Reply to ''Comment on 'Spectroscopic factors for bound *s*-wave states derived from neutron scattering lengths' ''

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In a recent comment Barker [Phys. Rev. C **56**, 3423 (1997), the preceding paper] criticized our procedure for the extraction of spectroscopic factors from neutron scattering lengths [Phys. Rev. C **55**, 1591 (1997)]. In this reply we compare the *R*-matrix analysis by Barker to our potential model calculation, and we discuss the applicability of both models for the extraction of spectroscopic factors. [S0556-2813(97)04012-0]

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In our first paper [1] we derived the spectroscopic factors (SF's) for bound *s*-wave states from the experimentally known neutron scattering lengths using a potential model. Barker, who used the *R*-matrix approach, criticized our procedure to be of ''doubtful validity and accuracy'' [2].

In the *R*-matrix theory there is a separation between the nuclear exterior region without a nuclear interaction of two colliding nuclei ($r>a$) and an interior compound region ($r<a$), where a is not much larger than the nuclear radius of the compound nucleus. Whereas the form of the wave functions in the exterior region is comprised of incoming and outgoing scattering waves, in the interior region the wave function is not known *a priori*; all that is known is that its magnitude and derivative must match smoothly onto the exterior waves across the boundary $r=a$ [3]. On the other hand, in the potential model the wave function is calculated from a given potential without any separation into an interior and exterior part.

In the *R*-matrix theory the nuclear phase shift in the one-level, one-channel approximation is given by the sum of a resonant part and a hard-sphere phase shift $\phi(E)$ [see Eq. (8) of Ref. [2]]:

$$\delta(E)=\arctan\frac{\frac{1}{2}\hat{\Gamma}(E)}{E_1+\Delta(E)-E}-\phi(E), \tag{1}$$

with the width $\hat{\Gamma}(E)$ and the shift function $\Delta(E)$. In the potential model we used

$$\delta(E)=\arctan\frac{\frac{1}{2}\widetilde{\Gamma}(E)}{E_B-E}, \tag{2}$$

with the width $\widetilde{\Gamma}(E)$. Both approaches have to describe experimental phase shifts. This means that the widths $\hat{\Gamma}$ in the *R*-matrix theory and $\widetilde{\Gamma}$ in the potential model are defined in a different way. This difference is important outside the resonance region $|E-E_R|>\Gamma$. In the potential model there exists no separation into an internal and external part. Therefore, an additional hard-sphere contribution has obviously not to be taken into account.

For *s*-wave neutrons at small energies (especially at the thermal energy $E=25$ meV where the experimental value of the phase shift is determined from the neutron scattering length b) the $\hat{\Gamma}(E)$, $\phi(E)$, and $\widetilde{\Gamma}(E)$ and therefore the phase shift $\delta(E)$ are proportional to the wave number $k=\sqrt{2ME/\hbar^2}$.

Originally, SF's are determined by the ratio of the measured transfer or capture reaction cross section to the cross section calculated in the direct reaction model (DRM):

$$C^2S_i=\sigma_i^{\rm expt}/\sigma_i^{\rm DRM}. \tag{3}$$

Theoretically, SF's are calculated from the shell model.

In the context of this work the SF's can be calculated by the ratio of the experimental resonance width $\Gamma^{\rm expt}$ to the calculated $\Gamma_{\rm s.p.}^{\rm calc}$. In the *R*-matrix theory this leads to [see also Eq. (10) of Ref. [2]]

$$C^2\hat{S}=\Gamma^{\rm expt}/\hat{\Gamma}_{\rm s.p.}^{\rm calc}=(\gamma^2)^{\rm expt}/(\hat{\gamma}^2)_{\rm s.p.}^{\rm calc}, \tag{4}$$

with the reduced widths γ^2. This *R*-matrix SF depends on the chosen interaction radius a. In the potential model we obtain for *s*-wave neutrons

$$C^2\widetilde{S}=\Gamma^{\rm expt}/\widetilde{\Gamma}_{\rm s.p.}^{\rm calc}=b^{\rm expt}/\widetilde{b}_{\rm s.p.}^{\rm calc}. \tag{5}$$

This SF depends on the shape of the chosen potential; however, because the strength of the potential is adjusted to the binding energy, the resulting dependence on the shape parameters is small.

The *physical* resonance energies and widths are the poles $E=E_{\rm res}-i\Gamma_{\rm res}/2$ of the scattering matrix. In the potential model these poles are given by the resonance energy $E_{\rm B}$ and width $\widetilde{\Gamma}$ in Eq. (2), whereas in the *R* matrix they are different from the resonance energy E_1 and width $\hat{\Gamma}$ in Eq. (1). One can see that the energy and width in the *R* matrix cannot serve as the estimate of such a physical width, since in this parametrization the resonance is a solution of the equation (see Ref. [4])

$$[E-E_1-\Delta(E)]^2+[\hat{\Gamma}(E)/2]^2=0. \quad (6)$$

The quantity $\Gamma_S(E)=\hat{\Gamma}(E)[1-\Delta'(E_{res})]^{-1}$ given in [4] would be such an estimate in the R-matrix theory. (The quantity $\Gamma_S(E)$ enters into the resonant factor of a reaction when written in the form $\Gamma_S(E)/\{(E-E_{res})^2+[\Gamma_S(E)/2]^2\}$.) The widths Γ_{res} and not $\hat{\Gamma}$ are related to the *physical* lifetimes and determine the SF in transfer and capture reactions [4]. Therefore, Eq. (4) seems to be not an appropriate definition of the SF.

In the following we will compare results for the systems $^{13}N=^{12}C\otimes p$ and $^{13}C=^{12}C\otimes n$ given by Barker in the framework of the R-matrix theory [5] and calculated in the potential model. This system was also chosen by Barker in his Comment [2] for the comparison of the two different approaches.

In Fig. 1 (upper part) we show phase shifts for s-wave scattering of protons of ^{12}C in the range up to 5 MeV. The calculated phase shifts show resonant behavior at $E_{c.m.}=421.4$ keV, and they agree well with the experimental data [6–9] above the resonance. The potential strength was adjusted to the energy of the quasibound $1/2^+$ state in ^{13}N at $E_x=2364.9$ keV ($E_{c.m.}=421.4$ keV). For the SF of this state we obtain $C^2\tilde{S}=0.881\pm0.022$ from $\tilde{\Gamma}^{calc}_{s.p.}=2[(d\delta/dE)|_{E=E_{res}}]^{-1}=36.0$ keV compared with $\Gamma^{expt}=31.7\pm0.8$ keV [10]. Barker and Ferdous [5] obtain results from $C^2\hat{S}\approx0.88$ for $a=4.2$ fm to $C^2\hat{S}\approx0.81$ for $a=7.0$ fm.

In Fig. 1 (lower part) we show phase shifts for s-wave scattering of neutrons of ^{12}C in the range up to 5 MeV. Again the calculated phase shifts agree well with the experimental data [12]. Of course, the potential strength was adjusted to the energy of the bound $1/2^+$ state in ^{13}C at $E_x=3089.4$ keV. Now we obtain $C^2\tilde{S}=0.966\pm0.015$ from $\tilde{b}^{calc}_{s.p.}=6.3562\pm0.0650$ fm compared with $b^{expt}=6.1487\pm0.0015$ fm [11]. (The uncertainty of the theoretical value comes from the two different parametrizations of the optical potential in Ref. [1].) From the R-matrix calculation [5] one gets results from $C^2\hat{S}\approx0.76$ for $a=4.2$ fm to $C^2\hat{S}\approx0.59$ for $a=7.0$ fm.

As expected from the isospin symmetry of the systems ^{13}N and ^{13}C the SF's derived in the potential model calculation are very similar whereas the R matrix leads to differences from 15% for $a=4.2$ fm to 30% for $a=7.0$ fm.

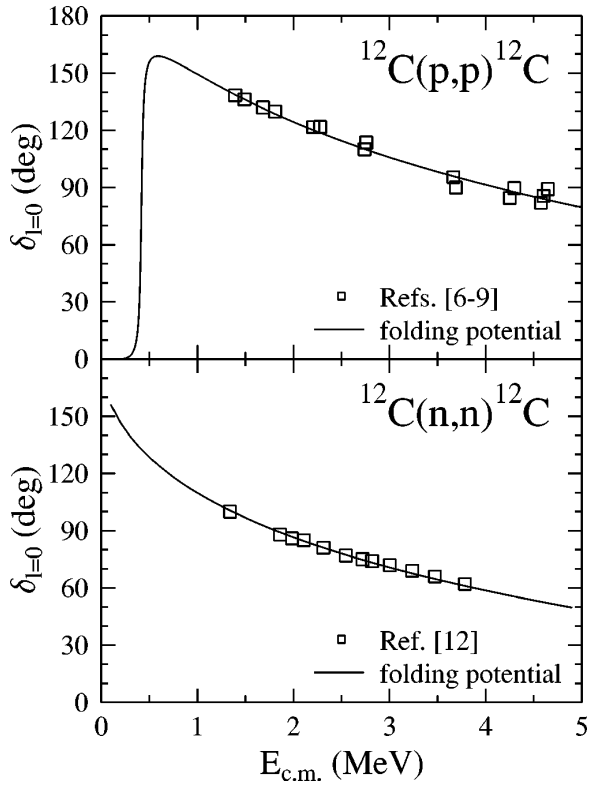

FIG. 1. s-wave phase shifts of proton (upper part) and neutron (lower part) scattering of ^{12}C. The calculation was performed using a folding potential with the strength adjusted to the $1/2^+$ (quasi) bound states of ^{13}N ($E_x=2364.9$ keV) and ^{13}C ($E_x=3089.4$ keV). A similar calculation with standard Woods-Saxon potentials ($R=R_0A_T^{1/3}$, $R_0=1.25$ fm, $a=0.65$ fm, and V_0 to be adjusted) leads to practically identical phase shifts. The experimental data points were taken from Refs. [6–9,12].

Shell model calculations for the $A=13$ system give $C^2S=0.85$ [1] and $C^2S=0.89$ [13]. The average of these values ($C^2S=0.87$) is in reasonable agreement with a weighted average of the results from our potential model ($C^2S=0.939\pm0.028$).

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