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Pion-nucleon scattering at low energies

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We study pion-nucleon scattering at tree level with a chiral Lagrangian of pions, nucleons, and Δ isobars using a K-matrix unitarization procedure. Evaluating the scattering amplitude to order Q^2, where Q is a generic small momentum scale, we obtain a good fit to the experimental phase shifts for pion center-of-mass kinetic energies up to 50 MeV. The fit can be extended to 150 MeV when we include the order-Q^3 contributions. Our results are independent of the off-shell Δ parameter.
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Pion-nucleon (πN) scattering is a fundamental hadronic process for which a large amount of data is available and it is important to understand this as completely as possible. Several relativistic models [1–5] exist which provide reasonably good fits to the experimental phase shifts. These models consider the N, π, and Δ-resonance fields, the isoscalar-scalar ϕ (in some cases implictly via a power series expansion), the ρ meson and sometimes higher resonances, although these play a minor role. Our interest here is to examine whether a model which contains the minimal number of fields, namely the N, π, and Δ, can yield equally good fits. Thus we effectively integrate out any other fields. For example, provided the center-of-mass (c.m.) energy is not too high, we can expand the ρ propagator as $(m_\rho^2-t)^{-1}=m_\rho^{-2}(1+t/m_\rho^2+\cdots)$, where the Mandelstam variable $t=(q-q')^2$ and q and q' are the initial and final pion c.m. four momenta. The series of terms can be absorbed into contact interactions in the Lagrangian and it is clearly important to employ the most general set of such contact interactions which is consistent with the symmetries of quantum chromodynamics.

While the Δ degree of freedom plays an important role in πN scattering, the Z parameter that specifies the form of the $\pi N\Delta$ vertex has been controversial, see the discussion of Benmerrouche *et al.* [6]. Most of the papers cited above fit the Z parameter to the πN data. This is unsatisfactory since, as we showed recently [7], the scattering is independent of Z if the Lagrangian contains the most general set of contact terms (we demonstrate this explicitly below). Thus results which depend on Z indicate that the contact terms have been implicitly constrained, whereas it is clearly preferable to employ a general Lagrangian and allow the data itself to impose constraints.

We would like to employ a Lagrangian which explicitly embodies chiral symmetry since this is known to be a fundamental symmetry at low energies. Such an approach was first taken by Peccei [8] to calculate the scattering lengths and this paper represents a modern extension of his work to study the phase shift data. In order to systematically enumerate the Lagrangian we can be guided by Weinberg's power counting arguments [9]. For this purpose we identify a generic small-momentum scale Q. This is of the order of the pion three-momentum or the pion mass and therefore much smaller than the scale of the nucleon or the Δ mass. Then according to the power counting, a Feynman tree diagram without loops contributes to πN scattering at order Q^ν with

$$\nu=1+\sum_i V_i(d_i+\tfrac{1}{2}n_i-2), \tag{1}$$

where V_i is the number of vertices of type i characterized by n_i baryon fields and d_i pion derivatives or m_π factors. This suggests that we associate $d_i+\frac{1}{2}n_i$ powers of Q to a term of type i in the Lagrangian [10]. Also, Krause [11] argues that $i\not{\mathcal{D}}-M$ is of $O(Q)$, as is a single factor of γ_5 [note $\gamma_\mu\gamma_5$ is of $O(1)$]. Although we naively count γ_5 as $O(Q)$ for organizing the Lagrangian, we shall show later that this counting is not precise. Chiral symmetry [$\mathrm{SU}(2)\otimes\mathrm{SU}(2)$], Lorentz invariance, and parity constrain the possible πN interactions and these can be found in Ref. [12]. For interactions involving the Δ isobar we use the notation of our previous paper [7] and follow the discussion therein. We write the Lagrangian up to quartic order as the sum of order Q^2, Q^3, and Q^4 parts: $\mathcal{L}=\mathcal{L}_2+\mathcal{L}_3+\mathcal{L}_4$.

The order Q^2 part of the Lagrangian is

$$\begin{aligned}\mathcal{L}_2=&\overline{N}(i\not{\mathcal{D}}+g_{\rm A}\gamma^\mu\gamma_5 a_\mu-M)N+\tfrac{1}{4}f_\pi^2\mathrm{tr}(\partial_\mu U^\dagger\partial^\mu U)\\&+\tfrac{1}{4}m_\pi^2 f_\pi^2\mathrm{tr}(U+U^\dagger-2)+\overline{\Delta_\mu^a}\Lambda_{ab}^{\mu\nu}\Delta_\nu^b+h_{\rm A}(\overline{\boldsymbol{\Delta}_\mu}\cdot\boldsymbol{a}_\nu\Theta^{\mu\nu}N\\&+\overline{N}\Theta^{\mu\nu}\boldsymbol{a}_\mu\cdot\boldsymbol{\Delta}_\nu)+\widetilde{h}_{\rm A}\overline{\Delta_\mu^a}\not{a}\gamma_5\Delta_a^\mu.\end{aligned} \tag{2}$$

where the pion field arises in $U(x)=\exp(2i\pi(x)/f_\pi)$ with $\pi\equiv\frac{1}{2}\boldsymbol{\pi}\cdot\boldsymbol{\tau}$ and the axial vector field $a_\mu=\partial_\mu\pi/f_\pi+\cdots$, while the vector field $v_\mu=-\frac{1}{2}i[\pi,\partial_\mu\pi]/f_\pi^2+\cdots$. The trace is taken over the isospin matrices and the covariant derivative of the nucleon field is $\mathcal{D}_\mu N=\partial_\mu N+iv_\mu N$. As regards the Δ, the kernel tensor in the kinetic-energy term is

$$\begin{aligned}\Lambda^{\mu\nu}=&-(i\not{\mathcal{D}}-M_\Delta)g^{\mu\nu}+i(\gamma^\mu\mathcal{D}^\nu+\gamma^\nu\mathcal{D}^\mu)\\&-\gamma^\mu(i\not{\mathcal{D}}+M_\Delta)\gamma^\nu.\end{aligned} \tag{3}$$

Here we have chosen the standard parameter $A=-1$, because it can be modified by redefinition of the Δ field with no physical consequences [13]. The covariant derivative is

$$\mathcal{D}_\mu\boldsymbol{\Delta}_\nu=\partial_\mu\boldsymbol{\Delta}_\nu+iv_\mu\boldsymbol{\Delta}_\nu-\boldsymbol{v}_\mu\times\boldsymbol{\Delta}_\nu, \tag{4}$$

in which $\mathbf{\Delta}_\mu = \mathbf{T}\Delta_\mu$, with T^a the standard 2×4 isospin $\frac{3}{2}$ to $\frac{1}{2}$ transition matrices. The off-shell Z parameter appears in $\Theta_{\mu\nu} = g_{\mu\nu} - (Z+\frac{1}{2})\gamma_\mu\gamma_\nu$. We have simplified the $\pi\Delta\Delta$ interaction in Eq. (2) by choosing the physically irrelevant parameters $Z_2 = -\frac{1}{2}$ and $Z_3 = 0$ (see Ref. [7]); this term does not contribute to the scattering amplitude at tree level.

The order Q^3 part of $\mathcal{L}$ is

$$\mathcal{L}_3 = \frac{\beta_\pi}{M}\overline{N}N\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) - \frac{\kappa_\pi}{M}\overline{N}v_{\mu\nu}\sigma^{\mu\nu}N + \frac{\kappa_1}{2M^2}i\overline{N}\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu N\mathrm{tr}(a^\mu a^\nu) + \frac{\kappa_2}{M}m_\pi^2\overline{N}N\ \mathrm{tr}(U+U^\dagger-2) + \cdots, \tag{5}$$

where the dots represent terms that do not contribute to the πN scattering amplitude up to $O(Q^3)$ and we have defined

$$\overleftrightarrow{\mathcal{D}}_\mu = \mathcal{D}_\mu - (\overleftarrow{\partial}_\mu - i v_\mu), \tag{6}$$

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + i[v_\mu, v_\nu]. \tag{7}$$

We have also applied naive dimensional analysis [14] to factor out the dimensional factors so that the parameters are expected to be of order unity.

Finally, the order Q^4 part of $\mathcal{L}$ is

$$\mathcal{L}_4 = \frac{\lambda_1}{M}m_\pi^2\overline{N}\gamma_5(U-U^\dagger)N + \frac{\lambda_2}{M^2}\overline{N}\gamma^\mu D^\nu v_{\mu\nu}\ N + \frac{\lambda_3}{M^2}m_\pi^2\overline{N}\gamma_\mu[a^\mu, U-U^\dagger]N + \frac{\lambda_4}{2M^3}i\overline{N}\sigma_{\rho\mu}\overleftrightarrow{\mathcal{D}}_\nu\ N\ \mathrm{tr}(a^\rho D^\mu a^\nu) + \frac{\lambda_5}{16M^4}i\overline{N}\gamma_\rho\{\overleftrightarrow{\mathcal{D}}_\mu, \overleftrightarrow{\mathcal{D}}_\nu\}\tau^a N\ \mathrm{tr}(\tau^a[D^\rho a^\mu, a^\nu]) + \cdots, \tag{8}$$

where the braces denote an anticommutator and

$$D_\mu a_\nu = \partial_\mu a_\nu + i[v_\mu, a_\nu], \quad D^\sigma v_{\mu\nu} = \partial^\sigma v_{\mu\nu} + i[v^\sigma, v_{\mu\nu}]. \tag{9}$$

Again the dots represent terms that do not contribute to the πN scattering amplitude up to $O(Q^3)$, such terms include the usual fourth-order pion Lagrangian.

Using the pion and nucleon equations of motion [9,15,16], we have simplified the contact terms listed in Ref. [12]. For example, we reduce the $O(Q^3)$ term $\overline{N}\overleftrightarrow{\mathcal{D}}_\mu\overleftrightarrow{\mathcal{D}}_\nu N\ \mathrm{tr}(a^\mu a^\nu)$ to the sum of the $O(Q^3)$ κ_1 term, the $O(Q^4)$ λ_4 term, and higher-order terms which we omit. As a result we have the minimum number of independent terms contributing to the πN scattering amplitude up to $O(Q^3)$. As we have remarked, the isoscalar-scalar ϕ and isovector-vector ρ fields as given in Ref. [10] have been integrated out. Their effects show up in the contact terms β_π, κ_2 and λ_2. For example, in terms of the $\rho\pi\pi$ coupling ($g_{\rho\pi\pi}$) and the ρNN coupling (g_ρ), the rho gives a contribution to the λ_2 parameter of $-2g_{\rho\pi\pi}g_\rho M^2 f_\pi^2/m_\rho^4$.

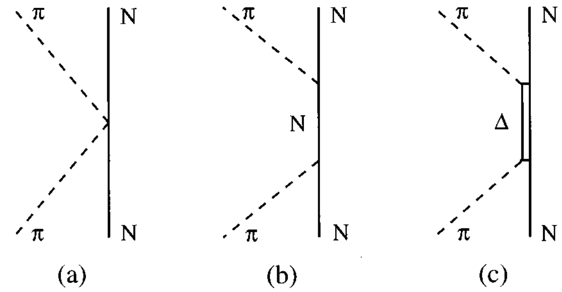

FIG. 1. Tree Feynman diagrams for πN scattering: (a) contact terms, (b) nucleon exchange, (c) Δ exchange. Crossed diagrams are not shown.

In Fig. 1 we show the tree Feynman diagrams for πN scattering. The crossed diagrams for Figs. 1(b) and 1(c) are suppressed. The Lagrangian $\mathcal{L}_2$ gives contributions to the T matrix of $O(Q)$ from all three diagrams; note that the contact diagram is due to the Weinberg term $-\overline{N}\gamma^\mu v_\mu N$. The interactions in $\mathcal{L}_3$ and $\mathcal{L}_4$ (except for the λ_1 term) give further contributions to Fig. 1(a) of order Q^2 and Q^3, respectively. In Fig. 1(b), each vertex can be either a pseudovector g_A vertex or a symmetry-breaking λ_1 vertex. As mentioned earlier, the appearance of γ_5 renders a λ_1 vertex of higher order than expected from the chiral counting of Eq. (1) so we have included it in $\mathcal{L}_4$. The reason for this extra power of Q results from the following relation:

$$\overline{u(p')}\gamma_5\frac{1}{\not{p}+\not{q}-M}\gamma_5 u(p) = -\frac{\overline{u(p')}\not{q}u(p)}{(p+q)^2-M^2}, \tag{10}$$

where $u(p)$ is the positive-energy free Dirac spinor. Thus, with one λ_1 and one g_A vertex, Fig. 1(b) is of $O(Q^3)$; we include this contribution. With both vertices of λ_1 type the result is of $O(Q^4)$, whereas, associating an extra factor of Q with each γ_5 as suggested by Ref. [11] and Eq. (8), we would expect $O(Q^5)$.

We follow the standard notation of Höhler [17] and Ericson and Weise [18] to write the T matrix as

$$T_{ba} \equiv \langle \pi_b|T|\pi_a\rangle = T^+\delta_{ab} + \tfrac{1}{2}[\tau_b, \tau_a]T^-, \tag{11}$$

where the isospin symmetric and antisymmetric amplitudes are

$$T^\pm = A^\pm + \tfrac{1}{2}(\not{q}+\not{q}')B^\pm. \tag{12}$$

Here $A^\pm$ and $B^\pm$ are functions of the Mandelstam invariant variables $s=(p+q)^2$, t, and $u=(p-q')^2$, where p is the initial nucleon c.m. momentum. They are given by the sum of the contributions from the contact terms in Fig. 1(a), the nucleon exchange in Fig. 1(b), and the Δ exchange in Fig. 1(c). The amplitudes arising from the contact terms are

$$A_C^+ = \frac{2}{Mf_\pi^2}[\beta_\pi(2m_\pi^2 - t) - 2\kappa_2 m_\pi^2 + \lambda_4\nu^2], \tag{13}$$

$$B_C^+ = \frac{1}{Mf_\pi^2}(\kappa_1 - 2\lambda_4)\nu, \tag{14}$$

$$A_C^- = -\frac{2\kappa_\pi}{f_\pi^2}\nu, \tag{15}$$

$$B_C^- = \frac{1}{2f_\pi^2}(1+4\kappa_\pi) - \frac{1}{M^2 f_\pi^2}\left(\frac{1}{2}\lambda_2 t + 4\lambda_3 m_\pi^2 - \lambda_5 \nu^2\right), \tag{16}$$

where $\nu=(s-u)/4M$. The contributions from the nucleon and Δ exchange are well known (see Ref. [17], for example). We list these contributions in the following for completeness. The amplitudes arising from nucleon exchange are

$$A_N^+ = \frac{M}{f_\pi^2} g_A\left(g_A - 4\lambda_1 \frac{m_\pi^2}{M^2}\right), \tag{17}$$

$$B_N^+ = \frac{M}{f_\pi^2} g_A\left(g_A - 4\lambda_1 \frac{m_\pi^2}{M^2}\right)\frac{\nu}{\nu_B^2-\nu^2}, \tag{18}$$

$$A_N^- = 0, \tag{19}$$

$$B_N^- = -\frac{g_A^2}{2f_\pi^2} + \frac{M}{f_\pi^2} g_A\left(g_A - 4\lambda_1 \frac{m_\pi^2}{M^2}\right)\frac{\nu_B}{\nu_B^2-\nu^2}, \tag{20}$$

where $\nu_B=(t-2m_\pi^2)/4M$. The amplitudes arising from Δ exchange are

$$A_\Delta^+ = \frac{2h_A^2}{9Mf_\pi^2}[\alpha_1 + \tfrac{3}{2}(M_\Delta+M)t]\frac{\nu_\Delta}{\nu_\Delta^2-\nu^2} - \frac{4h_A^2}{9M_\Delta f_\pi^2} \times\left[(E_\Delta+M)(2M_\Delta-M) + \left(2+\frac{M}{2M_\Delta}\right)m_\pi^2 - (2m_\pi^2-t)Y\right], \tag{21}$$

$$B_\Delta^+ = \frac{2h_A^2}{9Mf_\pi^2}[2(E_\Delta+M)(E_\Delta-2M)+\tfrac{3}{2}t]\frac{\nu}{\nu_\Delta^2-\nu^2} - \frac{16h_A^2}{9f_\pi^2}\frac{M}{M_\Delta^2}Z^2\nu, \tag{22}$$

$$A_\Delta^- = -\frac{h_A^2}{9Mf_\pi^2}[\alpha_1+\tfrac{3}{2}(M_\Delta+M)t]\frac{\nu}{\nu_\Delta^2-\nu^2} - \frac{8Mh_A^2}{9M_\Delta f_\pi^2}Y\nu, \tag{23}$$

$$B_\Delta^- = -\frac{h_A^2}{9Mf_\pi^2}[2(E_\Delta+M)(E_\Delta-2M)+\tfrac{3}{2}t]\frac{\nu_\Delta}{\nu_\Delta^2-\nu^2} + \frac{h_A^2}{9f_\pi^2}\left\{\left(1+\frac{M}{M_\Delta}\right)^2 + \frac{8M}{M_\Delta}Y + \frac{2}{M_\Delta^2}[(2m_\pi^2-t)Z^2 - 2m_\pi^2 Z]\right\}, \tag{24}$$

where $\nu_\Delta=(2M_\Delta^2-s-u)/4M$, $E_\Delta=(M_\Delta^2+M^2-m_\pi^2)/2M_\Delta$, and

$$\alpha_1 = 2(E_\Delta+M)[M_\Delta(2E_\Delta-M)+M(E_\Delta-2M)], \tag{25}$$

$$Y(Z) = \left(2+\frac{M}{M_\Delta}\right)Z^2 + \left(1+\frac{M}{M_\Delta}\right)Z. \tag{26}$$

Notice that in agreement with Ref. [7] only the nonpole terms in the Δ-exchange diagram involve the off-shell parameter Z. Therefore these contributions can be absorbed into the parameters of the contact terms according to

$$\beta_\pi(Z) = \beta_\pi(-\tfrac{1}{2}) - \frac{h_A^2}{18}\left[4Y(Z)+\frac{M}{M_\Delta}\right]\frac{M}{M_\Delta}, \tag{27}$$

$$\kappa_\pi(Z) = \kappa_\pi(-\tfrac{1}{2}) - \frac{h_A^2}{9}\left[4Y(Z)+\frac{M}{M_\Delta}\right]\frac{M}{M_\Delta}, \tag{28}$$

$$\kappa_1(Z) = \kappa_1(-\tfrac{1}{2}) + \frac{4h_A^2}{9}(4Z^2-1)\frac{M^2}{M_\Delta^2}, \tag{29}$$

$$\lambda_2(Z) = \lambda_2(-\tfrac{1}{2}) - \frac{h_A^2}{9}(4Z^2-1)\frac{M^2}{M_\Delta^2}, \tag{30}$$

$$\lambda_3(Z) = \lambda_3(-\tfrac{1}{2}) + \frac{h_A^2}{9}\left(Z^2-Z-\frac{3}{4}\right)\frac{M^2}{M_\Delta^2}. \tag{31}$$

We shall quote parameters obtained with $Z=-\frac{1}{2}$ and the parameters for other values of Z can be obtained from Eqs. (27) to (31). We have verified this numerically.

We use the standard labeling for isospin-spin partial wave channels, namely $\alpha\equiv(l,2I,2J)$ where l is the orbital angular momentum, I is the total isospin, and $J=l\pm\frac{1}{2}$ is the total angular momentum. The elastic-scattering amplitude

$$f_\alpha = \frac{1}{|\boldsymbol{q}|}e^{i\delta_\alpha}\sin\delta_\alpha \tag{32}$$

is obtained from the amplitudes $A^\pm$ and $B^\pm$ by the usual partial wave expansion [19]. Here δ_α is the phase shift of the α partial wave.

TABLE I. Parameters from fits to the S- and P-wave phase shifts.

Fit	β_π	κ_π	κ_1	κ_2	λ_1	λ_2	λ_3	λ_4	λ_5
$O(Q^2)$	−0.1960	0.5001	0.3061	−0.9328					
$O(Q^3)$	−0.1376	0.5301	0.7431	−0.5799	0.3650	−0.3239	−0.0401	0.6334	−0.4347

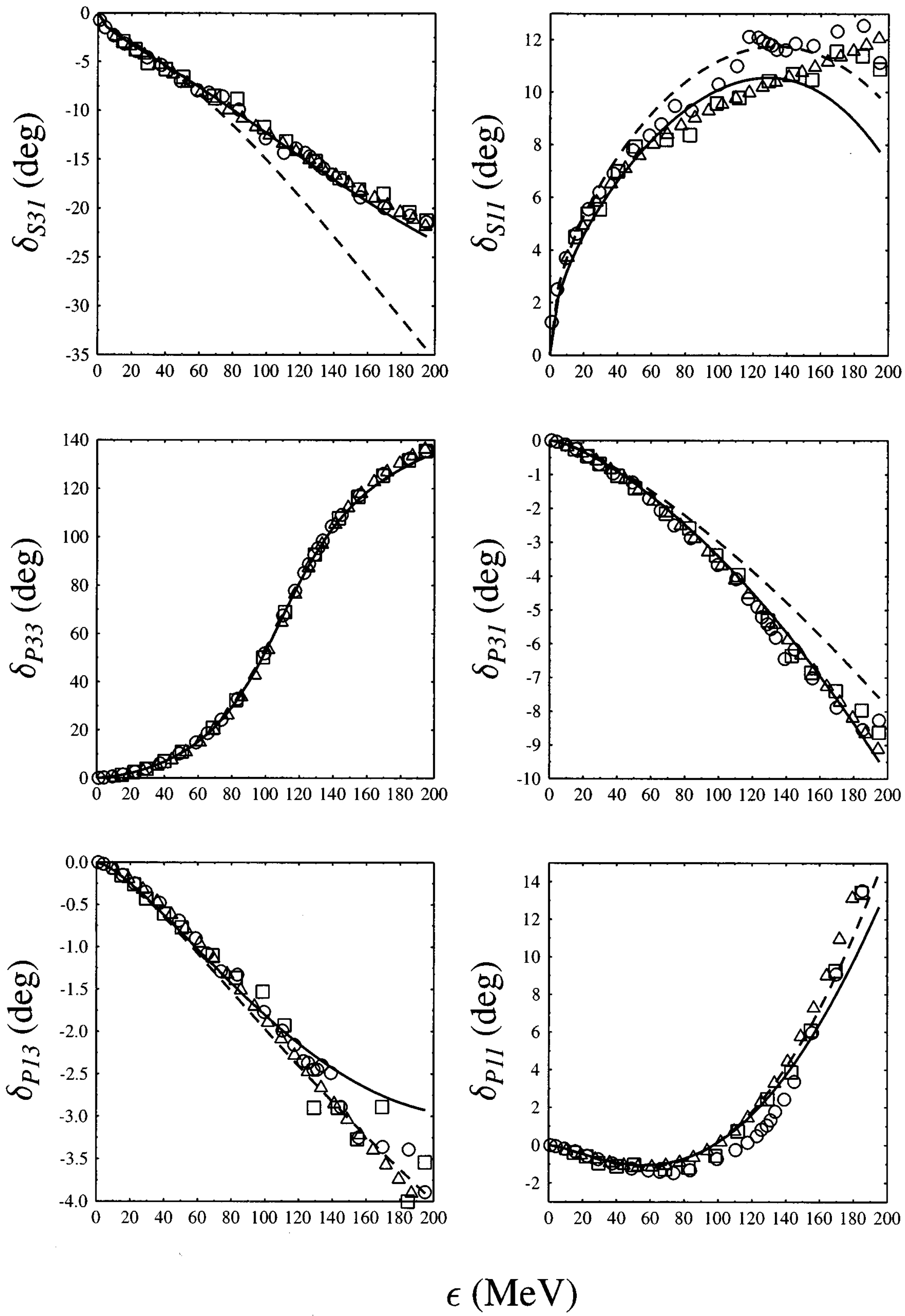

FIG. 2. The calculated S- and P-wave phase shifts as functions of the pion c.m. kinetic energy. The phase-shift data from Arndt [21] (triangles), Bugg [22] (squares), and Koch and Pietarinen [23] (circles) are also shown.

Unitarity requires f_α to take the complex structure in Eq. (32). However, f_α is real in a tree-level approximation to the scattering amplitude. We may recover unitarity by obtaining the phase shifts from two common methods. The first assumes that the calculated f_α is simply the real part of Eq. (32). The second introduces a K matrix given by [18]

$$f_\alpha = \frac{K_\alpha}{1 - i|\boldsymbol{q}|K_\alpha} \quad \text{where} \quad K_\alpha = \frac{1}{|\boldsymbol{q}|}\tan\delta_\alpha. \tag{33}$$

The calculated real tree-level amplitude f_α is then assumed to actually be K_α, which is true for $|\boldsymbol{q}|$ small enough. For sufficiently small phase shifts, the two methods yield the same answer because $\sin\delta_\alpha \approx \tan\delta_\alpha \approx \delta_\alpha$. However, near the resonance region where $\delta_\alpha \sim \pi/2$, the K-matrix method is preferred for the following simple reason. (We note that Goudsmit *et al.* [5] have proposed a justification for the K-matrix method.)

First, for energies near a resonance, the amplitude in the resonant channel takes the relativistic Breit-Wigner form. Taking the $P33$ channel as an example, we have [18]

$$|\boldsymbol{q}|f_{P33}^{\rm BW}=\frac{M_\Delta\Gamma_\Delta}{M_\Delta^2-s-iM_\Delta\Gamma_\Delta}, \tag{34}$$

where Γ_Δ is the Δ width. Equations (32) and (34) lead to

$$\tan\delta_{P33}=\frac{M_\Delta\Gamma_\Delta}{M_\Delta^2-s}. \tag{35}$$

Next, we expect that the tree-level amplitude can be obtained by setting the imaginary part of the denominator of Eq. (34) to zero:

$$|\boldsymbol{q}|f_{P33}^{\rm tree}=\frac{M_\Delta\Gamma_\Delta}{M_\Delta^2-s}, \tag{36}$$

and this is indeed obtained by retaining only the pole contribution of Eqs. (21)–(24) and using the partial wave expansion. Finally, given the tree amplitude Eq. (36), the correct phase shift of Eq. (35) is obtained by the K-matrix method. Thus, while the two methods do not differ for small phase shifts in the nonresonant channels, the K-matrix method is also good on resonance. We therefore use the K-matrix method here.

In our calculations we choose the standard values $M=939$ MeV, $M_\Delta=1232$ MeV, and $m_\pi=139$ MeV. We also take [20] $f_\pi=92.4$ MeV from charged pion decay, $g_{\rm A}=1.26$ from neutron β decay, and $h_{\rm A}=1.46$ from the Δ width, $\Gamma_\Delta=120$ MeV; allowing g_A and h_A to vary does not improve the fit. We first consider an $O(Q^2)$ approximation to the T matrix which neglects $\mathcal{L}_4$. The four parameters listed in Table I were obtained by a χ^2 fit to the data of Arndt [21] for pion c.m. kinetic energies between 10 and 150 MeV. Because negligible error bars are given in the data at low energies, we assign all the data points the same relative weight. In Fig. 2, we plot the calculated S- and P-wave phase shifts (dashed curves), along with the data to which we fit, as a function of the pion c.m. kinetic energy; we also display older data from Bugg [22] and from Koch and Pietarinen [23]. The calculation is in good agreement with the data up to 50 MeV, but beyond this energy the fit deteriorates for three of the partial waves. The value of χ^2 is unity for a relative weight of 15% which is a measure of the accuracy of the fit. The threshold (vanishing pion kinetic energy) S-wave scattering lengths (a_{2I}) and the P-wave scattering volumes (a_{2I2J}) are given in Table II. The difference between the data from Refs. [21] and [23] gives an indication of the error in the absence of a more reliable estimate. As regards theoretical predictions, apart from a_{13} which is closer to the older value [23], the $O(Q^2)$ results agree nicely with Ref. [21] which is to be expected since they are the zero-energy extrapolation of the data we have fitted.

TABLE II. The calculated S-wave scattering lengths and P-wave scattering volumes for the $O(Q^2)$ and $O(Q^3)$ fits compared with the data of Refs. [21] and [23]. The scattering lengths and volumes are in units of m_π^{-1} and m_π^{-3}, respectively.

Length/volume	$O(Q^2)$	$O(Q^3)$	Ref. [21]	Ref. [23]
a_1	0.169	0.144	0.175	0.173
a_3	−0.074	−0.087	−0.087	−0.101
a_{11}	−0.074	−0.071	−0.068	−0.081
a_{13}	−0.032	−0.031	−0.022	−0.030
a_{31}	−0.038	−0.040	−0.039	−0.045
a_{33}	0.212	0.209	0.209	0.214

We now include $\mathcal{L}_4$, which involves five additional parameters (λ_1 to λ_5), to take the tree approximation to $O(Q^3)$. The results are indicated by the solid curve in Fig. 2 which gives a good fit (with a relative weight of 8% for $\chi^2=1$) out to 150 MeV. In fact only the S_{11} and P_{13} phase shifts deviate significantly from the data in the 150–200 MeV range. Of course the rather precise agreement for δ_{P33} is strongly influenced by the phenomenological K-matrix unitarization. This forces the phase shift to be $\pi/2$ at $s=M_\Delta^2$ corresponding to a c.m. energy of 127 MeV. As regards the threshold results given in Table II, the predictions are a little closer to the data than at $O(Q^2)$ with the exception of a_1. In this connection it is instructive to examine the isoscalar and isovector S wave scattering lengths, (b_0, b_1). A recent determination [24] gave $(-0.008\pm0.007, -0.096\pm0.007)$ in units of m_π^{-1}, in substantial agreement with Refs. [21,23]; note that Arndt favors a value of b_0 consistent with zero. At $O(Q^2)$ we obtain $(0.007,-0.081)$ and at $O(Q^3)$ $(-0.010,-0.077)$. Thus the isoscalar b_0, which is zero in the chiral limit, has improved by going to $O(Q^3)$, while the magnitude of b_1 remains too small.

Apart from λ_3 which has little influence on the fit, the $O(Q^3)$ parameters listed in Table I are of order unity although Eqs. (27)–(31) show that, while the fit is independent of Z, the actual parameter values will depend on Z. The pseudoscalar coupling with parameter λ_1 allows the effective πNN coupling constant to be adjusted in the $O(Q^3)$ fit. From the Goldberger-Treiman relation, our values for $g_{\rm A}$ and f_π correspond to a πNN coupling, $g_{\pi NN}=12.8$ which is a little lower than the value of 13.1 obtained by Arndt *et al.* [25]. When the λ_1 term is included $g_{\pi NN}$ decreases slightly to 12.6. We will not comment on the sigma term since this requires extrapolation to the unphysical region which may not be reliable with this tree-level model.

With nine parameters our $O(Q^3)$ calculation deviates from the data only beyond 150 MeV c.m. energy. At the higher energies we do a little better than Goudsmit *et al.* [5] who have seven parameters and fit to 75 MeV. The calculation of Boffinger and Woolcock [2], which is an improved version of Ref. [1], contains ten parameters and produces a fit which is similar to ours but a little better at energies $\sim$200 MeV. The remaining models [3,4] have a larger number of parameters (14) and correspondingly fit to significiantly higher energies.

In conclusion, we have discussed a chiral Lagrangian involving just the basic N, π, and Δ fields, with a series of terms representing a momentum expansion. We find that a tree-level calculation with this model represents the data as well as other models with a similar number of parameters. Further we have confirmed by explicit calculation that the Z parameter of the $\pi N\Delta$ vertex is irrelevant if a sufficiently general Lagrangian is employed. Of course it would be more satisfactory if a unitary scattering amplitude emerged naturally, rather than being imposed phenomenologically. Such would be the case if loops were calculated in heavy baryon chiral perturbation theory and work in this direction is in progress.

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