} + \frac{5}{4} r^2 \frac{d^2}{dr^2} + \frac{1}{12} r^3 \frac{d^3}{dr^3}\right] V^4(r). \tag{5}$$

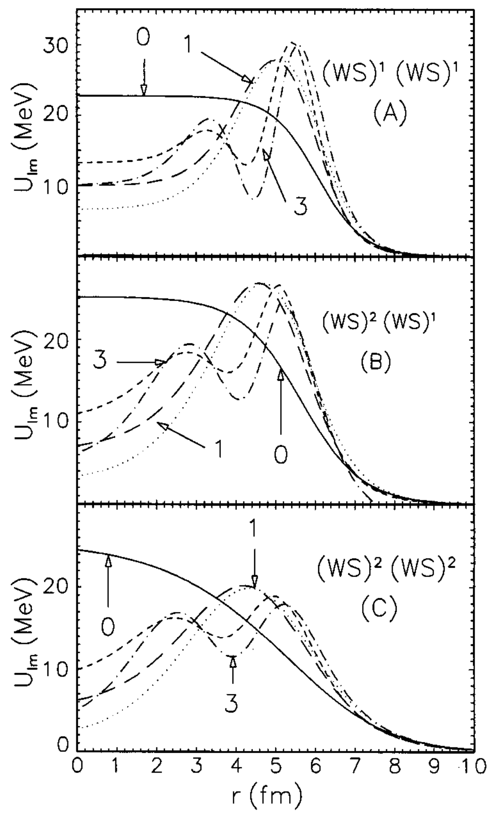

FIG. 1. The radial dependence of the imaginary parts of jth-order potentials $U_j(r)$. (The orders of potentials are shown by numbers.) Curves without (with) dots represent the calculations with (without) a Coulomb potential.

The first optical potential we have applied was a standard six-parameter Woods-Saxon (WS) optical potential [9] with first-order form factors for both the real and imaginary parts $(\mathrm{WS})^1(\mathrm{WS})^1$ which reads as follows:

$$V(r) = -V_R(r) - iV_I^{\mathrm{opt}}(r), \quad V_R(r) = V_R^{\mathrm{opt}}(r) - V_C, \tag{6}$$

where

$$V_{R,I}^{\mathrm{opt}}(r) = W_{R,I}\left(1 + \exp\frac{r - D_{R,I}}{a_{R,I}}\right)^{-1}, \quad D_{R,I} = r_{R,I}A^{1/3}. \tag{7}$$

However, since it is known from some microscopic calculations [12] that the radial shape of the real part of the alpha-nucleus optical potential is different from the WS shape and can be well parametrized by a squared-WS function, we used a squared-WS optical potential $(\mathrm{WS})^2(\mathrm{WS})^1$. Moreover, a more complicated optical potential was applied

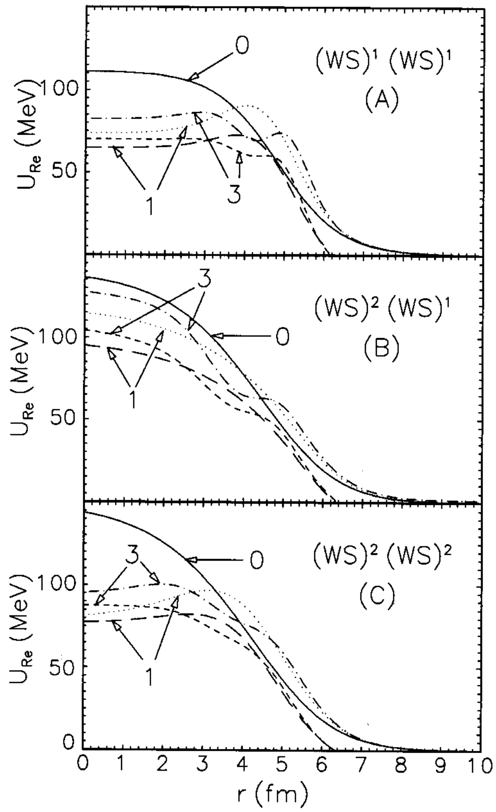

FIG. 2. Same as in Fig. 1 for the real parts of jth-order potentials $U_j(r)$.

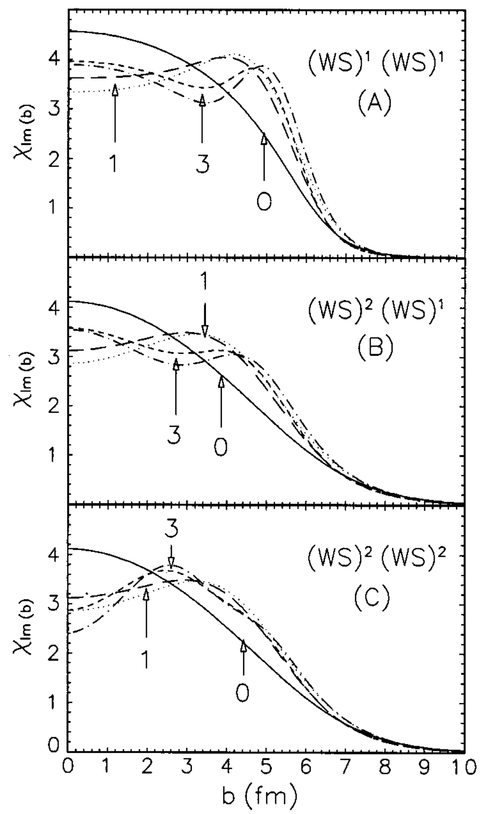

FIG. 3. Impact parameter dependence of the imaginary parts of jth-order phase shift functions $\chi_j(b)$. The notation as in Fig. 1.

with both squared real and imaginary form factors $(\mathrm{WS})^2(\mathrm{WS})^2$. The parameters of these potentials are given in Table I.

The Coulomb potential was taken to be a point α particle interacting with a uniformly charged spherical core of the radius R_C with $R_C = 1.34 A_T^{1/3}$ [12]:

$$V_C(r) = \frac{Z_P Z_T e^2}{2R_C}\left[3 - \left(\frac{r}{R_C}\right)^2\right], \quad r \leq R_C, \tag{8}$$

$$V_C(r) = \frac{Z_P Z_T e^2}{r}, \quad r > R_C. \tag{9}$$

To illustrate the effect of higher-order NE corrections, we have calculated both the real and imaginary parts up to the third-order effective potential $U(r)$ as given by Eq. (3), for our chosen set of arbitrary optical potentials. Figure 1 shows the imaginary part of these potentials in the eikonal $(j=0)$ and first- $(j=1)$ and third-order $(j=3)$ NE corrections. (For simplicity, the results of the second-order NE correction are not shown.) In addition, the effect of switching (on/off) the Coulomb interaction on the results was studied. It is to be noted that our results in Fig. 1(A) are consistent with those of Ref. [6]. We note that there is a dramatic change in the imaginary part in all orders of NE corrections for the whole set of optical potentials used. In the traditional optical model it is assumed that the imaginary part of the potential is responsible for the absorption process in a nuclear reaction, and its shape should not be affected by the real part.

As has been explained in Ref. [6], the decrease in the imaginary part of the first-order effective potential at small values of r is attributed to the term containing the product of the real and imaginary parts of the Woods-Saxon potential. (For the more complicated potentials used in the present work this is also true; see the dotted curves in Fig. 1.) However, in our higher-order NE corrections this simple picture is no longer valid since the imaginary part is rather a complicated function of both the real and imaginary parts of the used optical potential but the decrease in its value is also preserved for small values of r.

We add that the absorption gradually increases until its maximum value in the nuclear surface and substantially decreases in the central region of the nucleus. It is noticed that the maximum value of absorption for the third-order NE correction was greater and also displaced towards the nuclear surface as compared to the first-order one for the optical potential $(\mathrm{WS}^1)(\mathrm{WS}^1)$ [see Fig. 1(A)], while it was equal to or less than the first-order correction in the case of the $(\mathrm{WS})^2(\mathrm{WS})^1$ and $(\mathrm{WS})^2(\mathrm{WS})^2$ potentials, respectively. Moreover, the Coulomb potential plays an important role in the results, especially in the central region of the nucleus where its inclusion into the calculations lessens the absorption in the central region of the nucleus, as was also shown earlier in Ref. [6].

Figure 2 shows the real parts of the eikonal $(j=0)$ and first- $(j)=1$ and third-order $(j=3)$ NE corrections of the effective potentials for all arbitrary optical potentials chosen. Again, drastic changes were noticed, especially in the central region of the nucleus. It is to be noted that the third-order NE correction increases the depth of the real part of the optical potential as compared to the first-order NE correction. This property is preserved for the whole set of optical potentials.

However, to test the predictions of Eq. (3), in Fig. 3 we illustrate the impact parameter dependence of the imaginary parts of the phase shift function $\chi_j(b)$ for the whole set of selected optical potentials for different corrections. For all the types of potentials and orders of corrections used, it is observed that for smaller values of the impact parameter the inclusion of the Coulomb potential increases the corresponding imaginary part of the phase shift function as compared to its value when the Coulomb potential was eliminated. This fact shows the important role that the Coulomb potential plays, not only peripherally, but also in the interior region of the nucleus.

It is to be noted that the imaginary part of the phase shift function $\chi_j(b)$ increases, for all orders of NE corrections, especially for the third-order correction where the Coulomb potential was eliminated, at large values of the impact parameter b; i.e., the absorption increases. At the same time, for small values of b the absorption decreases (see Fig. 3). This confirms the predictions and the treatment based on the effective optical potential given by Eq. (3).

As a consequence of this drastic decrease in the imaginary part of the effective potential in the central region of the nucleus, the mean free path of the alpha particle becomes larger in this region and it can probe these deep nuclear levels and ‘‘see’’ the exotic and fine peculiarities of the nuclear structure, e.g., the variations of the density in the center of the nucleus, as discussed by Batty *et al.* [8].

[1] A. Bianconi and M. Radici, Phys. Rev. C **53**, 563 (1996).
[2] R. Bijker and A. Frank, Phys. Rev. C **52**, 831 (1995).
[3] M. H. Cha and Y. J. Kim, Phys. Rev. C **51**, 212 (1995).
[4] M. S. Hussein and M. P. Pato, Phys. Rev. C **51**, 2681 (1995).
[5] F. Carstoiu and R. J. Lombard, Phys. Rev. C **48**, 830 (1993).
[6] A. Ingemarsson and G. Fäldt, Phys. Rev. C **48**, R507 (1993).
[7] G. Fäldt, A. Ingemarsson, and J. Mahalanabis, Phys. Rev. C **46**, 1974 (1992).
[8] C. J. Batty, E. Friedman, H. J. Gils, and H. Rebel, Adv. Nucl. Phys. **19**, 1 (1989).
[9] J. Albinski *et al.*, Nucl. Phys. **A445**, 477 (1985).
[10] S. J. Wallace, Ann. Phys. (N.Y.) **78**, 190 (1973).
[11] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. G. Dunham (Interscience, New York, 1959), Vol. 1, p. 315.
[12] A. Budzanowski *et al.*, Phys. Rev. C **17**, 951 (1978).