Ioffe current constant of the Roper resonance from a relativistic three quark model

George L. Strobel[1,*] and K. V. Shitikova[2,†]
[1]*Physics Department, University of Georgia, Athens, Georgia 30602*
[2]*Physics Department, SUNY, Stony Brook, Stony Brook, New York 11794*
(Received 10 January 1997; revised manuscript received 13 March 1997)

The Ioffe current constants for the proton and Roper resonance are evaluated using a linear confining potential model parametrized to reproduce the proton magnetic moment, spin, and energy. The three-body Dirac equation, for the $(1/2^+)^3$ positive parity configuration, is solved in hypercentral approximation. Quark masses of the order of 9 MeV are needed to reproduce the proton magnetic moment. When the proton rms charge radius and magnetic moment are reproduced, the Roper has an Ioffe coupling constant about 25% larger than the proton. [S0556-2813(97)00107-6]

PACS number(s): 14.20.Dh, 11.10.Ef, 11.40.−q, 12.39.Ki

In a previous paper [1] the Ioffe current constant from a relativistic three quark model was studied. The Ioffe current constants for the proton were evaluated using a Poincare invariant solution of the three-body Dirac equation for the $(1/2^+)^3$ positive parity configuration. The results were comparable to the QCD sum rule predictions [2] and also with the random instanton liquid model [3,4]. These results stimulated us to a further investigation of the Roper resonance state properties of the nucleon.

The proton and Roper resonance are described here as three quark systems obeying the three-body Dirac equation dynamics. This approach has no unwanted center of mass motion in the system rest frame.

The proton is modeled as the ground state of the three-body Dirac equation solved in hypercentral approximation. Harmonic oscillator solutions have been found assuming equal mass quarks using quadratic central diagonal potentials [5,6]. Recently a dipole model of the proton was developed [7] that uses a linear confining and an attractive Coulombic potential. Using small quark masses on the order of 10 MeV, this model has a Dirac magnetic moment of 2.763 nm, very close to the experimental value for the proton. If these quarks are assumed to possess anomalous Pauli magnetic moments as per QED of $(\alpha/2\pi)(e/m)$, then this model can reproduce the proton magnetic moment.

The Ioffe current constant has been previously determined for the proton using the harmonic oscillator type solution for the composite three quark wave function components [1]. One needs the probability amplitude that all three quarks are at the center of mass of the nucleon simultaneously.

The constant was maximized at 0.14 GeV^3 with a quark mass of about 80 MeV, and decreases to 0.124 GeV^3 for massless quarks in that model. Here the Ioffe constant is determined using linear confining potentials and also estimated for the Roper resonance state of the nucleon. This model is solved here for an excited state containing an additonal hyperradial node in the hyperradial dependence of the composite three quark wave function compared to the ground state wave function. This excited state is identified here as the Roper resonance. The orbital quantum numbers of the three quarks are the same as for the proton ground state. For the Roper solution, each component of the composite wave function has an additonal node in the hyperradial part of the wave function that is not present in the ground state components.

The dipole model has three parameters, the energy, the quark mass, and the size parameter L. The Roper resonance state has the same values for the quark mass and size parameter, only the energy is different from the proton ground state. The size parameter is determined by matching the magnetic moment and the rms charge radius of the rest frame wave function for the proton to experiment.

The three-body Dirac equation is solved in hypercentral approximation. The six space coordinates necessary to specify the location of the particles are taken as a hyperradius ρ and five hyperangles Ω. The hyperradius is defined as

$$\rho^2 = r_1^2 + r_2^2 + r_3^2 = 2r^2/3. \tag{1}$$

The hypercentral approximation utilizes the hyperangular average of the $\Sigma_{i<j} V_{ij}(d_{ij})$ potential terms. The QCD equations for the quark potentials are nonlinear leading one to expect a complicated or nonexistant sum rule describing the potential from three interacting quarks in terms of potentials from a lesser number of quarks. Here the average over the color potential is taken as the model potential. This is a linear confining term proportional to the hyperradius, plus an attractive, so-called Coulombic term, proportional to the inverse of the hyperradius. The hyperangular reduction of these equations has been reported elsewhere [7–9].

The normalization of the wave function is now considered. Each configuration is here separately considered normalized to unity. After the hyperangular integration, the normalization for a configuration is

$$1 = (2/\pi^{3/2}) N^2 \Sigma_i \int \rho^5 d\rho (R_i)^2 / \Gamma[(\Lambda(i,i)+6)/2], \tag{2}$$

where Λ is twice K, and $\Gamma(n)$ is the gamma function of order n. The sum is over the eight components of the composite three quark wave function. K ranges from zero to

*Electronic address: gstrobel@hal.physast.uga.edu
†Pemanent address: Institute of Nuclear Physics, Moscow State University, Moscow, Russia.

three and is the single quark orbital angular momentum associated with each component.

For three identical particles, and with each particle with the same set of quantum numbers, one expects the components R_2, R_3, and R_5 to be equal, and also for the components R_4, R_6, and R_7 to equal each other. Then the wave function has only four unknown components, R_1, R_2, R_4, and R_8. For the $(1/2^+)^3$ configuration, including central diagonal interactions along the diagonal, results in the 4 by 4 Hamiltonian matrix that operates on the four unknown components:

$$\begin{bmatrix} (3M-E+V_1) & -D(5) & 0 & 0 \\ D(0) & (M-E+V_2) & -D(6)/2 & 0 \\ 0 & 2D(-1) & (-M-E+V_4) & -D(7)/5 \\ 0 & 0 & 3D(-2) & (-3M-E+V_8) \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_4 \\ R_8 \end{bmatrix} = 0. \tag{3}$$

This matrix operates on the hyperradial components R_1, R_2, R_4, and R_8. Solutions of this equation are found here of the form:

$$R_1 = A\exp(-Lr)(1-aLr),$$

$$R_2 = R_3 = R_5 = Br\exp(-Lr)(1-bLr),$$

$$R_4 = R_6 = R_7 = Cr^2\exp(-Lr)(1-cLr),$$

$$R_8 = Dr^3\exp(-Lr)(1-dLr), \tag{4}$$

where

$$A = 1,$$

$$B = -(E-3M)/6,$$

$$C = (E-M)(E-3M)/24, \tag{5}$$

$$D = -(E+M)(E-M)(E-3M)/48,$$

$$a = 1/3, \quad b = 1/4, \quad c = 1/5, \quad d = 1/6.$$

For the ground state solution, a, b, c, and d were zero. The system energy E is taken here as the Roper resonance energy. The normalization coefficient, N^2 of Eq. (2) differs for the Roper wave function compared to the proton ground state wave function. This difference follows from both the differing energies of the two states, and from the differing hyperradial dependences of the composite three quark wave function components. The potentials in terms of $y = Lr$, and the proton and Roper energy Ep and E, respectively, are found to be

Normalized potential	Proton	Roper
V_1/L	$(Ep-3M)y/6L$	$(E-3M)y/8L$
V_2/L	$(E_p-M)y/8L-6L/y(E_p-3M)$	$(E-M)y/10L-8L/y(E-3M)$
V_4/L	$(E-M)/y/10L-8L/y(E_p-M)$	$(E+M)y/12L-10L/y(E-M)$
V_8/L	$(Ep+3M)/L-6L/y(Ep+M)$	$(E+3M)/L-7.2L/y(E+M)$

The potentials are quite similar in shape and magnitude. The potentials show some energy dependence as the Roper state potentials are systematically more positive than for the ground state proton case. QCD theory leads one to expect the quark-quark potentials would depend on the distribution of the quarks, and therefore to some state dependence of the potentials.

This solution has three parameters, the system energy E, the size parameter L, and the quark mass M. E is set to the Roper resonance energy. The Dirac magnetic moment for the proton is maximized for massless or small mass quarks, with a size parameter L of about 0.29 GeV. The maximum calculated Dirac magnetic moment is 2.763 for massless quarks, and 2.7472 for 9 MeV quarks. These values are close to, but less than, the experimental proton magnetic moment. The difference is attributed to an anomalous Pauli magnetic moment for each quark. The anomalous magnetic moment of a bound quark is calculated following Miller [10]. Including an anomalous magnetic moment as per QED of $(\alpha/2\pi)(e/m)$ from the bound quarks, the proton magnetic moment is reproduced for a 9 MeV quark mass, when the Dirac magnetic moment is maximized. The model quark mass required for the Dirac plus anomalous magnetic moments to reproduce the proton experimental value varies from about 4 to 9 MeV for any size parameter below 0.36 GeV. Such values are in agreement with the Particle Data Group [11] which determine quark masses by other means. The proton probability density has a single peak, while the Roper case has two peaks, with a central minimum. The central minimum is not zero for the Roper as the various components vanish at different hyperradial values. The Roper is predicted to be about

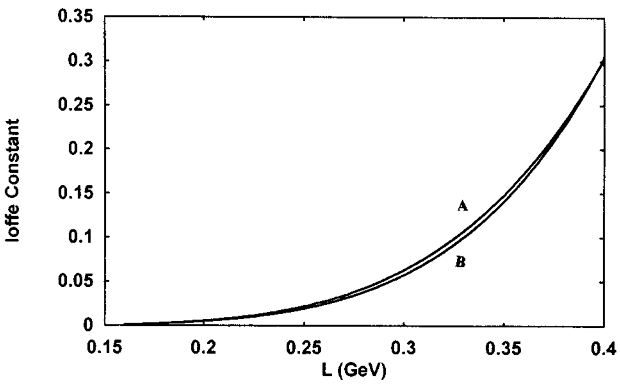

FIG. 1. The first Ioffe coupling constant vs the size parameter L of the dipole model. The curves cross at $L=0.396$ GeV. Proton curve, A; Roper curve, B.

25–35% larger than the proton, depending on the size parameter.

The details of evaluating the Ioffe current are the same as in [1]. We finally obtain, for the Ioffe current constant,

$$I_1 = 10AN/\pi^{9/4}\sqrt{3}. \tag{6}$$

This Ioffe coupling constant depends only on the first component as all other components vanish at the origin for the $(1/2^+)^3$ configuration. The coupling constant also depends linearly on the normalization constant of the composite three quark wave function.

The component constants by themselves suggest that the normalized first component would be smaller for the Roper than for the proton. Due to the interplay of the node in the wave function with the normalization this turns out to depend on the size parameter. For size parameters smaller than 0.396 GeV, it is correct, but the reverse holds for larger size parameters.

In Fig. 1 are shown the Roper and proton Ioffe coupling constants versus the size parameter of the dipole model. The quark mass is assumed to be 9 MeV. The coupling constants have a negligible variation for smaller quark masses, for a given size parameter. The Ioffe coupling constant increases at about the third power of the size parameter in the dipole model. For size parameters below 0.396 GeV, the Roper Ioffe constant is smaller than that of the proton.

For quark masses of about 9 MeV or less, the linear confining potential can reproduce both the proton magnetic moment and the rms proton charge radius. The rms charge radius restricts the size parameters to about 0.536 GeV. Thus for parameters which reproduce both the proton magnetic moment and the rms charge radius, the model predicts the Ioffe coupling constant for the proton of 1.234 GeV^3 and for the Roper, 1.524 GeV^3.

The Ioffe coupling constant for the Roper resonance is predicted by this model as about the same but slightly more than for the proton, using size parameters in this range for each state. For a size parameter of $L=0.396$ GeV, the dipole model predicts equal Ioffe constants for the Roper and the proton.

The linear confining potential model of the proton has been extended to include a breathing mode excitation characterized by a node in the hyperradial dependence of the composite three quark wave function. The Roper resonance is identified as this excited state. This model has three parameters, the energy, the quark mass, and a size parameter. The quark mass and the size parameter are taken the same for both states, with only the energy varying from the ground to the excited state. The ground state has been parametrized to reproduce the magnetic moment of the proton. Assuming an anomalous magnetic moment of the quarks as in QED of $(\alpha/2\pi)(e/m)$, this requires a quark mass of 9 MeV. This assumes equal mass quarks in the proton. The rms charge radius of the proton is reproduced with size parameters about 0.536 GeV. The Roper has a slightly smaller Ioffe coupling constant than the proton if the size parameter is below 0.396 GeV. A size parameter of 0.536 GeV and a quark mass about that of the electron will simultaneously reproduce the proton rms charge radius and magnetic moment. For these parameters the Ioffe coupling constant is 1.234 and 1.524 GeV^3 for the proton and Roper, respectively.

The relativistic potential quark model is able to be parametrized to successfully predict Ioffe current constants in agreement with QCD sum rule predictions. The linear confining potential model, with parameters that reproduce the proton magnetic moment, predicts Ioffe coupling constants comparable to but larger than random instanton liquid model estimates. Simultaneously reproducing the rms charge radius and magnetic moment of the proton results in Ioffe constants larger than predicted by the Gaussian model.

We are indebted to E. Shuryak for very useful discussions. One of us (K.V.S.) would like to thank the theory group of SUNY for the hospitality extended during a stay there. One of us (G.L.S.) wishes to thank Lawrence Livermore Laboratory for a pleasant extended stay. Part of this work was done under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract No W-7405-ENG-48.

[1] G. L. Strobel and K. V. Shitikova, Phys. Rev. C **54**, 888 (1996).
[2] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981).
[3] E. V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993).
[4] T. Schafer, E. V. Shuryak, and J. J. M. Verbaarschot, Nucl. Phys. **B412**, 143 (1994).
[5] G. L. Strobel and C. A. Hughes, Few-Body Syst. **2**, 155 (1987).
[6] G. L. Strobel and T. Pfenninger, Phys. Lett. B **195**, 7 (1987).
[7] G. L. Strobel, Int. J. Theor. Phys. **35**, 2475 (1996).
[8] G. L. Strobel, Hadronic J. **9**, 181 (1986).
[9] G. L. Strobel, Few-Body Syst. **21**, 1 (1996).
[10] L. D. Miller, Ann. Phys. (N.Y.) **91**, 40 (1975).
[11] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).