# Friction coefficient for deep-inelastic heavy-ion collisions

G. G. Adamian,<sup>1,2</sup> R. V. Jolos,<sup>1</sup> A. K. Nasirov,<sup>1,2</sup> and A. I. Muminov<sup>2</sup>

<sup>1</sup>Joint Institute for Nuclear Research, Dubna, 141980 Russia

<sup>2</sup>Heavy Ion Physics Department, Institute of Nuclear Physics, 702132 Ulugbek, Tashkent, Uzbekistan

(Received 20 September 1996)

Based on the microscopic model, the friction coefficient for the relative motion of nuclei in deep-inelastic heavy-ion collisions is calculated. An advantage of the suggested method is that it allows one to consider the relative motion of nuclei and the intrinsic motion self-consistently. The radial dependence of the friction coefficient is studied and the results are compared with those found by other methods. It was demonstrated that the kinetic energy dissipation in deep-inelastic heavy-ion collisions is a gradual process which takes up a significant part of a reaction time. A decrease of the radial friction coefficient with a heating of nuclei is shown. [S0556-2813(97)02706-4]

PACS number(s): 25.70.Lm, 21.65.+f, 24.10.-i, 25.70.Jj

# I. INTRODUCTION

Nuclear friction is an important ingredient of theoretical approaches to a variety of nuclear physics phenomena, such as dynamic thresholds for compound nucleus formation [1-5], enhancement of neutron emission prior to fission [6,7], width of mass and charge distributions in deep-inelastic heavy-ion reactions [8], and the width of giant resonances [9]. There are many experimental results on deep-inelastic heavy-ion collisions (DIC's) and fusion-fission reactions which need the introduction of the nuclear friction concept for their interpretation. This stresses the importance of understanding the nature of nuclear friction.

The present paper is devoted to calculations of the friction coefficient for DIC's. Its appearance is stimulated not only by the possibility to perform more exact calculations than earlier, but also by the new experimental results which require a more detailed microscopic theory for their interpretation than was necessary before.

Different theoretical approaches to this problem are known. The majority of them are based on the assumption that the dissipative mechanism is of a one-body nature [10-12]. These models differ in the structure of the intrinsic excitations that are taken into account. The friction is arisen from scattering of the nucleon in one nucleus with the moving one-body potential of the other nucleus. The application of the independent-particle model to calculate entrance channel dynamics was discussed in linear response theory [13,14]. In frame of validity of the presented method the calculated total kinetic energy loss was poor. The main collective modes which were responsible for dissipation of relative energy and the problems of relation between cutoff time and relaxation times were explored as well. The same onebody mechanism of dissipation was used in [15] for the microscopic calculations of the friction coefficient for radial and deforming motions adopting the linear response theory discussed above. The main difference was in the use of the two-center shell model. It was found that this model gave a physically plausible value for the friction coefficient as a whole. The dependence of the friction coefficients on the cutoff time, on the deformation and mass asymmetry of nuclear system were presented. A free parameter  $\Gamma$ , as the smearing width, was introduced to take into account the effect of finite collision time and finite decay time of particlehole excitations. The last was taken into account by those authors adopting the cutoff procedure obtained from [16] to reevaluate the friction coefficients for deep-inelastic collisions microscopically in the framework of the linear response theory [17]. The results were sensitive to temperature of nuclei.

Some of the models do not include nucleon exchange and consider only particle-hole excitations with both the particle and the hole belonging to the same nucleus [18] or vice versa [10]. The models also differ in the approximations they use for including the finite decay time of one-particle–one-hole (1p-1h) excitations in more complicated configurations (2p-2h, and so on). Many approaches [12,14,19,20] implicitly use the statistical assumption of rapid equilibration of the noncollective intrinsic degrees of freedom and therefore are not applicable to the description of the initial phase of the reactions where the main part of kinetic energy dissipation takes place.

The contribution of the actual 1p-1h state or more complicated ones to the dissipation process depends on the occupation numbers of the single-particle states and their evolution during the reaction. However, in the calculations of the nuclear friction coefficient performed up to now, the statistical assumption on the excitation energy distribution is realized in the usual way, meaning the introduction of temperature and correspondingly, of the Fermi occupation numbers at the very beginning of the reaction. In principle, temperature introduced in this way is a time-dependent quantity. However, in practice, a change of temperature and evolution of the relevant collective variables, i.e., relative distance and deformation in [17] and elongation in [21], are not selfconsistently considered. A dependence on temperature is studied when the collective variables under discussion are fixed or vice versa. It is clear that the self-consistent calculations are impossible in the frame of these models because they do not consider an intrinsic excitation explicitly. So the temperature must be considered as a measure of response of a system to an effect of external forces.

Thus, the time dependence of the single-particle occupation numbers was not taken into account. Only in the ap-

373

© 1997 The American Physical Society

proach based on the dissipative diabatic dynamics (DDD) [22,23] was the evolution of the single-particle occupation numbers taken into account, but under the assumption of diabaticity. There are some doubts, however, about the validity of the DDD concept. It is also known from the calculations of inelastic processes in nucleus-nucleus collisions that appreciable energy dissipation takes place even before the first crossing of the single-particle levels near the Fermi surface [24].

We should also mention the approach suggested in [25], where relative and intrinsic motion were consistently treated in a time-dependent theory of heavy-ion collisions. The authors presumed neither weak coupling between the relative motion and the intrinsic excitations nor the canonical distribution function for the density operator of intrinsic motion. However, the analytical expressions for the friction tensor and other characteristics of the energy transport obtained in [25] were not applied to calculate them.

Thus, it is the aim of the present paper to take into account the time evolution of the single-particle occupation numbers during the reaction by numerical solution of the master equation for them and, based on this result, to perform calculations for the friction coefficient. Since the occupation numbers found in this way correspond to the current kinetic energy losses, this means that the relative and intrinsic motions are considered self-consistently. Our model makes it possible to take into account explicitly the influence of the nuclear shell structure on the collision process. Moreover, we improved the single-particle approximation by a phenomenological allowance for the residual interaction which is treated in the so-called  $\tau$  approximation. The radial friction coefficient is calculated as a function of the mass and charge of the reaction participants.

The general formalism is given in Sec. II. The results of the calculations are presented in Sec. III. A summary is given in Sec. IV.

### **II. BASIC FORMALISM**

It is convenient to start with the total Hamiltonian of a dinuclear system written in the form

$$\hat{H} = \hat{H}_{\text{rel}}(\mathbf{R}; \mathbf{P}) + \hat{H}_{\text{in}}(\xi) + \delta \hat{V}(\mathbf{R}, \xi), \qquad (1)$$

where the Hamiltonian of a relative motion,

$$\hat{H}_{\rm rel}(\mathbf{R};\mathbf{P}) = \frac{\hat{\mathbf{P}}^2}{2\,\mu} + \hat{\mathcal{V}}(\hat{\mathbf{R}}),\tag{2}$$

consists of the kinetic energy operator and the nucleusnucleus interaction potential  $\hat{\mathcal{V}}(\hat{\mathbf{R}})$ . Here,  $\hat{\mathbf{R}}$  is the relative distance between the centers of mass of the fragments,  $\hat{\mathbf{P}}$  is the conjugate momentum, and  $\mu$  is the reduced mass of the system;  $\xi$  is a set of relevant intrinsic variables. The last two terms in Eq. (1) describe the internal motion of nuclei and the coupling between the relative and internal motions (for details, see [26,27]). It is clear that the coupling term leads to a dissipation of the kinetic energy into the energy of internal nucleon motion. Neglecting at the moment the residual nucleon-nucleon interaction, whose effect will be included later, we take a sum of the last two terms (1) as a single-particle Hamiltonian of a dinuclear system

$$\hat{\mathcal{H}}_{in}(\xi) + \delta \hat{V}(\mathbf{R}, \xi) = \hat{\mathcal{H}}(\mathbf{R}(t), \xi) + h_{\text{residual}},$$
$$\hat{\mathcal{H}}[\mathbf{R}(t)] = \sum_{i=1}^{A} \left( \frac{-\hbar^2}{2m} \Delta_i + \hat{V}_P[\mathbf{r}_i - \mathbf{R}(t)] + \hat{V}_T(\mathbf{r}_i) \right), \quad (3)$$

where *m* is the nucleon mass and  $A = A_P + A_T$  is the total number of nucleons in the system.

Then, in the second quantization representation, the Hamiltonian  $\hat{\mathcal{H}}(\mathbf{R}(t), \xi)$  can be written as

$$\hat{\mathcal{H}}[\mathbf{R}(t),\xi] = \sum_{P} \varepsilon_{P} a_{P}^{+} a_{P} + \sum_{T} \varepsilon_{T} a_{T}^{+} a_{T}$$
$$+ \sum_{i,i'} V_{ii'}(\mathbf{R}(t)) a_{i}^{+} a_{i'}, \qquad (4)$$

where

$$\sum_{i,i'} V_{ii'}[\mathbf{R}(t)] a_i^+ a_{i'} = \sum_{P,P'} \Lambda_{PP'}^{(T)}[\mathbf{R}(t)] a_P^+ a_{P'} + \sum_{T,T'} \Lambda_{TT'}^{(P)}[\mathbf{R}(t)] a_T^+ a_{T'} + \sum_{T,P} g_{PT}[\mathbf{R}(t)] (a_P^+ a_T + \text{H.c.}).$$
(5)

Here  $P \equiv (n_P, j_P, l_P, m_P)$  and  $T \equiv (n_T, j_T, l_T, m_T)$  are the sets of quantum numbers characterizing the single-particle state in an isolated projectile and the target nuclei, respectively. The single-particle basis is constructed from the asymptotic wave vectors of the single-particle states of the noninteracting nuclei—the projectile ion  $|P\rangle$  and the target nucleus  $|T\rangle$  in the form

$$|\widetilde{P}\rangle = |P\rangle - \frac{1}{2} \sum_{T} |T\rangle \langle T|P\rangle,$$
 (6a)

$$|\tilde{T}\rangle = |T\rangle - \frac{1}{2} \sum_{P} |P\rangle \langle P|T\rangle.$$
 (6b)

For this basis set, the orthogonality condition is satisfied up to terms linear in  $\langle P|T \rangle$ . Then

$$\Lambda_{PP'}^{(T)}[\mathbf{R}(t)] = \langle P | V_T(\mathbf{r}) | P' \rangle, \qquad (7a)$$

$$\Lambda_{TT'}^{(P)}[\mathbf{R}(t)] = \langle T | V_P[\mathbf{r} - \mathbf{R}(t)] | T' \rangle, \qquad (7b)$$

$$g_{PT}[\mathbf{R}(t)] = \frac{1}{2} \langle P | V_P[\mathbf{r} - \mathbf{R}(t)] + V_T(\mathbf{r}) | T \rangle.$$
(7c)

The nondiagonal matrix elements  $\Lambda_{PP'}^{(T)}$  ( $\Lambda_{TT'}^{(P)}$ ) generate the particle-hole transitions in the projectile (target) nucleus. The matrix elements  $g_{PT}$  are responsible for the nucleon exchange between reaction partners. These matrix elements

were calculated using the approach proposed in [28,29]. In Eq. (4),  $\varepsilon_{P(T)}$  are the single-particle energies of nonperturbed states in the projectile (target) nucleus. The coupling between the intrinsic nuclear degrees of freedom and the collective variable **R** is introduced by the **R** dependence of the sum of the single-particle potentials in Eq. (3). Since the trajectory calculation shows that the relative distance **R**(*t*) between the centers of the interacting nuclei could not be less than the sum of their radii, the tail of the partner single-particle potentials can be considered as a perturbation disturbing the asymptotic single-particle wave functions and their energies.

It is convenient to include the diagonal matrix elements of  $V_{ii'}[\mathbf{R}(t)]$  in  $H_{in}$ , introducing the renormalized  $\mathbf{R}(t)$ -dependent single-particle energies

$$\widetilde{\varepsilon}_{P}[\mathbf{R}(t)] = \varepsilon_{P} + \langle P | V_{T}(\mathbf{r}) | P \rangle, \qquad (8a)$$

$$\widetilde{\varepsilon}_{T}[\mathbf{R}(t)] = \varepsilon_{T} + \langle T | V_{P}[\mathbf{r} - \mathbf{R}(t)] | T \rangle.$$
(8b)

When the nuclear forces begin to act between the colliding nuclei, the velocity of their relative motion can be considered as a small quantity compared to the Fermi velocity. Then the speed of the nucleons is mainly associated with their intrinsic motion. Since the relative (collective) motion is rather slow compared to the intrinsic one, the perturbation of the intrinsic motion produced by changing the coupling to the relative motion  $(\mathbf{R})$  can be assumed to be small during some small time interval  $\Delta t$  of an arbitrarily chosen time t [14]. The small parameter in our consideration  $\Delta t$  thus characterizes the time interval during which the R-dependent mean field of the combined dinuclear system changes so little that we can neglect the effect of this changing on the intrinsic motion. At the same time, the characteristic time  $\Delta t$  cannot be taken smaller than the relaxation time of the mean field. The situation described above is suitable for applying the linear response theory to a description of dissipative heavy-ion collisions [14]. For this reason, we start from the expression for the friction coefficient of the radial motion obtained in that approach [14],

$$\gamma_{RR}[R(t)] = \sum_{ik} \left| \frac{\partial V_{ik}(\mathbf{R})}{\partial R} \right|^2 B_{ik}^{(1)}(t), \qquad (9)$$

$$B_{ik}^{(n)}(t) = \frac{2}{\hbar} \int_{t-\Delta t}^{t} dt' \frac{(t'-t)^n}{n!} \exp\left(\frac{t'-t}{\tau_{ik}}\right) \\ \times \sin\{\widetilde{\omega}_{ki}[\mathbf{R}(t')](t-t')\}[\widetilde{n}_k(t') - \widetilde{n}_i(t')],$$
(10)

where  $\tau_{ij} = \tau_i \tau_k / (\tau_i + \tau_k)$ ;  $\tau_i$  is the parameter describing the damping of the single-particle motion. The expression for  $\tau_i$  is derived in the theory of quantum liquids [30,31] (see the Appendix);  $\hbar \tilde{\omega}_{ij} = \tilde{\varepsilon}_i(\mathbf{R}) - \tilde{\varepsilon}_j(\mathbf{\bar{R}})$  is the energy of the single-particle transition in one of the nuclei as well as between the interacting nuclei. The important ingredients of this formula are the occupation numbers of the single-particle states  $\tilde{n}_i(t)$ . Since the excitation energy of the interacting nuclei changes significantly during the course of the collision, it is necessary to take into account the time dependence of the

occupation numbers. The importance of this point was already stressed in [14]. At the same time, new experimental data indicate that the assumption of the fast statistical equilibration of the excitation energy during the collision time, i.e., an introduction of a time-dependent temperature and Fermi occupation numbers is not adequate for the physical picture. As already mentioned in the Introduction, the calculations of  $\gamma_{RR}$  performed up to now have been done under the assumption that the occupation numbers can be taken as the Fermi occupation numbers

$$n_i = \{1 + \exp[(E_i - \lambda)/\Theta]\}^{-1}$$

where  $\Theta$  is the temperature corresponding to the total excitation energy of a dinuclear system. To find the timedependent occupation numbers  $n_j(t)$ , we developed in [26,27,32] a method which is described briefly below for completeness of the presentation.

Since explicit allowance for the residual interaction requires extensive calculations, it is customary to take the twoparticle collision integral into account in linearized form ( $\tau$ approximation). Then, the equation for the single-particle density matrix  $\tilde{n}$  takes the form [26,27,32]

$$i\hbar \frac{\partial \widetilde{n}(t)}{\partial t} = \{ \hat{\mathcal{H}}[\mathbf{R}(t)], \widetilde{n}(t) \} - \frac{i\hbar}{\tau} \{ \widetilde{n}(t) - \widetilde{n}^{\text{eq}}[\mathbf{R}(t)] \},$$
(11)

where  $\tilde{n}^{\text{eq}}[\mathbf{R}(t)]$  is a local quasiequilibrium distribution, i.e., a Fermi distribution with the temperature T(t) corresponding to the excitation energy at the internuclear distance  $\mathbf{R}(t)$ . Substituting our Hamiltonian (4) into Eq. (11), we have

$$i\hbar \frac{\partial \widetilde{n_i(t)}}{\partial t} = \sum_k \{ V_{ik} [\mathbf{R}(t)] \widetilde{n_{ki}}(t) - V_{ki} [\mathbf{R}(t)] \widetilde{n_{ik}}(t) \} - \frac{i\hbar}{\tau_i} [\widetilde{n_i}(t) - \widetilde{n_i}^{\text{eq}}(t)], \qquad (12)$$

where  $\tilde{n_i}$  is a diagonal matrix element of the density matrix. The approximate equation for nondiagonal matrix elements takes the form

$$i\hbar \frac{\partial \widetilde{n}_{ik}(t)}{\partial t} = \hbar \left[ \widetilde{\omega}_{ik}(\mathbf{R}(t)) - \frac{2i}{\tau_{ik}} \right] \widetilde{n}_{ik}(t) + V_{ki}[\mathbf{R}(t)][\widetilde{n}_{k}(t) - \widetilde{n}_{i}(t)], \quad (13)$$

where we have used the notations  $\widetilde{\omega}_{ik} = [\widetilde{\varepsilon}_i - \widetilde{\varepsilon}_k]/\hbar$ .

Assuming incoherence in the phases of the nondiagonal matrix elements, we use the following approximation to simplify Eq. (13):

$$\sum_{k'} V_{k'i}[\mathbf{R}(t)]n_{k'k}(t) - \sum_{i'} V_{ki'}[\mathbf{R}(t)]n_{ii'}(t)$$
$$\approx V_{ki}[\overline{\mathbf{R}}(t)][n_k(t) - n_i(t)].$$

As formulated above, we shall consider the solution of Eqs. (12) and (13) for a small time interval  $\Delta t$  of an arbitrarily chosen time *t*. Then the solution of Eq. (13) can be written as

$$\widetilde{n}_{ik}(\overline{t}) = \frac{1}{i\hbar} \int_{t}^{\overline{t}} dt' V_{ik}[\mathbf{R}(t')] \\ \times \exp\left\{i \int_{t'}^{\overline{t}} dt'' \left[\widetilde{\omega}_{ki}[\mathbf{R}(t')] + \frac{i}{\tau_{ik}}\right]\right\} \\ \times [\widetilde{n}_{k}(t') - \widetilde{n}_{i}(t')], \qquad (14)$$

where  $t \le \overline{t} \le t + \Delta t$ . Substituting this result into Eq. (12) and transforming this equation to an integral, we obtain

$$\widetilde{n}_{i}(\overline{t}) = \exp\left(\frac{t-\overline{t}}{\tau_{i}}\right) \times \left\{\widetilde{n}_{i}(t) + \frac{1}{\tau_{i}}\int_{t}^{\overline{t}}dt' \,\widetilde{n}_{i}^{\text{eq}}[\mathbf{R}(t')]\exp\left(\frac{t'-t}{\tau_{i}}\right) \qquad (15)$$

$$+\sum_{k}\int_{t}^{\overline{t}}dt'\int_{t}^{t'}dt''\Omega_{ik}(t',t'')\exp\left(\frac{t''-t}{\tau_{ik}}\right)[\widetilde{n_{k}}(t'') - \widetilde{n_{i}}(t'')]\bigg\},$$
(16)

where

$$\Omega_{ik}(t,t') = \frac{2}{\hbar^2} \operatorname{Re} \left\{ V_{ik}[\mathbf{R}(t)] V_{ki}[\mathbf{R}(t')] \times \exp \left[ i \int_{t'}^{t} dt'' \widetilde{\omega}_{ki}[\mathbf{R}(t'')] \right] \right\}.$$

The formal solution of Eq. (15) is

$$\widetilde{n_{i}}(\overline{t}) = \widetilde{n_{i}^{\text{eq}}}[\mathbf{R}(\overline{t})] \left[1 - \exp\left(\frac{-\Delta t}{\tau_{i}}\right)\right] + n_{i}(\overline{t}) \exp\left(\frac{-\Delta t}{\tau_{i}}\right),$$
(17)

where

$$n_{i}(\vec{t}) = \widetilde{n}_{i}(t) + \sum_{k} \int_{t}^{\vec{t}} dt' \Omega_{ik}(t', t') \\ \times \frac{\sin\{\widetilde{\omega}_{ki}[\mathbf{R}(t')](t'-t)\}}{\widetilde{\omega}_{ki}[\mathbf{R}(t')]} [\widetilde{n}_{k}(t') - \widetilde{n}_{i}(t')].$$
(18)

In fact, Eqs. (17) and (18) present an integral equation for  $\tilde{n}_i(t)$ .

#### **III. RESULTS AND DISCUSSION**

In this section, we present the results of the calculations of the radial friction coefficient and the kinetic energy losses as functions of the internucleus distance for trajectories corresponding to DIC. The initial projectile energy, atomic masses, and charges of the colliding nuclei are the initial information used in the calculations. The single-particle potentials of the colliding nuclei are taken in the Woods-Saxon form with the parameters  $r_0=1.18$  fm and a=0.54 fm. The characteristic time parameter  $\Delta t$  introduced above is taken to



FIG. 1. The radial friction coefficient calculated according to the present model for the approach phase of deep-inelastic <sup>64</sup>Zn(440 MeV) + <sup>196</sup>Pt collision as a function of *R* (solid curve). Presented for comparison are results of the classical model calculations by Gross and Kalinowski [33] (dashed curve with stars); the results of the microscopic model developed in [17] obtained with a fixed temperature of the nuclei  $\Theta$  (long-dashed curve,  $\Theta = 2.0$  MeV; dashed curve,  $\Theta = 1.0$  MeV; dotted curve –  $\Theta = 0.5$  MeV).

be equal to  $(0.8-1.0) \times 10^{-22}$  s.

The relative motion equations for the internuclear distance  $\mathbf{R}(t)$  and the conjugate momentum  $\mathbf{P}(t)$ ,

$$\dot{\mathbf{R}} = \boldsymbol{\nabla}_{P}(\boldsymbol{H}_{\text{rel}} + \langle t | \hat{\boldsymbol{V}}_{\text{int}} | t \rangle), \qquad (19)$$

$$\dot{\mathbf{P}} = -\nabla_R (H_{\text{rel}} + \langle t | \hat{V}_{\text{int}} | t \rangle), \qquad (20)$$

where  $\langle t | \cdots | t \rangle$  means averaging over the intrinsic state at the moment *t* and the single-particle occupation numbers  $\tilde{n}_i(t)$ , Eqs. (17) and (18) have been solved numerically. This transforms the differential equations into finite difference equations with the time step  $\Delta t$ , and the initial conditions R(0)=20 fm and  $\tilde{n}_i(0)=1$  or 0 for occupied and unoccupied states of the noninteracting nuclei, respectively. Matrix elements  $\Lambda_{PP'}^{(T)}$ ,  $\Lambda_{TT'}^{(P)}$ , and  $g_{PT}$  are calculated using the procedure developed in [26,27].

The description of the relative motion depends on the nucleus-nucleus interaction potential, which is determined by a double folding of the effective nuclear and Coulomb interactions of the nucleons with the nuclear densities of the interacting nuclei. Because of nucleon exchange and particlehole excitations, the nuclear densities of the colliding nuclei evolve during the reaction and the nucleus-nucleus potential correspondingly changes. This effect is included in our calculations.

As an example of the calculations, we present the radial friction coefficient for the  $^{64}$ Zn(440 MeV) +  $^{196}$ Pt collision in Fig. 1 as a function of *R* for the approach phase of the reaction (solid curve). The results of the classical model calculations of Gross and Kalinowski [33] (dashed curves with stars) are shown for comparison, together with the results of the calculations based on the microscopic model developed in [17], which are obtained with a constant temperature (dashed curves without stars). The last curves correspond to different temperatures, which increase from 0.5 MeV (bot-



FIG. 2. The radial friction coefficients  $\gamma_{RR}$  as a function of *R* for both the exit (solid curve) and approach (dashed curve) stages of the <sup>64</sup>Zn(440 MeV) + <sup>196</sup>Pt reaction.

tom curve) to 2 MeV (upper curve) in Fig. 1. It can be seen that the difference between our results and the results of [33] increases with the increase in the overlapping of the colliding nuclei. Comparing our results with the results of [17], we can see that our radial friction coefficient increasingly coincides as R decreases. This is caused by the fact that our matrix elements of the nucleon intrinsic transitions are larger than ones in [17] at small values of R. Notice, in particular, the qualitative difference in the calculation methods. In the wellknown classical case presented in [33], the shell structure of the interacting nuclei is not taken into account. In [17], the shell structure was included into consideration, although it was assumed that the intrinsic states do not change over time and the single-particle occupation numbers are characterized by a fixed temperature. In contrast to [17], our method allows us to include the time dependence of single-particle occupation numbers in the consideration and avoid the averaging procedure which makes it impossible to perform consistent calculations of the evolution of the collective and intrinsic variables.

To demonstrate the importance of the shell effects, we perform the calculations for two reactions in which nearly the same number of nucleons are included. In Figs. 2 and 3, the radial friction coefficient  $\gamma_{RR}$  is given as a function of



FIG. 3. The same as in Fig. 2 but for the  ${}^{56}$ Fe(480 MeV) +  ${}^{208}$ Pb reaction.



FIG. 4. Dependence of the total kinetic energy losses calculated by our method (solid curve) and on the basis of the classical model [32] realized with the code TRAJEC [33] for the  $^{64}$ Zn(440 MeV) +  $^{196}$ Pt reaction. The solid and dashed arrows indicate the moment corresponding to the turning point of the trajectory in the present and the classical model calculations, respectively.

R for both the approach (solid curve) and exit (dashed curve) stages of the  ${}^{64}Zn(440 \text{ MeV}) + {}^{196}Pt$  and  ${}^{56}Fe(480 \text{ MeV}) +$ <sup>208</sup>Pb reactions. It can be seen that  $\gamma_{RR}$  takes a smaller value for the exit stage in comparison with that of the approach one. This result demonstrates the decrease in the radial friction coefficient with heating of the nuclei. Such behavior is in contrast to that obtained in [17] in the coherent term approximation (dashed lines in Fig. 1). The effect of the shell structure and heating of nuclei can be seen in comparison Figs. 2 and 3. It is interesting that for the second  ${}^{56}$ Fe(480 MeV) + <sup>208</sup>Pb reaction, the difference between values of  $\gamma_{RR}$  for the exit and approach stages is larger. This effect is explained by the difference in the single-particle level scheme of the nuclei participating in the reaction, namely, by a difference in the energy gap between the occupied and unoccupied levels, which is larger in <sup>208</sup>Pb than in <sup>196</sup>Pt, and in <sup>56</sup>Fe than in <sup>64</sup>Zn. With the temperature increase, the energy gap between the occupied and unoccupied singleparticle states is smoothed out. As a result, the excitation energy per particle transition becomes smaller. It decreases the friction coefficient compared to those at lower temperature. However, the temperature increase should be comparable with the energy gap to produce the effect.

To illustrate the dynamics of the relative motion, we demonstrate in Figs. 4 and 5 the dependence of the total kinetic energy losses on the current time of the collision [R(t=0)=20 fm]. The calculations are done in the framework of our model (solid curve) and based on the classical model of [33] realized with the code TRAJEC [34] for the  $^{64}$ Zn(440 MeV) +  $^{196}$ Pt and  $^{56}$ Fe(480 MeV) +  $^{208}$ Pb reactions, respectively. The solid and dashed arrows indicate the moments corresponding to the turning points of the trajectories in our model and in the classical model calculations, respectively. It can be seen that, in contrast to the results of the classical model [33] calculations, where the majority of kinetic energy is dissipated at the very beginning of the reaction during a short time interval of the order of  $0.4 \times 10^{-21}$  s, in our model, this process takes a significantly larger time. According to the classical model, the absence of



FIG. 5. The same as in Fig. 4 but for the  ${}^{56}$ Fe(480 MeV) +  ${}^{208}$ Pb reaction.

correlations between the nucleon exchange and the total kinetic energy losses leads to intensive energy dissipation and the small nucleon exchange simultaneously. An application of this classic model to calculate the charge variance  $\sigma_Z^2$  was unsuccessful [8]. The friction coefficient obtained in [33] significantly exceeds our friction coefficient where the interacting nuclei strongly overlapped. The shortness of the characteristic time for the kinetic energy dissipation is explained by the large values of the friction coefficient used in the classical phenomenological model (Fig. 1).

In contrast to the results of the classical model calculations, which support the idea of fast kinetic energy losses and thermalization of the excitation energy at the beginning of the reaction, our calculations support the idea of a gradual kinetic energy dissipation. This conclusion is in line with the results of analysis of mechanism of very heavy-ion collisions [35]. Kinetic energy of the relative motion is found to be dissipated as nucleons are exchanged, indicating that the time scales of both processes are similar.

The effect of the redistribution of the particles over the single-particle states is considered to be a response of the nuclei to the relative motion. That appears as friction and in the dynamic change of the nucleus-nucleus potential. In Fig. 6, we show correction to the nucleus-nucleus interaction potential generated by the rearrangement of the nuclear densities during the approach phase of the reaction for the collision of  ${}^{64}Ni(320 \text{ MeV}) + {}^{208}Pb$ . An expression for this correction is obtained early as an effect of multinucleon transfer [36,27] and particle-hole excitations [27] on the relative motion due to the coupling term (5). It can be seen that the correction increases in absolute value with a decrease of the internucleus distance R. This dynamic effect can be important when calculating massive nuclei collisions where  $Z_1 \cdot Z_2$  is large and the pocket in the nucleus-nucleus potential is too shallow to provide for the capture of nuclei in the entrance channel. The nucleus-nucleus interaction potentials found by the sudden approximation (dotted line) and those calculated with the dynamic correction (dashed line), which is discussed just above, are shown in Fig. 7. The available energy of a relative motion, which is equal to the difference between the initial kinetic energy  $E_{c.m.}$  and the total kinetic energy loss (TKEL), is also presented (solid line) as a func-



FIG. 6. Correction to the nucleus-nucleus interaction potential generated by a rearrangement of the nuclear densities during the approach phase of the reaction for the collision of  $^{64}$ Ni(320 MeV) +  $^{208}$ Pb.

tion of the relative distance R. It can be seen that the correction to the sudden approximation, produced by a dynamic rearrangement of the particle distributions in the interacting nuclei, is comparable to the depth of the pocket in the nucleus-nucleus potential, particularly in the case of massive nuclear systems. Thus, taking this effect into account, we can determine the dynamic critical value of the orbital angular momentum, which is usually used together with the width of the angular momentum distribution as input information to study a behavior of the compound nucleus formed in a collision. It is well known that the mean value and width of the compound nucleus angular momentum distribution is a crucial parameter for statistical models.

# **IV. SUMMARY**

In conclusion, we have calculated the friction coefficient for DIC, based on the microscopic model of the structure of the colliding nuclei, and thereby avoid the assumption of a



FIG. 7. The nucleus-nucleus interaction potential V(R) obtained in the sudden approximation (dotted line) and with the dynamic correction (dashed line) presented in Fig. 6. An available energy of the relative motion ( $E_{c.m.}-TKEL$ ) is given as a function of the relative distance *R* (solid line) for the <sup>64</sup>Ni(320 MeV) + <sup>208</sup>Pb reaction.

fast statistical equilibrium of the dissipated kinetic energy. Our results demonstrate the importance of considering the friction coefficient as an exact dynamic function of the single-particle occupation numbers. The decrease in the radial friction coefficient with the heating of the nuclei has been shown. Such behavior is in contrast to that obtained in [15]. We have demonstrated that the kinetic energy dissipation in DIC is a gradual process which takes up a significant amount of a reaction time.

### ACKNOWLEDGMENTS

We are grateful to Dr. H. Hofmann and Dr. F. Ivanyuk for the valuable discussions and suggestions which stimulated the writing of this paper. We also wish to thank Dr. N. V. Antonenko for the fruitful discussions. The authors (G.G.A., R.V.J., and A.K.N.) are grateful to the International Science Foundation (Grant No. RFJ-000) and the Russian Foundation for Basic Research (Grant No. 95-02-05684) for financial support.

#### APPENDIX

The value of  $\tau_i$  is calculated using the results of the theory of quantum liquids [30,31]

$$\frac{1}{\tau_i^{(\alpha)}} = \frac{\sqrt{2}\pi}{32\hbar\varepsilon_{F_K}^{(\alpha)}} \left[ (f_K - g)^2 + \frac{1}{2}(f_K + g)^2 \right] \\ \times \left[ (\pi\Theta_K)^2 + (\widetilde{\varepsilon}_i - \lambda_K^{(\alpha)})^2 \right] \left[ 1 + \exp\left(\frac{\lambda_K^{(\alpha)} - \widetilde{\varepsilon}_i}{\Theta_K}\right) \right]^{-1},$$
(A1)

[1] J. O. Newton, Sov. J. Part. Nucl. 21, 349 (1990).

- [2] D. J. Hinde, Nucl. Phys. A553, 255c (1993).
- [3] M. Thoenessen, E. Ramakrishnan, J. P. Beene, F. F. Bertrand, M. L. Halbert, D. J. Horen, P. E. Mueller, and R. L. Varner, Phys. Rev. C 51, 3148 (1995).
- [4] J. L. Baretto, N. G. Nicolis, D. G. Sarantites, R. J. Charity, L. G. Sobotka, D. W. Stracener, D. C. Hensley, J. R. Beene, C. Baktash, M. L. Halbert, and M. Thoennessen, Phys. Rev. C 51, 2584 (1995).
- [5] D. J. Hofmann, B. B. Back, and P. Paul, Phys. Rev. C 51, 2597 (1995).
- [6] D. Hilsher and H. Rossner, Ann. Phys. (Paris) 17, 471 (1992).
- [7] I. I. Gontchar, Sov. J. Part. Nucl. 26, 922 (1995).
- [8] W. U. Schröder and J. R. Huizenga, in *Treatise on Heavy-Ion Science*, edited by D.A. Bromley (Plenum, New York, 1984), Vol. 2, p. 115.
- [9] J. R. Nix and A. J. Sierk, Los Alamos National Laboratory Report LA-UR-87-133, 1987.
- [10] J. Randrup, Nucl. Phys. A307, 319 (1978); A327, 490 (1979).
- [11] H. Feldmeier, Rep. Prog. Phys. 50, 1 (1987).
- [12] J. Blocki, Y. Boneh, J. R. Nix, J. Randrup, M. Robel, A. J. Sierk, and W. J. Swiatecki, Ann. Phys. (N.Y.) **112**, 356 (1978); **113**, 330 (1978); **125**, 193 (1980).
- [13] P. J. Johansen, P. J. Siemens, A. S. Jensen, and Helmut Hof-

where

$$\Theta_{K}(t) = 3.46 \sqrt{\frac{E_{K}^{*}(t)}{\langle A_{K}(t) \rangle}}$$

is the effective temperature determined by the amount of intrinsic excitation energy  $E_K^* = E_K^{*(Z)} + E_K^{*(N)}$ ;  $\langle A_K(t) \rangle = \langle Z_K(t) \rangle + \langle N_K(t) \rangle$ ,  $\lambda_K^{(\alpha)}(t)$ , and  $E_K^{*(\alpha)}(t)$  are the mass number, chemical potential, and intrinsic excitation energies for the proton ( $\alpha = Z$ ) and neutron ( $\alpha = N$ ) subsystems of the nucleus K(K = P, T), respectively (for details, see [26]). Furthermore, the finite size of nuclei and the available difference between the numbers of neutrons and protons need to use the following expressions for the Fermi energies [31]:

$$\varepsilon_{F_{K}}^{(Z)} = \varepsilon_{F} \left[ 1 - \frac{2}{3} (1 + 2f') \frac{\langle N_{K} \rangle - \langle Z_{K} \rangle}{\langle A_{K} \rangle} \right],$$
  

$$\varepsilon_{F_{K}}^{(N)} = \varepsilon_{F} \left[ 1 + \frac{2}{3} (1 + 2f') \frac{\langle N_{K} \rangle - \langle Z_{K} \rangle}{\langle A_{K} \rangle} \right], \quad (A2)$$

where  $\epsilon_F = 37$  MeV,

$$f_{K} = f_{\rm in} - \frac{2}{\langle A_{K} \rangle^{1/3}} (f_{\rm in} - f_{\rm ex}),$$
  
$$f'_{K} = f'_{\rm in} - \frac{2}{\langle A_{K} \rangle^{1/3}} (f'_{\rm in} - f'_{\rm ex}), \qquad (A3)$$

and  $f_{in}=0.09$ ,  $f'_{in}=0.42$ ,  $f_{ex}=-2.59$ ,  $f'_{ex}=0.54$ , g=0.7 are the constants of the effective nucleon-nucleon interaction.

mann, Nucl. Phys. A288, 152 (1977).

- [14] H. Hofmann and P. J. Siemens, Nucl. Phys. A257, 165 (1976);
   A275, 464 (1977).
- [15] A. Iwamoto, K. Harada, S. Yamaji, and S. Yoshida, Z. Phys. A 302, 149 (1981).
- [16] A. S. Jensen, J. Leffers, K. Reese, H. Hofmann, and P. J. Siemens, Phys. Lett. 117B, 5 (1982).
- [17] S. Yamaji and A. Iwamoto, Z. Phys. A 313, 161 (1983).
- [18] R. V. Jolos, R. Schmidt, and J. Teichert, Nucl. Phys. A429, 139 (1984).
- [19] H. A. Weidenmüller, Prog. Part. Nucl. Phys. 3, 49 (1980).
- [20] A. Gobbi and W. Nörenberg, in *Heavy-Ion Collisions*, edited by R. Bock (North-Holland, Amsterdam, 1980), Vol. II.
- [21] F.A. Ivanyuk, H. Hofmann, V.V. Pashkevich, and S. Yamaji, Report RIKEN-AF-NP-232, 2-1, 1996.
- [22] A. Lukasiak, W. Cassing, and W. Nörenberg, Nucl. Phys. A426, 181 (1984).
- [23] D. Berdichevsky, A. Lukasiak, W. Nörenberg, and P. Rozmej, Nucl. Phys. A499, 609 (1989).
- [24] H. J. Krappe, Nucl. Phys. A505, 417 (1989).
- [25] N. Takigawa, K. Niita, Y. Okhuhara, and Y. Yoshida, Nucl. Phys. **A371**, 130 (1981).
- [26] G. G. Adamian, R. V. Jolos, and A. K. Nasirov, Z. Phys. A 347, 203 (1994).

- [27] G. G. Adamian, N. V. Antonenko, R. V. Jolos, and A. K. Nasirov, Phys. Part. Nuclei A25, 583 (1994).
- [28] G. G. Adamian, R. V. Jolos, and A. K. Nasirov, Sov. J. Nucl. Phys. 55, 660 (1992).
- [29] G. G. Adamian, N. V. Antonenko, R. V. Jolos, and A. K. Nasirov, Nucl. Phys. A551, 321 (1993).
- [30] D. Pines and P. Noziéres, *Theory of Quantum Liquids* (Benjamin, New York, 1966).
- [31] A. B. Migdal, Theory of the Finite Fermi-Systems and Prop-

erties of Atomic Nuclei (Nauka, Moscow, 1983).

- [32] G. G. Adamian, R. V. Jolos, A. I. Muminov, and A. K. Nasirov, Phys. Rev. C 53, 871 (1996).
- [33] D. H. E. Gross and H. Kalinowski, Phys. Rep. 45, 175 (1978).
- [34] R. Schmidt, Part. Nuclei 13, 1203 (1982).
- [35] W. U. Schröder, J. R. Birkelund, J. R. Huizenga, K. L. Wolf, and V. E. Viola, Jr., Phys. Rep. 45, 301 (1987).
- [36] R. V. Jolos and A. K. Nasirov, Sov. J. Nucl. Phys. 45, 805 (1987).