# Reexamination of the variable moment of inertia nuclear softness model

J. B. Gupta, A. K. Kavathekar, and Y. P. Sabharwal
*Ramjas College, University of Delhi, Delhi-110 007, India*
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The three parameter variable moment of inertia (VMI) nuclear softness model named VMINS3 is shown to be adequate in reproducing the main features of the VMI model. Its failure for deformed nuclei reported by earlier workers is shown to be the result of a wrong approach in the calculation. The variation of the softness parameter $\sigma$ and of the stretching parameter $K=(\frac{1}{2})C\theta_0^2$ with increasing deformation of the nuclear core is now consistent with results of the variable moment of inertia model. [S0556-2813(97)00312-9]

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Batra and Gupta [1] studied the ground-state bands in even-even nuclei by reformulating the variable moment of inertia (VMI) model of Mariscotti *et al.* [2] in terms of nuclear softness. In VMI one expresses the energy as a sum of the rotational and the potential energy:

$$E(J)=\frac{\hbar^2 J(J+1)}{2\theta_J}+(1/2)C(\theta_J-\theta_0)^2 \tag{1}$$

and treats the moment of inertia $\theta=\theta_J$ itself as a variable, dependent on $J$. Here, $\theta_J$ is determined from the equilibrium condition

$$\left.\frac{dE}{d\theta}\right|_J=0 \tag{2}$$

which yields a cubic equation in $\theta_J$. The ground-state moment of inertia $\theta_0$ and the stretching constant $C$ not being known *ab initio*, the solution of the cubic equation is somewhat involved for which several alternative approaches have been adopted [1,2]. Following the success of the earlier proposed nuclear softness model [3] in which the moment of inertia was expressed as a polynomial in $J$ in the single term energy formula

$$E(J)=\frac{\hbar^2 J(J+1)}{2\theta_0(1+\sigma_1 J+\sigma_2 J^2+\cdots)}, \tag{3}$$

Batra and Gupta replaced the equilibrium condition (2) for determining $\theta_J$ in Eq. (1) by the Taylor series expansion of $\theta_J$ about its ground-state value $\theta_0$ for $J=0$ as in the denominator of Eq. (3). By retaining only the first order term in $J$, viz., $\theta_J=\theta_0(1+\sigma J)$, the VMI model Eq. (1) was simplified [1] to

$$E(J)=\frac{AJ(J+1)}{1+\sigma J}+BJ^2 \tag{4}$$

where $A=(\frac{1}{2})\hbar^2/\theta_0$ and $B=K\sigma^2=(\frac{1}{2})C\theta_0{}^2\sigma^2$. As it involves three parameters, namely, softness parameter $\sigma$, the ground-state moment of inertia $\theta_0$, and the stretching constant $C$, it was called the VMINS3 model. If the second order softness parameter $\sigma_2$ is also retained, the model was called VMINS4. Batra and Gupta [1] also claimed the range of validity of the VMINS model to be $2.0\leqslant R_{42}\leqslant 3.33$, where $R_{42}=E(4_1{}^+)/E(2_1{}^+)$.

However, in their calculation of VMINS3 model results, Batra and Gupta observed that even though the VMINS3 model gave better energy values compared to the two parameter VMI model for softly deformed nuclei such as $^{126}$Ba and $^{192}$Os nuclei, it gave rather poor results (energy values) for deformed nuclei such as $^{162}$Dy, $^{164}$Er, $^{174}$Yb, $^{172}$Hf, and $^{174}$W. Observing that this may be due to the large values of $\sigma_1$ (obtained in the calculation), they justified the use of the VMINS4 model [1] which gave level energies comparable to the VMI model [2] and to the NS3 model [3] for the nuclei under consideration.

In fact, in their plot for some 130 nuclei (Fig. 1 of [1]) they obtained an unexpected trend of variation of $\sigma_1$ with the deformation measure $R_{42}$, the value of $\sigma_1$ increasing with $R_{42}$ for $R_{42}>3.0$. Similar trend for the Yb isotopes differed from the continuous decrease of $\sigma_1$ with increasing $R_{42}$ in the other approaches, e.g., NS3, VMI, etc. (see Fig. 2 of [1]). Since the VMINS model was intended to be an improvement over the NS model, the above results were somewhat perplexing. Hence we have carefully worked out the results of VMINS3 model using Eq. (4), following their procedure of solving the first three energy level equations for the three coefficients $\sigma$, $\theta_0$, and $C$.

In such an approach as an intermediate step (by eliminating $B$ and $A$ from Eq. (4) for $J=2$, 4, 6) one obtains a quadratic equation in $\sigma$ $(=\sigma_1)$

$$a_1\sigma^2+b_1\sigma+c_1=0 \tag{5}$$

[the coefficients $a_1$, $b_1$, $c_1$ being known in terms of $E(2_1)$, $E(4_1)$, and $E(6_1)$] which yields two real roots. A complex root if obtained, implies the inapplicability of VMINS3 to the given nucleus (also see below). For a proper choice of $\sigma$ value we set a constraint on it to yield a positive value of the coefficients $B$ and $K$ in Eq. (4), since $C$, $\theta_0$, and $\sigma$ are all positive. Also, out of two roots (if both yield positive $B$), the smaller one was preferred, since a lower $\sigma$ represents a smaller correction to $\theta_0$.

While our results for softly deformed nuclei coincide with those of Ref. [1], altogether different results were obtained for well-deformed nuclei. This is infact the region in which VMINS3 results were reported to be relatively poor [1]. Our

 

TABLE I. The parameters of the models.

| | $\theta_0$ (MeV)$^{-1}$ | | | $\sigma_1$ | | |
|---|---|---|---|---|---|---|
| Nucleus | VMINS3[a] | VMINS4[a] | Present | VMINS3[a] | NS3[a] | Present |
| $^{162}$Dy | 48.49 | 31.80 | 38.26 | 0.4382 | 0.0021 | 0.0121 |
| $^{164}$Er | 43.74 | 27.29 | 34.0 | 0.3508 | 0.0036 | 0.0173 |
| $^{174}$Yb | 48.41 | 31.72 | 39.9 | 0.5717 | 0.0010 | 0.0071 |
| $^{172}$Hf | 41.77 | 30.38 | 32.9 | 0.2567 | 0.0070 | 0.0268 |
| $^{174}$W | 31.65 | 29.43 | 28.8 | 0.1239 | 0.0176 | 0.0631 |

[a]Reference [1].

values are rather closer to those of the nuclear softness (NS2 and NS3) model [3]. The large value of $\sigma$ obtained in Ref. [1] for the deformed nuclei was unphysical. In Table I we report our values of $\sigma$ which are quite small for the typical well-deformed nuclei of Ref. [1]. Similarly, our values of the moment of inertia $\theta_0$ are smaller than those of Ref. [1] and lie closer to those of NS3 [1] (see Table I). This also yields the calculated energies of ground band levels much closer to experiment (Table II) than obtained previously [1]. The improvement in the value is even upto 400 keV at 18+, e.g., in $^{172}$Hf. The other (larger) value of $\sigma$ of course reproduces the results of Ref. [1].

We also extended the calculation to all the nuclei in the $Z=54$–76, $N=66$–126 region. The wrong trend of the variation of $\sigma_1$ with $R_{42}$ at large values of $R_{42}$ obtained in [1] is now corrected (see Fig. 1). As expected, now the deformation parameter $\sigma$ decreases regularly with increasing $R_{42}$. Note that in a few cases, e.g., at $N=80$, 84, 86 both roots in Eq. (5) yield negative $B$ in Eq. (5) and $\sigma$ values lie much off the general trend [in specific cases the roots of Eq. (5) were complex]. These data lie outside the scope of VMINS3 and were excluded from smooth curve in the graph. This also implies that in general the VMINS3 model is not useful for $R_{42}<2.4$. The smooth curve in Fig. 1 is the least square fit to the data using a quadratic in $R_{42}$. It represents the data better than the linear fit.

Also, the $K=(1/2)C\theta_0^2$ value in VMINS3 no longer decreases with $R_{42}$ for $R_{42}>3.0$ as wrongly obtained in Fig. 1 of [1]. There is a slow increase in the value of $K$ upto $R_{42}=3.0$ (Fig. 2), and a sharp exponential rise beyond it (Fig. 3). This sharp rise for $R_{42}>3.0$ corresponds to the very small

TABLE II. Ground-state band energies (in keV) for a few typical nuclei. The dashes refer to the energies used in determining the three parameters of Eq. (4).

| Model | $E2$ | $E4$ | $E6$ | $E8$ | $E10$ | $E12$ | $E14$ | $E16$ | $E18$ |
|---|---|---|---|---|---|---|---|---|---|
| Dy-162 | | | | | | | | | |
| Expt. | 80.7 | 265.7 | 548.5 | 921 | 1375 | 1903 | 2494 | 3143 | 3836 |
| VMINS3[a] | | | | 928 | 1403 | 1974 | 2640 | 3402 | 4260 |
| VMINS3[b] | | | | 923 | 1385 | 1927 | 2547 | 3240 | 4001 |
| Er-164 | | | | | | | | | |
| Expt. | 91.4 | 299.5 | 614.4 | 1025 | 1518 | 2083 | 2703 | 3263 | 3769 |
| VMINS3[a] | | | | 1034 | 1556 | 2181 | 2909 | 3740 | 4673 |
| VMINS3[b] | | | | 1028 | 1531 | 2119 | 2784 | 3522 | 4327 |
| Yb-174 | | | | | | | | | |
| Expt. | 76.5 | 253.1 | 526.0 | 890 | 1336 | 1861 | 2457 | 3117 | 3836 |
| VMINS3[a] | | | | 895 | 1358 | 1917 | 2572 | 3321 | 4166 |
| VMINS3[b] | | | | 891 | 1346 | 1886 | 2510 | 3212 | 3992 |
| Hf-172 | | | | | | | | | |
| Expt. | 95.3 | 309.3 | 628.1 | 1037 | 1521 | 2065 | 2654 | 3278 | 3920 |
| VMINS3[a] | | | | 1047 | 1564 | 2179 | 2890 | 3697 | 4600 |
| VMINS3[b] | | | | 1040 | 1536 | 2107 | 2746 | 3449 | 4209 |
| W-174 | | | | | | | | | |
| Expt. | 111.9 | 355.0 | 704.0 | 1137 | 1635 | 2186 | 2780 | 3392 | 3973 |
| VMINS3[a] | | | | 1146 | 1675 | 2285 | 2974 | 3741 | 4583 |
| VMINS3[b] | | | | 1142 | 1658 | 2242 | 2890 | 3597 | 4358 |

[a]VMINS3 results are from [1].
[b]Improved results of VMINS3 as explained in text.

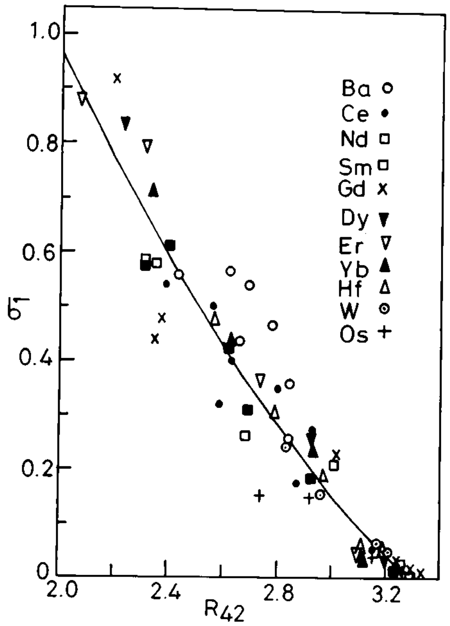


FIG. 1. The softness parameter $\sigma_1$ versus the energy ratio $R_{42}$. The smooth curve here is the least square fit to the data on a quadratic expression as explained in the text.

value of $\sigma$ (less than 0.05) so that $B=K\sigma^2$ in the potential energy term remains finite.

The value of the stiffness constant $C$ of Eq. (1) no longer decreases with increasing neutron number $N$ as obtained in Fig. 4 of [1] for Yb and W isotopes (see our Fig. 4 for Yb) and has a trend similar to the VMI model [2]. Thus our work removes the anomalies noted for the VMINS3 model in Ref. [1], and the predictions of the VMINS3 model are in reasonable agreement with other solutions of VMI.

Since improved results on level energies in Ref. [1] were obtained in the four-parameter version VMINS4, using large values of $\sigma$ for the few deformed nuclei cited in Table I of [1], we need to analyze results of VNINS4. As an analytical solution was not obtainable, we worked for a numerical solution. Here, we adopted the Newton-Raphson method of solving the $F(X)=0$ problem for the four parameters $A$,

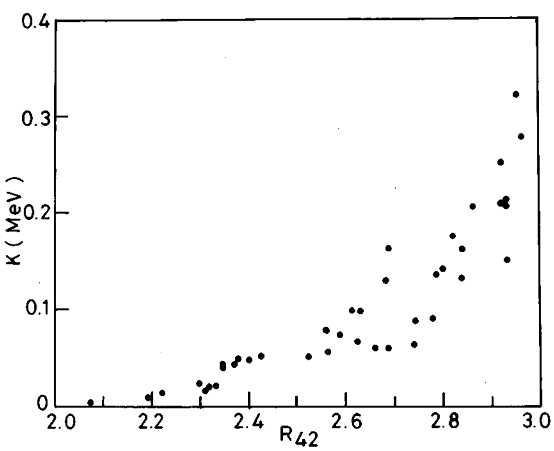


FIG. 2. The constant $K$ versus $R_{42}$ for nuclei with $R_{42}$ less than 3.0.

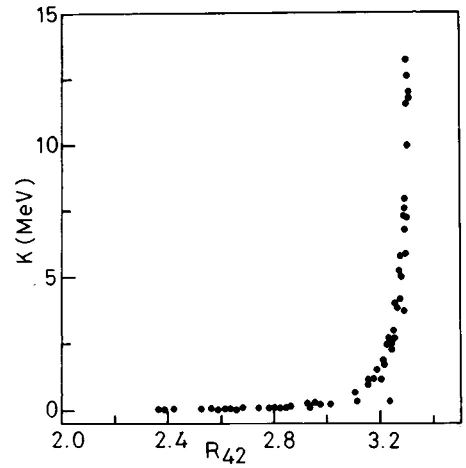


FIG. 3. The same as in Fig. 2 but on a condensed $Y$ scale for all nuclei. The $K$ values of Fig. 2 now overlap with the $X$ axis and $K$ values for larger $R_{42}$ increase exponentially.

$B(=K\sigma^2)$, $\sigma_1$, and $\sigma_2$. A test of the method was performed for all nuclei by applying the method to the VMINS3 expression, wherein the numerical solutions agreed with the analytic ones.

While convergence was readily attained for the relatively softer nuclei with $\sigma=\sigma_1>0.2$, it was often hard to attain the same for very small $\sigma$ values say around 0.05 or less. Of the two solutions, one with smaller $\sigma$ conformed to the trend of $\sigma$ decreasing with increasing $R_{42}$, but the other value was often very large ($\sigma \geqslant 1.0$), the latter value was reported for the nuclei considered in Ref. [1]. In some cases the two solutions were for negative and positive $\sigma_2$ yielding larger and smaller values for $\sigma_1$, respectively. The one with smaller $\sigma_1$ was preferred as in VMINS3. For the values of $\sigma_1$ obtained in VMINS4 no correlation could be obtained

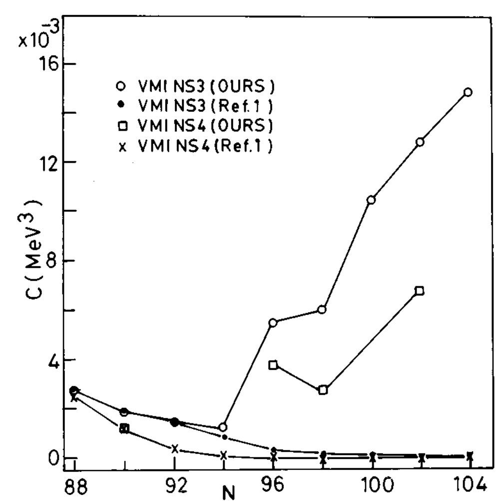


FIG. 4. The stiffness constant $C$ for Yb isotopes versus neutron number $N$ in the VMINS3 and VMINS4 models. The values of $C$ increase with $N$ corresponding to the deformed nuclei. For VMINS4 some values are not available (see text). The values in Ref. [1] are shown for comparison. The latter values decrease with increasing $N$ and correspond to larger value of $\sigma_1$.

between $\sigma_1$ and $R_{42}$ and thus a plot of such values has no relevance. In Ref. [1] no plot for VMINS4 $\sigma_1$ was given.

The values of stiffness constant $C$ obtained in VMINS4 for Yb isotopes are shown in Fig. 4. No convergence could be obtained in the numerical solution for small $\sigma_1$ in some cases in the VMINS4 model. Hence these data are missing in the graph.

In VMINS4 one does obtain better energy fits, but considering the difficulties cited above, it does not offer a simplification of the VMI model calculation and the results of VMINS3 should, therefore, be adequate. Thus our work shows the utility and the limitations of the VMINS model adoption of VMI model.

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