Lifetime of bound states of negative pions and neutrons

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We estimate the lifetime of systems consisting of negative pions and neutrons (pineuts). We consider the direct decay of the pion as well as second-order diagrams involving the intermediate decay processes $n \to p + e^- + \overline{\nu}$ and $\pi^- \to \pi^0 + e^- + \overline{\nu}$ by assuming a simple single-particle model. We found that the dominant process is that with an intermediate neutron decay which leads to a lifetime of around 10^{-8} sec. [S0556-2813(97)03112-9]

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I. INTRODUCTION

Bound states of Z negative pions and N neutrons (pineuts) have been predicted for a variety of values of Z and N assuming reasonable forms for the various interactions between the particles [1,2]. The experimental searches of these systems up to now have given negative results [3–11] since they have been restricted to the systems with $Z \leqslant 2$ while as shown in [2] the minimum number of pions that are required in order to bind the system is $Z=5$.

The average density of the pineuts is about 3 times nuclear matter density. Then, in order to build such a bound state it is necesary to have neutron matter at that highly dense state. In nature this can be found at the core of neutron stars, where also negative pions are produced via the decay $n \to p + \pi^-$ [12,13]. Thus, in the neutron star core the formation of pineuts is possible, although in a probably small amount. Nevertheless, the very presence of pineuts in the neutron star core must modify the solution to the Oppenheimer-Volkoff equation [14]. We are presently investigating this subject [15].

Pineuts have isospin $I = N/2 + Z$ and they are stable since they cannot decay by electromagnetic or strong interactions. Their only possible modes of decay are through weak interactions. Thus, in this work we will investigate what are the dominant weak decay modes in order to give estimates of the lifetime of pineuts with values of $N=6$ or 7 and $Z=6$ or 7.

In Sec. II we will describe the theoretical model used to obtain the binding energies and wave functions of pineuts. In Sec. III we will write down the amplitudes for the decay rate of the various processes and in Sec. IV we discuss the results obtained from these amplitudes and give our conclusions.

II. THE PINEUT'S MODEL

The theoretical model of Refs. [1,2] consists of a prescription to calculate (a) the interaction of the pion with the neutrons, (b) the interaction between the neutrons themeselves, and (c) the Coulomb interaction between the pions. The strong interaction between the pions is neglected since two negative pions must necessarily be in an isospin 2 state. Therefore, since the $\pi\pi$ phase shifts for isospin 2 are very small [16] one expects this interaction to be negligible.

A. Binding energies

The energy of a pineut consisting of N neutrons and Z pions is

$$E = E_N - ZB + E_{\mathrm{Coul}}, \tag{1}$$

where E_N is the self-energy of the N neutrons, B is the binding energy of each pion, and E_{Coul} is the repulsive Coulomb energy of the Z pions.

The strong interaction between a negative pion and a piece of neutron matter is described by the solution of the truncated Klein-Gordon equation

$$\psi(\vec{k}) = \frac{2\omega}{k_0^2 - k^2} \int d\vec{k}'\, V(\vec{k},\vec{k}')\, \psi(\vec{k}'), \tag{2}$$

where

$$\omega = m_{\pi^-} - B, \tag{3}$$

with m_{π^-} the mass of the pion and B the binding energy, and

$$k_0^2 = \omega^2 - m_{\pi^-}{}^2. \tag{4}$$

The potential of a negative pion with a piece of neutron matter is given in momentum space by

$$V(\vec{k},\vec{k}') = \frac{4\pi}{2m_{\pi^-}} (\beta_0 + \beta_1 \vec{k}\cdot\vec{k}')\, \rho_n(\vec{k}-\vec{k}') \times \frac{M_n^2}{M_n^2 + k^2}\, \frac{M_n^2}{M_n^2 + k'^2}, \tag{5}$$

where

$$\rho_n(\vec{k}-\vec{k}') = \int d\vec{r} e^{i(\vec{k}-\vec{k}')\cdot\vec{r}} \rho_n(r), \tag{6}$$

and $\rho_n(r)$ is the (spherically symmetric) neutron density. The parameters β_0 and β_1 are obtained by taking the limit of the optical potential of pionic atoms when the proton density tends to zero [1,2]. Equation (5) contains also a cutoff with a cutoff parameter M_n equal to the mass of the neutron as it is appropriate for the πN amplitude with isospin 3/2.

TABLE I. Parameters r_0 and d of the neutron density and energy of the pineut E for different values of N and Z.

(N,Z)	r_0(fm)	d(fm)	E(MeV)
(5,6)	0.599	0.292	−93.3
(5,7)	0.621	0.269	−139.6
(6,5)	0.447	0.409	−52.2
(6,6)	0.496	0.377	−98.2
(6,7)	0.532	0.350	−145.9
(7,5)	0.302	0.493	−51.7
(7,6)	0.375	0.457	−98.3
(7,7)	0.424	0.428	−146.6

The self-energy of a piece of neutron matter is calculated from the results of infinite neutron matter by applying the local density approximation as shown in [1]. Since the pions are in an S state, this gives rise to a spherically symmetric charge density $\rho_Q(r) = -eZ|\psi(\vec{r})|^2$. Thus, the Coulomb energy is calculated in the standard way by considering a spherical shell of radius r with a differential of charge $dq = \rho_Q(r)4\pi r^2 dr$ and integrating from 0 to ∞ as shown in [2].

A Fermi distribution is assumed for the density of the neutrons, i.e.,

$$\rho_n(r) = \frac{\rho_0}{1+e^{(r-R)/d}}, \tag{7}$$

with

$$R = r_0 N^{1/3}. \tag{8}$$

The parameter ρ_0 in Eq. (7) is determined by the condition

$$\int d\vec{r}\rho_n(r) = N, \tag{9}$$

while the other two parameters r_0 and d will be determined as follows. The energy of the pineut is given by Eq. (1) and is a function of the two parameters r_0 and d of the density of the piece of neutron matter, i.e.,

$$E(r_0,d) = E_N(r_0,d) - ZB(r_0,d) + E_{\mathrm{Coul}}(r_0,d); \tag{10}$$

therefore, one has to find the values of the parameters r_0 and d that minimize the energy. Thus, the energy of the physical pineut is obtained by applying the variational conditions

$$\frac{\partial E(r_0,d)}{\partial r_0} = 0, \tag{11}$$

$$\frac{\partial E(r_0,d)}{\partial d} = 0, \tag{12}$$

which determine simultaneously the parameters r_0 and d as well as the energy of the pineut. We give in Table I these values for the pineuts that are going to be needed in the calculations of this work.

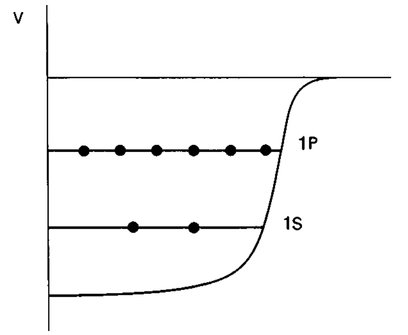

FIG. 1. Single-particle model of the neutrons in a pineut.

B. Vertex functions

Since all the pions are in the ground state which is an S wave, the spin of the pineut is determined by the neutrons. Thus, we will assume that the pineuts with N = even have spin 0 and those with N = odd have spin 1/2. Moreover, following the analogy with the nuclear shell model, we will assume that the neutrons in a pineut are in single-particle states as depicted in Fig. 1. The wave functions for the neutrons in the S and P shells are then given by

$$\phi_0(r) = \frac{1}{\sqrt{N_s}} \frac{\alpha}{\sqrt{r^2+\alpha^2}} \sqrt{4\pi\rho_n(r)}, \tag{13}$$

$$\phi_1(r) = \frac{1}{\sqrt{N_p}} \frac{r}{\sqrt{r^2+\alpha^2}} \sqrt{4\pi\rho_n(r)}, \tag{14}$$

where

$$N_s + N_p = N. \tag{15}$$

These wave functions satisfy

$$N_s\phi_0^2(r) + N_p\phi_1^2(r) = 4\pi\rho_n(r), \tag{16}$$

while the normalization condition

$$\int_0^\infty r^2 dr\, \phi_0^2(r) = 1, \tag{17}$$

determines the constant α. From Eqs. (9), (15), and (17) we see that the P-state wave function is automatically normalized as

$$\int_0^\infty r^2 dr\, \phi_1^2(r) = 1. \tag{18}$$

In Figs. 2(a) and 2(b) we show the vertices $A\pi B$ and AnB, where A and B are pineuts on the mass shell, and in vertex 2(a) the pion is off-mass-shell while in vertex 2(b) the neutron is off-mass-shell. In vertex 2(a) we will assume that pineut A is a bound state of a π^- and pineut B. Similarly, in vertex 2(b) pineut A is a bound state of a neutron and pineut B. In vertex 2(b) pineuts A and B will be assumed to be in the ground state so that according to the single-particle model of Fig. 1 only the P-shell neutrons will be considered.

The vertex 2(a) when pineuts A and B have spin 0 is written as

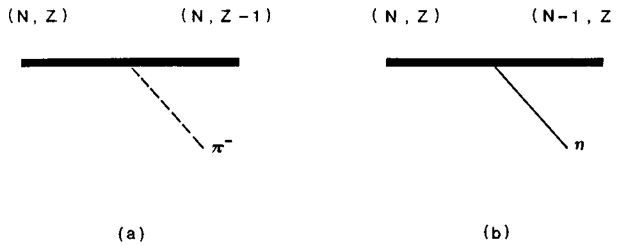

FIG. 2. The $A\pi B$ and AnB vertices with pineuts A and B on the mass shell.

$$V_{A\pi B}(k_\pi^2)=2\pi\left[\frac{M_A E_B}{E_\pi}\right]^{1/2}(k_\pi^2-m_\pi^2)\psi(p), \qquad (19)$$

where k_π^2 is the pion's four-momentum squared, $\psi(p)$ is the radial bound-state wave function which will be taken as the solution of Eq. (2), p is the $\pi-B$ relative three-momentum in the center of mass of the pair which is a Lorentz invariant given by

$$p=\left[\frac{(M_A^2+M_B^2-k_\pi^2)}{4M_A^2}-M_B^2\right]^{1/2}, \qquad (20)$$

and

$$E_B=\sqrt{M_B^2+p^2}, \qquad (21)$$

$$E_\pi=\sqrt{m_\pi^2+p^2}. \qquad (22)$$

In the case when pineuts A and B have spin 1/2 the vertex 2(a) is written as

$$U_{A\pi B}(k_\pi^2)=\pi\left[\frac{E_B}{E_\pi M_B}\right]^{1/2}(k_\pi^2-m_\pi^2)\psi(p). \qquad (23)$$

In the AnB vertex shown in Fig. 2(b) pineut A contains N neutrons while pineut B contains $N-1$ neutrons. If N is even then pineut A has spin 0 and pineut B has spin 1/2 and the vertex AnB is written as

$$V_{AnB}(k_n^2)=2\pi\sqrt{2M_A}\frac{\sqrt{E_nE_B(E_n+M_n)(E_B+M_B)}}{E_n+M_n+E_B+M_B}$$
$$\times(M_A-E_n-E_B)\frac{\phi_1(p)}{p}. \qquad (24)$$

Here k_n^2 is the neutron's four-momentum squared, $\phi_1(p)$ is the P-wave radial bound-state wave function in momentum space which is given in coordinates space by Eq. (14), p is the $n-B$ relative three-momentum in the center of mass of the pair which is given by

$$p=\left[\frac{(M_A^2+M_B^2-k_n^2)}{4M_A^2}-M_B^2\right]^{1/2}, \qquad (25)$$

and

$$E_B=\sqrt{M_B^2+p^2}, \qquad (26)$$

$$E_n=\sqrt{M_n^2+p^2}. \qquad (27)$$

If N is odd then pineut A has spin 1/2 and pineut B has spin 0. Thus, in this case the AnB vertex will be given by

$$U_{AnB}(k_n^2)=-2\pi\sqrt{2E_nE_B(E_n+M_n)}$$
$$\times(M_A-E_n-E_B)\frac{\phi_1(p)}{p}\gamma_5. \qquad (28)$$

The vertices V_{AnB} and U_{AnB} are different since in Eq. (24) $\phi_1(p)$ is the bound-state wave function of two spin-1/2 particles while in Eq. (28) $\phi_1(p)$ is the bound-state wave function of spin-0 and spin-1/2 particles. The vertices Eq. (24) and Eq. (28) have been constructed in analogy to the dnn vertex given by Refs. [17,18].

We show in Fig. 3 the pionic wave function $\psi(p)$ and the neutronic wave function $\phi_1(p)/p$ for the cases that are going to be needed in this work. The pionic wave functions $\psi(p)$ corresponding to $(N,Z)=(6,7)$ and $(7,6)$ lie in between those shown in Fig. 3(a).

III. WEAK DECAY MODES

We show in Fig. 4 the weak decay mechanisms that will be considered in this work. We will discuss separately the two cases corresponding to the initial pineut having spin 0 (even number of neutrons) or spin 1/2 (odd number of neutrons).

A. Initial pineut with spin 0

Following the rules of Ref. [19] the transition rate for any of the processes depicted in Fig. 4 in the case when the initial pineut has spin 0 is written as

$$d\omega=\frac{m_2m_3}{2M_AE_1(2\pi)^3}\sum_{\text{spin}}|M|^2\frac{\vec{k}_1^2|\vec{k}_3|d|\vec{k}_1|d\cos\theta}{|\vec{k}_3|+|\vec{k}_1|\cos\theta+E_2}, \qquad (29)$$

where

$$E_i=\sqrt{\vec{k}_i^2+m_i^2}, \qquad (30)$$

with m_i the mass of the particle with momentum $\vec{k}_i$ in Fig. 4 (notice that the mass m_3 of the antineutrino will be canceled by a similar mass in the spinor of this particle) and where

$$\cos\theta=\frac{\vec{k}_1\cdot\vec{k}_3}{|\vec{k}_1||\vec{k}_3|}. \qquad (31)$$

The simplest decay mechanism that one can think of is that in which one of the negative pions inside the pineut decays into a lepton-antilepton pair as shown in Figs. 4(a) and 4(b). The process 4(a) is the dominant decay mechanism when the pion is free and it determines its lifetime of 2.6 $\times10^{-8}$ sec. This process is not allowed for a pineut which is in its ground state due to energy conservation as can be seen from Table I. Process 4(a), however, could in principle still go on if the decaying pion were in an excited state (which we will not study here). The invariant amplitude M for this process is given by

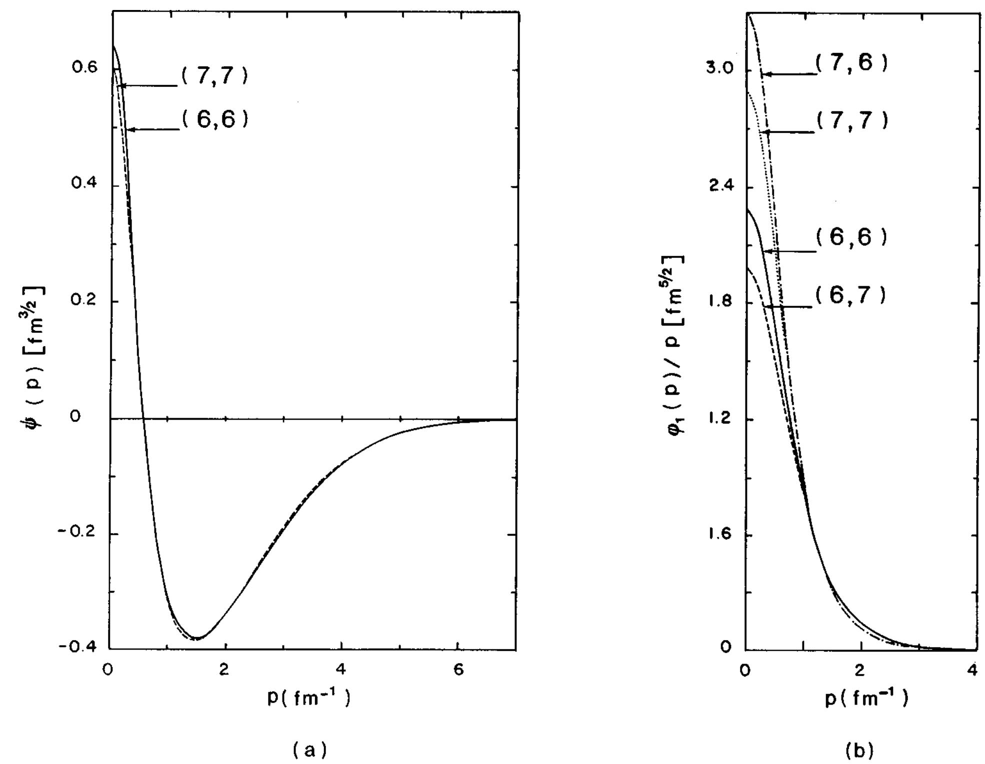

FIG. 3. Pineut wave functions: (a) pionic wave function $\psi(p)$ for (N,Z) = (6,6) and (7,7), and (b) neutronic wave function $\phi_1(p)/p$ for (N,Z) = (6,6), (6,7), (7,6), and (7,7).

$$M_{4(\mathrm{b})}=ZG_F f_\pi \overline{u}_2 \not{k}_\pi(1-\gamma_5)v_3\frac{1}{k_\pi^2-m_\pi^2}V_{A\pi B}(k_\pi^2), \tag{32}$$

where G_F is the Fermi constant, f_π is the pion decay constant, and $V_{A\pi B}$ is the $A\pi B$ vertex defined by Eq. (19). Equation (32) resembles single pion exchange in beta neutron decay [20].

It has been suggested by Lipkin [21] that the electron-decay mode which is helicity suppressed in the case of a free pion may be greatly enhanced in the case of a pineut since in

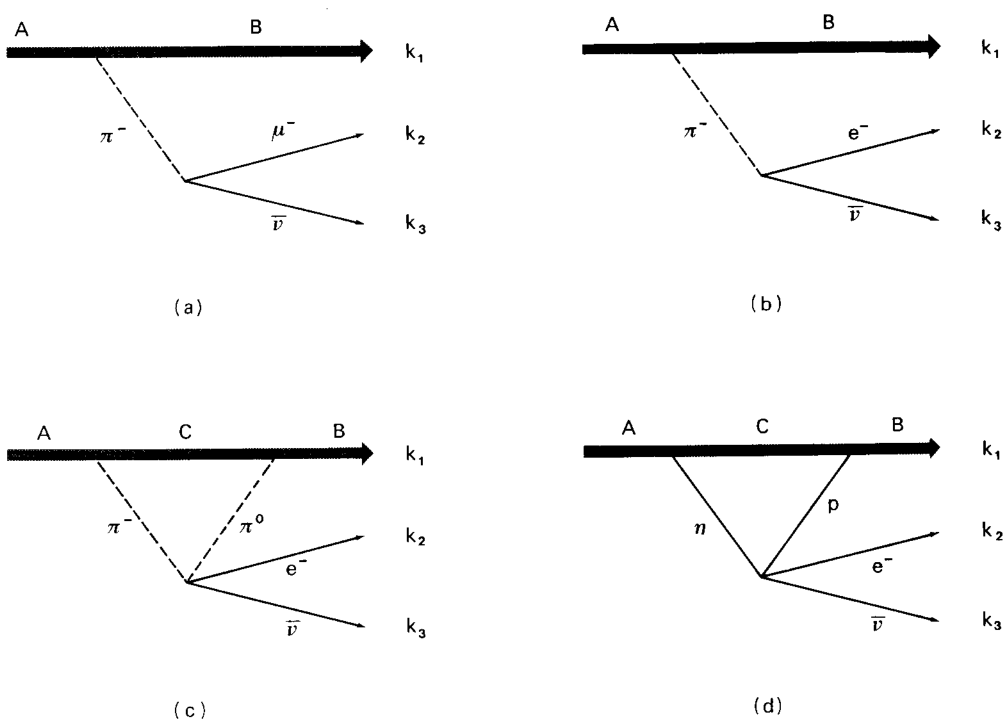

FIG. 4. Weak decay mechanisms of a pineut: (a) and (b) first-order processes and (c) and (d) second-order processes.

that case angular momentum can be transfered to the surrounding nucleons. Therefore, following the suggestion of Lipkin we have also considered the processes depicted in Figs. 4(c) and 4(d). The amplitude of diagram 4(c) is

$$M_{4(c)}=\frac{Z}{(2\pi)^4}\int d^4k_C\frac{1}{k_C^2-M_C^2+i\epsilon}V_{B\pi C}(k_{\pi^0}^2)\times\frac{1}{k_{\pi^0}^2-m_{\pi^0}^2+i\epsilon}F_{\pi^-\to\pi^0+\overline{\nu}+e^-}\frac{1}{k_{\pi^-}^2-m_{\pi^-}^2+i\epsilon}\times V_{A\pi C}(k_{\pi^-}^2), \quad (33)$$

where according to the CVC hypothesis, the elementary decay amplitude $F_{\pi^-\to\pi^0+\overline{\nu}+e^-}$ is given by

$$F_{\pi^-\to\pi^0+\overline{\nu}+e^-}=\frac{G_Ff}{\sqrt{2}}\overline{u}_2(\not{k}_{\pi^-}+\not{k}_{\pi^0})(1-\gamma_5)v_3, \quad (34)$$

with $f=\sqrt{2}\cos\theta_C=1.37$ in order to describe the free decay rate (here θ_C is the Cabibbo angle) [20]. We will evaluate this expression using the spectator-on-mass-shell approximation [22]. The spectator particle in this case is the pineut C and we can write the propagator of this particle as

$$\frac{1}{k_C^2-M_C^2+i\epsilon}=\frac{1}{(k_{C0}-E_C+i\epsilon)(k_{C0}+E_C-i\epsilon)}, \quad (35)$$

where k_{C0} is the fourth component of k_C and

$$E_C=\sqrt{\vec{k}_C^2+M_C^2}. \quad (36)$$

In order to integrate over k_{C0} we close the contour from below in the complex k_{C0} plane so that the propagator (35) will contribute with the pole at $k_{C0}=E_C-i\epsilon$. Thus, if we neglect the contribution of any other pole, Eq. (33) takes the form

$$M_{4(c)}=\frac{Z}{(2\pi)^3}\int\frac{d\vec{k}_C}{2E_C}V_{B\pi C}(k_{\pi^0}^2)\frac{1}{k_{\pi^0}^2-m_{\pi^0}^2+i\epsilon}\times F_{\pi^-\to\pi^0+\overline{\nu}+e^-}\frac{1}{k_{\pi^-}^2-m_{\pi^-}^2+i\epsilon}V_{A\pi C}(k_{\pi^-}^2). \quad (37)$$

Notice that in Eq. (37) particle C has now been put on the mass shell so that the vertex $V_{A\pi C}$ is given by Eq. (19) where both pineuts are on the mass shell. The two denominators in Eq. (37) never vanish so that we may as well drop the $i\epsilon$ there. Since the pineuts that appear in Fig. 4(c) are $A=(N,Z)$, $C=(N,Z-1)$, and $B=(N,Z-1)$, we will assume $B=C$. The vertex $V_{B\pi C}$ will be taken of the same form as given by Eqs. (19)–(22) except that we have here $M_C=M_B$ and we must include the isospin Clebsch-Gordan coefficient $C_{0,-I+1,-I+1}^{1,I-1,I-1}$ with $I=N/2+Z$. For the vertex $V_{A\pi C}$ the corresponding Clebsch-Gordan coefficient $C_{-1,-I+1,-I}^{1,I-1,I}$ is equal to 1.

In order to evaluate diagram 4(d) we will assume that pineuts A and B have spin 0 and pineut C has spin 1/2. Thus, the amplitude of diagram 4(d) is given by

$$M_{4(d)}=\frac{N_p}{(2\pi)^4}\int d^4k_C\frac{\not{k}_C+M_C}{k_C^2-M_C^2+i\epsilon}\times V_{BpC}(k_p^2)\frac{\not{k}_p+M_p}{k_p^2-M_p^2+i\epsilon}F_{n\to p+\overline{\nu}+e^-}\times\frac{\not{k}_n+M_n}{k_n^2-M_n^2+i\epsilon}V_{AnC}(k_n^2), \quad (38)$$

where the elementary decay amplitude $F_{n\to p+\overline{\nu}+e^-}$ is given by

$$F_{n\to p+\overline{\nu}+e^-}=\frac{G_F}{\sqrt{2}}\gamma^\mu(1-\alpha\gamma_5)\overline{u}_2\gamma_\mu(1-\gamma_5)v_3, \quad (39)$$

with $\alpha=1.21$ in order to describe the free decay rate (where α measures the rate of the axial-vector coupling to vector coupling) [20]. If we now apply the spectator-on-mass-shell approximation [22], Eq. (38) becomes

$$M_{4(d)}=\frac{N_p}{(2\pi)^3}\int\frac{M_C}{E_C}d\vec{k}_C\overline{v}_CV_{BpC}(k_p^2)\frac{\not{k}_p+M_p}{k_p^2-M_p^2+i\epsilon}\times F_{n\to p+\overline{\nu}+e^-}\frac{\not{k}_n+M_n}{k_n^2-M_n^2+i\epsilon}V_{AnC}(k_n^2)v_C, \quad (40)$$

where $v_C=i\gamma_2u_C^*$ is a charge conjugate spinor [19]. The vertex V_{AnC} is given by Eq. (24) where both pineuts are on the mass shell. The pineuts that appear in Fig. 4(d) are $A=(N,Z)$, $C=(N-1,Z)$, and $B=(N,Z-1)$. The vertex V_{BpC} will be taken of the same form as given by Eq. (24) except that here we must include the isospin Clebsch-Gordan coefficient $C_{1/2,-I+1/2,-I+1}^{1/2,I-1/2,I-1}$ with $I=N/2+Z$. For the vertex V_{AnC} the corresponding Clebsch-Gordan coefficient $C_{-1/2,-I+1/2,-I}^{1/2,I-1/2,I}$ is equal to 1.

B. Initial pineut with spin 1/2

If the initial pineut has spin 1/2 the transition rate is written as

$$d\omega=\frac{m_1m_2m_3}{E_1(2\pi)^3}\sum_{\text{spin}}|M|^2\frac{\vec{k}_1^2|\vec{k}_3|d|\vec{k}_1|d\cos\theta}{|\vec{k}_3|+|\vec{k}_1|\cos\theta+E_2}, \quad (41)$$

with E_i and $\cos\theta$ defined by Eqs. (30) and (31).

The amplitude for the decay process depicted in Figs. 4(a) and 4(b) is given by

$$M_{4(b)}=ZG_Ff_\pi\overline{u}_2\not{k}_\pi(1-\gamma_5)v_3\frac{1}{k_\pi^2-m_\pi^2}\overline{u}_BU_{A\pi B}(k_\pi^2)u_A, \quad (42)$$

where $U_{A\pi B}$ is the $A\pi B$ vertex defined by Eq. (23).

Following similar arguments as those given in the previous subsection, the amplitude of the process 4(c) in the spectator-on-mass-shell approximation is given by

TABLE II. Decay rate ω in sec^{-1} of the various processes depicted in Figs. 4(b), 4(c), and 4(d) for pineuts with different values of (N,Z).

(N,Z)	$\omega(\text{sec}^{-1})$ 4(b)	4(c)	4(d)
(6,6)	1.60×10^2	1.88×10^5	1.45×10^7
(6,7)	1.93×10^2	2.29×10^5	1.33×10^7
(7,6)	1.50×10^2	2.10×10^5	3.08×10^7
(7,7)	1.82×10^2	2.55×10^5	2.83×10^7

$$M_{4(c)}=\frac{Z}{(2\pi)^3}\int \frac{M_C}{E_C}d\vec{k}_C\overline{u}_B U_{B\pi C}(k_{\pi^0}^2)u_C\frac{1}{k_{\pi^0}^2-m_{\pi^0}^2+i\epsilon}$$
$$\times F_{\pi^-\to\pi^0+\overline{\nu}+e^-}\frac{1}{k_{\pi^-}^2-m_{\pi^-}^2+i\epsilon}$$
$$\times\overline{u}_C U_{A\pi C}(k_{\pi^-}^2)u_A, \tag{43}$$

with $U_{A\pi C}(k_{\pi^-}^2)$ the $A\pi C$ vertex defined by Eq. (23).

Finally, the amplitude of the process depicted in Fig. 4(d) is given within the spectator-on-mass-shell approximation as

$$M_{4(d)}=\frac{N_p}{(2\pi)^3}\int \frac{d\vec{k}_C}{2E_C}\overline{u}_B U_{BpC}(k_p^2)\frac{\not{k}_p+M_p}{k_p^2-M_p^2+i\epsilon}$$
$$\times F_{n\to p+\overline{\nu}+e^-}\frac{\not{k}_n+M_n}{k_n^2-M_n^2+i\epsilon}U_{AnC}(k_n^2)u_A, \tag{44}$$

where $U_{AnC}(k_n^2)$ is the AnC vertex defined by Eq. (28).

IV. RESULTS AND CONCLUSIONS

We evaluated numerically the decay amplitudes derived in the previous section and calculated the total decay rate by integrating over $d|\vec{k}_1|$ and $d\cos\theta$ in Eqs. (29) and (41). We show in Table II our results for the decay rates of pineuts with $N=6,7$ and $Z=6,7$ considering the different decay mechanisms depicted in Fig. 4.

As shown in Table II, the direct decay mechanism depicted in Fig. 4(b) has the smallest decay rate. The process 4(c) with an intermediate pion decay has a decay rate which is about three orders of magnitude larger than the direct decay process. However, the process 4(d) with an intermediate neutron decay is the dominant one since it is about two orders of magnitude larger than the process 4(c). Thus, the process depicted in Fig. 4(d) will be the one that determines the lifetime of the pineut. Using the decay rates of the fourth column of Table II we see that the lifetime of these pineuts will go from $\tau=3.24\times10^{-8}$ sec for $(N,Z)=(7,6)$ up to $\tau=6.92\times10^{-8}$ sec for $(N,Z)=(6,7)$.

The results of Table II also confirm the suggestion of Lipkin [21] that processes involving more than one hadron will dominate over the direct decay process and will lead to an enhancement of the decay rate.

Notice that processes where the final pineut B is left in an excited state (which in principle are allowed by our single-particle model) are expected to lead to smaller decay rates since in that case the kinetic energy of the three final particles is smaller and consequently the available phase space will be reduced.

Finally, we would like to say a few words about the process 4(d) when the decaying neutron is in the S shell in Fig. 1. In that case we do not know the wave functions and binding energies of the intermediate pineut C which will be in an excited state. However, we can estimate those processes if we calculate them using the same binding energy as for the ground state together with the S-state wave function given by Eq. (13) (and appropriate vertices). We obtain that the decay rate is of the form $\omega_S\sim(N_S/N_P)^2\omega_P$, where ω_P are the decay rates of Table II so that the decay rates ω_S will be rougly between 4 and 6 times smaller than ω_P.

In summary we have presented calculations for the pineuts lifetime, considering various weak decay modes, and confirmed the estimated enhancement suggested by Lipkin [21]. The pineuts lifetimes range from 1.24 to 2.8 times the free pion lifetime. Our calculations are based on the theoretical model of Refs. [1,2] and the assumption of a single-particle model. It is interesting that the lifetime is of the order of the pion lifetime but proceeds from a diagram where a neutron is decaying.

ACKNOWLEDGMENT

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