Attractive central potential in the SU(3) Skyrme model

Hideyuki Abe
Institute of Physics, University of Tokyo, Komaba, Tokyo 153, Japan

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The interaction between the hyperon and the nucleon is investigated in the SU(3) Skyrme model. The static potential, which is expanded in terms of the modified SU(3) rotation matrices, is obtained for several orientations with the Atiyah-Manton ansatz. The interaction is calculated for the NN, ΛN, and ΣN systems. The medium-range attraction of the central potential between Λ and N is obtained by considering the Λ-Σ mixing through the intermediate state. [S0556-2813(97)04212-X]

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I. INTRODUCTION

It is widely accepted that quantum chromodynamics (QCD) is the fundamental theory for the strong interaction. The high-energy behavior of QCD is well described by the perturbed QCD because of its asymptotic freedom. It explains the experimental data such as those in the deep inelastic scattering process. On the other hand, it is difficult to describe the low energy properties of QCD because the effective coupling constant increases with the decreasing momentum as the renormalization group analysis suggests.

't Hooft proposed to use the inverse number of colors $1/N_{\rm C}$ as an expansion parameter by generalizing QCD to the SU($N_{\rm C}$) gauge theory [1]. In the large $N_{\rm C}$ limit, it becomes the theory of the weakly interacting meson. Witten pointed out that the baryons should appear as topological solitons [2]. The Skyrme model is recognized as an effective theory of QCD in the large $N_{\rm C}$ limit although the Skyrme Lagrangian is not derived from QCD directly [3]. The baryon number is introduced into the Skyrme model from a topological point of view. Skyrme proposed a stable configuration called the hedgehog ansatz. The quantization is performed by introducing the collective coordinates, the flavor rotation of the hedgehog configuration [4]. The Skyrme model explains the static properties of the baryon such as the charge radius and the magnetic moment with 30% accuracy.

The product ansatz is a two-skyrmion configuration proposed by Skyrme [3]. It is a good approximation so long as the two skyrmions are separated in the long distance. The numerical simulation is a direct method to obtain the exact two-baryon configuration [5]. There is another way to describe the skyrmion configuration with a few parameters. The Atiyah-Manton ansatz is constructed from the instanton configuration in the SU(2) gauge theory [6,7]. The stable configuration with the torus shape can be described by this ansatz. Even if the skyrmion configuration is obtained, there is a problem that the attraction in the central potential at the intermediate range is absent. It is being solved by considering the N-Δ mixing through the intermediate state [8,9], the finite-$N_{\rm C}$ effect [10], the higher-order terms generated by ω-meson, and the radial excitation. By diagonalizing the potential between the NN and NΔ states for each channel, one can construct the better eigenstate. It amounts to the N-Δ mixing. The finite $N_{\rm C}$ effects are often considered together with the NN-NΔ mixing. Since the Skyrme model is recognized as an effective theory in the large $N_{\rm C}$ limit, the finite $N_{\rm C}$ correction is required [11].

The Skyrme model is extended into the SU(3) flavor symmetry. There are two approaches to deal with the extra strange degrees of freedom. One is the bound state approach [12] in which the symmetry breaking is regarded as large. The K-meson is introduced as a small fluctuation from the SU(2) symmetry. Another is the collective coordinate method which is based on the SU(3) symmetry. In this method, the symmetry breaking is taken to be small. The symmetry breaking is treated perturbatively [13,14]. Yabu and Ando unified these two approaches by the exact treatment of the symmetry breaking [15]. The Yabu-Ando approach reproduces the mass splitting of the baryons in the same multiplet. In the two-baryon case, only the product ansatz has been investigated because of the complexity of the numerical simulation of the SU(3) Skyrme model [16,17].

In this paper, we investigate the interaction between the hyperon and the nucleon in the SU(3) Skyrme model. The Atiyah-Manton ansatz extended to the SU(3) symmetry is adopted as the two-baryon configuration. The static potential is expanded in the modified SU(3) rotational matrices. We obtain the interaction between the baryons by integrating the static potential with the initial and final wave functions over the Euler angles. To obtain the attractive force in the central channel of the Λ-N interaction, we take account of the ΛN-ΣN mixing through the intermediate state.

In Sec. II, we construct the two-baryon configuration by the Atiyah-Manton ansatz. In Sec. III, we express the potential in the modified SU(3) rotational matrices and obtain its matrix element between the baryons. In Sec. IV, we consider the ΛN-ΣN mixing through the intermediate state together with the finite $N_{\rm C}$ effects. In Sec. V, we discuss our results.

II. TWO-BARYON CONFIGURATION

Let us consider the nonlinear field of the pseudoscalar meson U within the flavor SU(3) symmetry. The action of the SU(3) Skyrme model is given by

$$S=\int dt(L_2+L_4+L_{\rm SB})+N_{\rm C}\Gamma, \tag{1}$$

where

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$$L_2=\int d^3x\frac{F_\pi^2}{16}\mathrm{tr}(\partial_\mu U^\dagger\partial^\mu U), \tag{2a}$$

$$L_4=\int d^3x\frac{1}{32e^2}\mathrm{tr}[U^\dagger(\partial_\mu U),U^\dagger(\partial_\nu U)]^2, \tag{2b}$$

$$L_{\mathrm{SB}}=\int d^3x\left\{\frac{F_\pi^2}{32}(m_\pi^2+m_\eta^2)\mathrm{tr}(U+U^\dagger-2)+\frac{\sqrt{3}F_\pi^2}{24}(m_\pi^2-m_{\mathrm{K}}^2)\mathrm{tr}(\lambda_8(U+U^\dagger))\right\}, \tag{2c}$$

$$\Gamma=-\frac{i}{240\pi^2}\int_Q d^5x\,\epsilon^{ijklm}\mathrm{tr}(U^\dagger(\partial_i U)U^\dagger(\partial_j U)\times U^\dagger(\partial_k U)U^\dagger(\partial_l U)U^\dagger(\partial_m U)). \tag{2d}$$

The summation over the repeated indices is assumed and λ_a denote the Gell-Mann matrices. The symmetry breaking part of the Lagrangian (2c) reproduces the mass terms expanded in the pseudoscalar meson fields with the Gell-Mann-Okubo relation $m_\pi^2+3m_\eta^2-4m_{\mathrm{K}}^2=0$. In the Wess-Zumino-Witten term (2d), the integration is taken over the five-dimensional disc Q the boundary of which is the usual spacetime. The length and the meson mass are often measured in the unit $1/(eF_\pi)$, and the energy in (F_π/e), called the Skyrme units.

The hedgehog configuration is also stable in the SU(3) Skyrme model. The quantization is done with respect to the collective coordinates expressed as the rotation of the hedgehog configuration,

$$U(x,t)=A(t)U_{\mathrm{H}}(x)A^\dagger(t). \tag{3}$$

It is difficult to construct the two-baryon configuration in the general form. The product ansatz is used as the first approximation. The product ansatz holds when the two skyrmions are separated in the long distance.

The Atiyah-Manton ansatz is another method to construct the two-baryon configuration, which has been used in the SU(2) Skyrme model. It is obtained from the instanton, a topological configuration of the gauge field defined in the Euclidean spacetime,

$$U_{\mathrm{AM}}(x)=P\ \exp\left(-\int_{-\infty}^{\infty}dtA_4(x,t)\right). \tag{4}$$

The instanton configuration given by 't Hooft [18] is expressed as

$$A_4=\frac{i}{2}\tau\cdot\partial\ln\rho, \tag{5}$$

$$\rho=1+\frac{\lambda_1^2}{(t-T_1)^2+(x-X_1)^2}+\frac{\lambda_2^2}{(t-T_2)^2+(x-X_2)^2}, \tag{6}$$

where (T_i,X_i) and λ_i are the instanton coordinate and the spreading of the ith instanton, respectively. Jackiw-Nohl-Rebbi (JNR) proposed the more general form of the instanton configuration [19]. The two-instanton superpotential is expressed as

$$\rho=\frac{\lambda_1^2}{(t-T_1)^2+(x-X_1)^2}+\frac{\lambda_2^2}{(t-T_2)^2+(x-X_2)^2}+\frac{\lambda_3^2}{(t-T_3)^2+(x-X_3)^2}. \tag{7}$$

From the skyrmion point of view, it can describe the stable configuration with the torus shape. However, it is difficult to apply the JNR form to the SU(3) Skyrme model because of the complex relation between the instanton parameters (T_i,X_i) and the skyrmion position. Therefore, we concentrate our efforts on the 't Hooft form.

To apply the above method to our problem, we extend the Atiyah-Manton ansatz to the SU(3) symmetry. We can change Eq. (5) into the form

$$A_4=-\frac{i}{2}(\tau_1\cdot\partial_1+\tau_2\cdot\partial_2)\ln\rho, \tag{8}$$

because the differentiation with respect to the spatial variables can be separated into that with the instanton coordinates. Now, we extend the gauge group from SU(2) to SU(3) by replacing the SU(2) τ-matrices with the generators of the SU(3) group $\tau_{1a}=A\lambda_a A^\dagger$ and $\tau_{2a}=B\lambda_a B^\dagger$ different for each instanton coordinate,

$$A_4=-i\tau_1\cdot(x-X_1)\frac{\lambda_1^2 s_2^2}{s_1^2\{(s_1^2+\lambda_1^2)(s_2^2+\lambda_2^2)-\lambda_1^2\lambda_2^2\}}-i\tau_2\cdot(x-X_2)\frac{\lambda_2^2 s_1^2}{s_2^2\{(s_1^2+\lambda_1^2)(s_2^2+\lambda_2^2)-\lambda_1^2\lambda_2^2\}}, \tag{9}$$

where we have used the notations

$$s_1^2=(t-T_1)^2+(x-X_1)^2, \tag{10a}$$

$$s_2^2=(t-T_2)^2+(x-X_2)^2. \tag{10b}$$

The Atiyah-Manton configuration does not have the exponential damping behavior of the massive meson in the long distance. We introduce the additional parameters for the exponential damping by the substitution in Eq. (9),

$$\lambda_i^2\to\lambda_i^2(1+\mu_i|x-X_i|)e^{-\mu_i|x-X_i|}. \tag{11}$$

This substitution improves the long-distance behavior of the Atiyah-Manton configuration for the massive case. It corresponds to the hedgehog solution with the profile function

$$F(r)=\pi\left(1-\frac{r}{\sqrt{r^2+\lambda^2(1+\mu r)e^{-\mu r}}}\right). \tag{12}$$

The long-distance behavior of the profile function leads to

$$F(r)\to\frac{\pi\lambda^2}{2r^2}(1+\mu r)e^{-\mu r}. \tag{13}$$

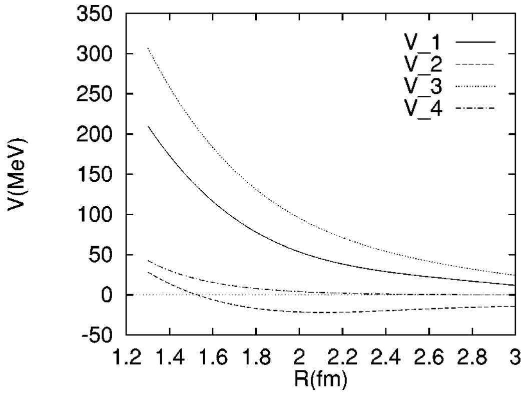

FIG. 1. Static potential in MeV as a function of the separation between the two baryons R(fm) V_1, V_2, V_3, V_4 for the relative orientation $C=1$, $e^{-(\pi/2)\lambda_2}$, $e^{-(\pi/2)\lambda_3}$, $e^{-(\pi/2)\lambda_4}$.

In spite of the above modifications, the baryon number of the Atiyah-Manton configuration is still conserved. Indeed, the baryon number is confirmed to be two within 1% discrepancy by the numerical simulation. We use the third parameter set in Ref. [15] (F_π=82.9 MeV, e=4.87, m_{K}=769 MeV) throughout this paper. The subtraction of the vacuum-like energy does not matter because we use the energy difference between the two configurations. There is ambiguity in determining the separation of the two skyrmions for the generated configuration. We adopt the separation between the two baryons as

$$R=2\left[\int d^3xB_0\left(z^2-\frac{1}{2}(x^2+y^2)\right)\right]^{1/2}, \tag{14}$$

where B_0 is the baryon number density [7]. We perform the numerical simulation by taking the orientations of the individual skyrmions as $A=\sqrt{C}^\dagger$ and $B=\sqrt{C}$ which ensures the symmetry under the exchange between the two skyrmions. We determine the instanton parameters $\lambda_{1,2},\mu_{1,2}$. From the symmetry under the exchange of the two skyrmions, we require that they should be equal for both skyrmions. It turns out that $\lambda=2.6,\mu=0.342$ for the relative orientation $C=1$ and $\lambda=2.2,\mu=1.369$ for $C=e^{-i(\pi/2)\lambda_4}$ by minimizing the static energy. These parameters give the classical mass $M=39.9$ in the Skyrme unit which is consistent with the exact value 39.849 estimated by the numerical simulation in Ref. [15]. The static potentials V_1, V_2, V_3, and V_4 for the orientations $C=1$, $\exp(-i(\pi/2)\lambda_2)$, $\exp(-i(\pi/2)\lambda_3)$, and $\exp(-i(\pi/2)\lambda_4)$, respectively are shown in Fig. 1.

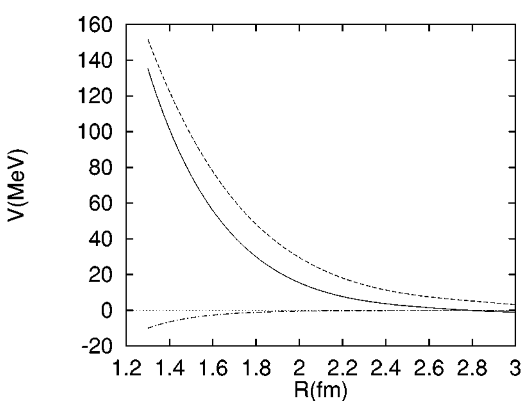

FIG. 2. Central part of the NN-potential V_{C} in MeV as a function of the separation R(fm), dashed curve denotes the product ansatz and dot-dashed curve denotes the one-boson exchange model.

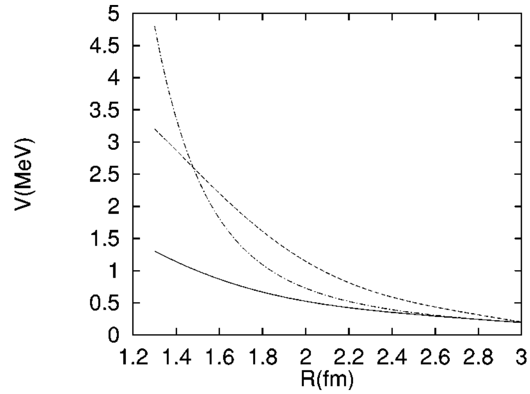

FIG. 3. Spin-isospin part of the NN-potential $V_{\sigma\tau}$.

III. HYPERON-NUCLEON INTERACTION

The static potential is generally expanded in the SU(3) rotation matrices [17]. From the symmetry of the solution, the static potential is reduced to the form

$$V(R,C)=\sum_{\lambda,S_z} V^{\lambda}_{SS_z}(R)D^{\lambda}_{(0SS_z)(0SS_z)}(C), \tag{15}$$

with the condition $V^{\lambda}_{SS_z}=V^{\lambda *}_{S,-S_z}$.

In the symmetry-breaking case, it should be expanded in the modified rotation matrices rather than the nonbreaking ones because the mixing of a certain representation with its higher ones is not so small as to be neglected. The SU(3) rotation is parametrized by the SU(3) Euler angles,

$$A=ue^{-i\nu\lambda_4}e^{-i(\rho/\sqrt{3})\lambda_8}u', \tag{16}$$

where the matrices u and u' are expressed in the usual SU(2) Euler angles [20]. The wave function for the baryon belonging to the multiplet λ is given by Yabu and Ando as the modified SU(3) rotation matrix,

$$\Psi^{\lambda}_{(YII_z)(Y_{\mathrm{R}}SS_z)}(A)=\sqrt{N_\lambda}\widetilde{D}^{\lambda}_{(YII_z)(Y_{\mathrm{R}}S,-S_z)}(A), \tag{17}$$

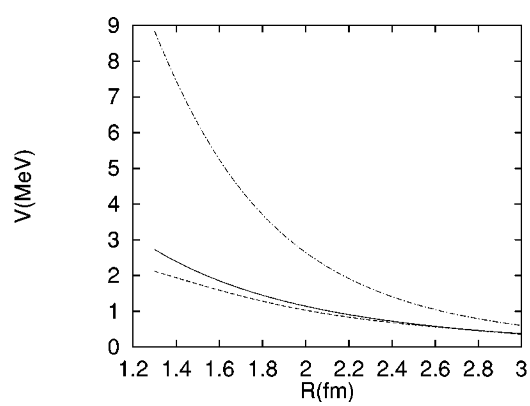

FIG. 4. Tensor-isospin part of the NN-potential $V_{\mathrm{T}\tau}$.

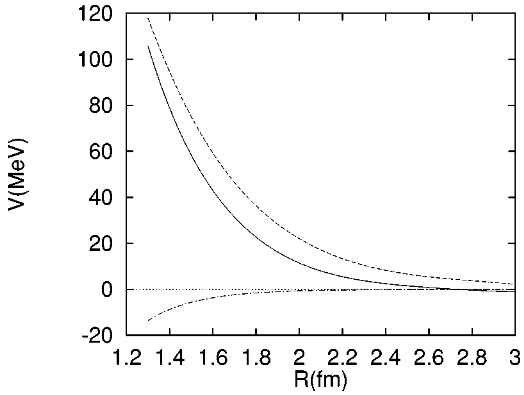

FIG. 5. Central part of the ΛN-potential V_{C}.

$$\widetilde{D}^{\lambda}_{(YII_z)(Y_{\mathrm{R}}S,-S_z)}(A)=D^{I}_{I_zM_{\mathrm{L}}}(u)f^{\lambda}_{(YIM_{\mathrm{L}})(Y_{\mathrm{R}}SM_{\mathrm{R}})}(\nu)\times e^{-i\rho Y_{\mathrm{R}}}D^{S}_{M_{\mathrm{R}},-S_z}(u'), \qquad (18)$$

where $D^{I}_{I_zM_{\mathrm{L}}}$ and $D^{S}_{M_{\mathrm{R}},-S_z}$ stand for the usual SU(2) rotation matrices and N_λ is the multiplicity of the representation λ. The properties of the symmetry breaking are contained in the strange-mixing function $f(\nu)$. A subsidiary condition derived from the Wess-Zumino term is imposed on the physical states,

$$Y_R=1. \qquad (19)$$

We can determine the coefficients $V^{\lambda}_{SS_z}$ in the static potential (15) by observing it for several relative orientations. Once the static potential is given, the interaction between the hyperon and the nucleon is obtained by integrating the static potential between the initial and the final wave functions over all orientations,

$$V^{\mathrm{YN}}(R)=\int dAdB\Psi^{*}_{Y'}(A)\Psi^{*}_{N'}(B)\times V(R,A^{\dagger}B)\Psi_{Y}(A)\Psi_{N}(B). \qquad (20)$$

where we adopt the direct product of the Yabu-Ando wave function as the two-baryon state for the first approximation. The interaction is obtained in the form

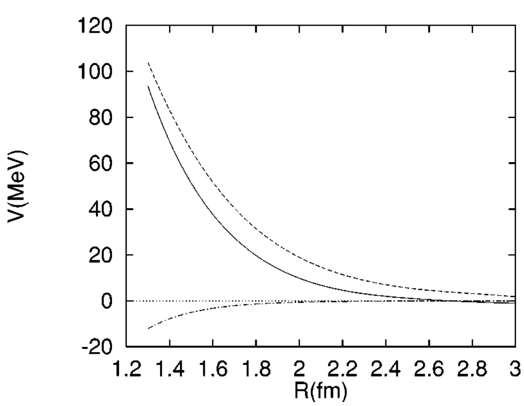

FIG. 6. Central part of the ΣN-potential V_{C}.

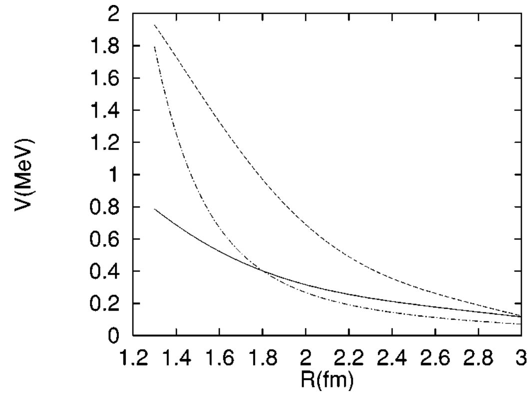

FIG. 7. Spin-isospin part of ΣN-potential $V_{\sigma\tau}$.

$$V^{\mathrm{NN}}=V^{\mathrm{NN}}_{\mathrm{C}}+V^{\mathrm{NN}}_{\tau}(\tau_1\cdot\tau_2)+V^{\mathrm{NN}}_{\sigma}(\sigma_1\cdot\sigma_2)+V^{\mathrm{NN}}_{\mathrm{T}}S_{12}+V^{\mathrm{NN}}_{\sigma\tau}(\sigma_1\cdot\sigma_2)(\tau_1\cdot\tau_2)+V^{\mathrm{NN}}_{\mathrm{T}\tau}S_{12}(\tau_1\cdot\tau_2), \qquad (21)$$

for the NN interaction,

$$V^{\Lambda\mathrm{N}}=V^{\Lambda\mathrm{N}}_{\mathrm{C}}+V^{\Lambda\mathrm{N}}_{\sigma}(\sigma_1\cdot\sigma_2)+V^{\Lambda\mathrm{N}}_{\mathrm{T}}S_{12}, \qquad (22)$$

for the ΛN interaction, and

$$V^{\Sigma\mathrm{N}}=V^{\Sigma\mathrm{N}}_{\mathrm{C}}+V^{\Sigma\mathrm{N}}_{\tau}(T_1\cdot\tau_2)+V^{\Sigma\mathrm{N}}_{\sigma}(\sigma_1\cdot\sigma_2)+V^{\Sigma\mathrm{N}}_{\mathrm{T}}S_{12}+V^{\Sigma\mathrm{N}}_{\sigma\tau}(\sigma_1\cdot\sigma_2)(T_1\cdot\tau_2)+V^{\Sigma\mathrm{N}}_{\mathrm{T}\tau}S_{12}(T_1\cdot\tau_2), \qquad (23)$$

for the ΣN interaction. Since the baryons stay in the z axis, the tensor operator is defined by $S_{12}=3\sigma_{1z}\sigma_{2z}-\sigma_1\cdot\sigma_2$. The potentials V_1, V_2, V_3, V_4 for the relative orientations determine the coefficients V^{1}_{00}, V^{8}_{00}, V^{8}_{10}, V^{8}_{11} in the static potential (15) expanded up to the octet representation. The graphs of the interaction $V^{\mathrm{NN}}_{\mathrm{C}}$, $V^{\mathrm{NN}}_{\sigma\tau}$, $V^{\mathrm{NN}}_{\mathrm{T}\tau}$, $V^{\Lambda\mathrm{N}}_{\mathrm{C}}$, $V^{\Sigma\mathrm{N}}_{\mathrm{C}}$, $V^{\Sigma\mathrm{N}}_{\sigma\tau}$, and $V^{\Sigma\mathrm{N}}_{\mathrm{T}\tau}$ are shown in Figs. 2–8. The central potential V_{C} of the NN, ΛN, ΣN systems is still repulsive, while the Atiyah-Manton ansatz tends to show the less repulsive force than the product ansatz [16,17]. For the spin-isospin part $V_{\sigma\tau}$, the results show a good agreement with the Nijmegen potential (model D), the one-boson-exchange potential developed by the Nijmegen group [21] at the range $R>1.6$ fm. The behavior of the tensor-isospin part $V_{\mathrm{T}\tau}$ is consistent with the Nijmegen model.

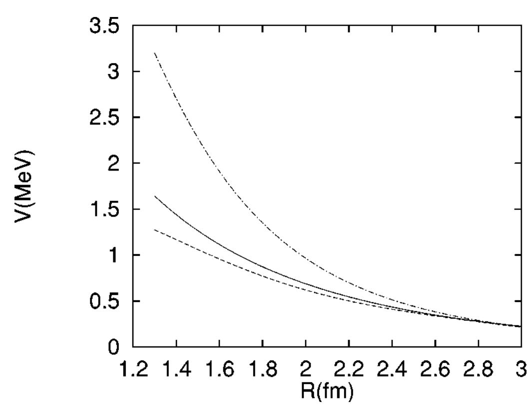

FIG. 8. Tensor-isospin part of the ΣN-potential $V_{\mathrm{T}\tau}$.

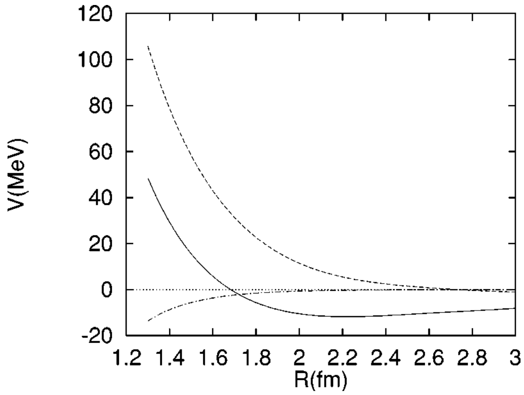

FIG. 9. Central part of the ΛN-potential V_{C} with ΛN-ΣN mixing, solid curve stands for that with the Λ-Σ mixing, dashed one for the naive estimation, and dot-dashed one for the one-boson-exchange model.

IV. ΛN-ΣN MIXING

It is well known that the naive estimation of the interaction is insufficient to give the central attraction at the intermediate range. In the SU(2) case, the Δ-N mixing is taken into account. The direct product of the wave functions as the two-baryon eigenstate becomes worse when the separation between the skyrmions decreases. The candidate for the SU(3) symmetry is the Λ-Σ mixing. We consider the ΛN-ΣN mixing in the intermediate state together with the finite N_{C} effects. It is shown by the fact that the off-diagonal element of the potential survives,

$$V^{\Lambda\mathrm{N}-\Sigma\mathrm{N}} = (V_{\tau}^{\Lambda\mathrm{N}-\Sigma\mathrm{N}} + V_{\sigma\tau}^{\Lambda\mathrm{N}-\Sigma\mathrm{N}}(\sigma_1\cdot\sigma_2) + V_{\mathrm{T}\tau}^{\Lambda\mathrm{N}-\Sigma\mathrm{N}}S_{12})O_{\mathrm{T}}, \tag{24}$$

where

$$O_{\mathrm{T}} = -\sqrt{2/3}T_{2+}\delta_{I'_{1z},1} + 2/\sqrt{3}T_{2z}\delta_{I'_{1z},0} + \sqrt{2/3}T_{2-}\delta_{I'_{1z},-1}. \tag{25}$$

It is found that the isospin is conserved under the baryon-baryon interaction. The total spin is not an invariant of the hyperon-nucleon system whereas the projection of the spin in the z direction is still conserved. The potentials with respect to the total spin and isospin states are written as

$$V_{\Lambda\Lambda} = V_{\mathrm{C}}^{\Lambda\mathrm{N}} - 3V_{\sigma}^{\Lambda\mathrm{N}}, \tag{26a}$$

$$V_{\Sigma\Sigma} = (V_{\mathrm{C}}^{\Sigma\mathrm{N}} - 2V_{\tau}^{\Sigma\mathrm{N}}) - 3(V_{\sigma}^{\Sigma\mathrm{N}} - 2V_{\sigma\tau}^{\Sigma\mathrm{N}}), \tag{26b}$$

$$V_{\Lambda\Sigma} = \frac{5}{3}(V_{\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}} - 3V_{\sigma\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}}), \tag{26c}$$

for the $I=1/2, S=0$ channel,

$$V_{\Lambda\Lambda} = V_{\mathrm{C}}^{\Lambda\mathrm{N}} + V_{\sigma}^{\Lambda\mathrm{N}} - 4V_{\mathrm{T}}^{\Lambda\mathrm{N}}, \tag{27a}$$

$$V_{\Sigma\Sigma} = (V_{\mathrm{C}}^{\Sigma\mathrm{N}} - 2V_{\tau}^{\Sigma\mathrm{N}}) + (V_{\sigma}^{\Sigma\mathrm{N}} - 2V_{\sigma\tau}^{\Sigma\mathrm{N}}) - 4(V_{\mathrm{T}}^{\Sigma\mathrm{N}} - 2V_{\mathrm{T}\tau}^{\Sigma\mathrm{N}}), \tag{27b}$$

$$V_{\Lambda\Sigma} = \frac{5}{3}(V_{\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}} + V_{\sigma\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}}) - 4V_{\mathrm{T}\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}}, \tag{27c}$$

for the $I=1/2, S=1, S_z=0$ channel, and

$$V_{\Lambda\Lambda} = V_{\mathrm{C}}^{\Lambda\mathrm{N}} + V_{\sigma}^{\Lambda\mathrm{N}} + 2V_{\mathrm{T}}^{\Lambda\mathrm{N}}, \tag{28a}$$

$$V_{\Sigma\Sigma} = (V_{\mathrm{C}}^{\Sigma\mathrm{N}} - 2V_{\tau}^{\Sigma\mathrm{N}}) + (V_{\sigma}^{\Sigma\mathrm{N}} - 2V_{\sigma\tau}^{\Sigma\mathrm{N}}) + 2(V_{\mathrm{T}}^{\Sigma\mathrm{N}} - 2V_{\mathrm{T}\tau}^{\Sigma\mathrm{N}}), \tag{28b}$$

$$V_{\Lambda\Sigma} = \frac{5}{3}(V_{\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}} + V_{\sigma\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}}) + 2(V_{\mathrm{T}\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}}), \tag{28c}$$

for the $I=1/2, S=1, S_z=\pm 1$ channel. The nonzero off-diagonal matrix element $V^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}}$ shows that the direct product of the single-baryon wave functions is not a good eigenstate for the two-baryon system. One can obtain the better two-baryon state by diagonalizing the matrix

$$\begin{pmatrix} V_{\Lambda\Lambda} & V_{\Lambda\Sigma} \\ V_{\Lambda\Sigma} & V_{\Sigma\Sigma} \end{pmatrix}$$

for each channel. After the diagonalization, the lowest eigenvalue is adopted for the ΛN central potential. The finite N_{C} effects should be taken into consideration together with the Λ-Σ mixing because the Skyrme model is recognized as an effective theory in the large N_{C} limit. The spin-isospin matrix elements are enhanced in the $N_{\mathrm{C}}=3$ case compared with those in the large N_{C} limit by the factors 20/9 for $V_{\sigma\tau}^{\Sigma\mathrm{N}}$, $V_{\mathrm{T}\tau}^{\Sigma\mathrm{N}}$ and $(20\sqrt{3})/9$ for $V_{\sigma\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}}$, $V_{\mathrm{T}\tau}^{\Lambda\mathrm{N}\text{-}\Sigma\mathrm{N}}$, from the analysis of the quark hedgehog model as in Ref. [10]. The graph of the Λ-N interaction in the central channel $V_{\mathrm{C}}^{\Lambda\mathrm{N}}$ with the ΛN-ΣN mixing is shown in Fig. 9. The attractive force at the intermediate range appears by taking account of the ΛN-ΣN mixing through the intermediate state together with the finite N_{C} effects.

V. DISCUSSION

We discuss the results in this section. The static potential is obtained from the Atiyah-Manton ansatz extended to the SU(3) symmetry. The Atiyah-Manton ansatz gives a lower energy than the product ansatz at the intermediate range. Since the symmetry breaking is not small, the static potential is expanded in the modified SU(3) rotation matrices up to the octet representation. We have obtained the baryon-baryon interaction by integrating the static potential between the two-baryon states over the Euler angles. In the naive estimation, we have not obtained the intermediate attraction of the central force although the result from the Atiyah-Manton ansatz is less repulsive than the product ansatz. To improve the estimation of the central potential, we have to take account of the several effects as in the SU(2) case. One of such effects is introduced by considering the mixing with the higher excitations, the Δ-N mixing in the SU(2) case, through the intermediate state. The candidate within the SU(3) symmetry is the mixing between Σ and Λ [22]. The effects from the mixing of these particles are expected to play a significant role in the hyperon-nucleon interaction because the mass difference between Λ and Σ, $m_{\Sigma}-m_{\Lambda}\simeq 80$ MeV, is smaller than that between N and Δ, $m_{\Delta}-m_{\mathrm{N}}\simeq 300$ MeV.

The central potential between Λ and N with the ΛN-ΣN mixing shows the attraction at the intermediate range. This result is consistent with the one-boson-exchange model. The direct product of the two single-baryon states is not a good eigenstate when the two skyrmions close together. It is suggested by the nonvanishing off-diagonal matrix element between the Λ-N and Σ-N states. By diagonalizing this matrix for each channel, one can obtain the better eigenstate of the two-baryon system. The lowest eigenvalue is adpoted for the ΛN potential. This procedure amounts to taking account of the ΛN-ΣN mixing through the intermediate state. The finite N_C effects is considered together with the Λ-Σ mixing. The finite N_C correction is estimated from the analysis of quark hedgehog model.

The spin-isospin part $V_{\sigma\tau}$ and the tensor-isospin part $V_{T\tau}$ shows a consistent behavior with the one-boson-exchange model at the range $R>1.6$ fm. This implies that the long-range force which is dominated by the π-exchange reproduces the one-boson-exchange potential well.

In the present paper, we have observed that the naive estimation of the interaction between the hyperon and the nucleon does not show the attractive central force at the intermediate range. The Atiyah-Manton ansatz is adopted to improve the medium-range behavior of the skyrmion configuration rather than the product ansatz. The ΛN-ΣN mixing in the intermediate state is taken into account. The finite N_C effects are included from the quark hedgehog model. After these treatments are taken, the hyperon-nucleon interaction shows the central attraction which is consistent with the one boson exchange model. Therefore, we conclude that the configuration with a certain accuracy, the ΛN-ΣN mixing, and the finite N_C effects are required for the attractive force in the Λ-N interaction.

Finally, we discuss the validity of the Atiyah-Manton configuration based on the 't Hooft instanton. In this paper, we have adopted the 't Hooft form as a starting point of the SU(3) skyrmion configuration. On the other hand, the Jackiw-Nohl-Rebbi form gives the more general configuration. It can describe the stable configuration with the torus shape. Indeed, it is significant to reproduce such a configuration in the SU(3) model as well. In this field, the stable point in the manifold of the Atiyah-Manton configuration is investigated. By quantizing the fluctuation around it, one constructs the quantum state which has the same quantum number as the deuteron [23]. However, the JNR form is difficult to handle the modification of the long-distance behavior caused by the mass of the pseudoscalar meson. At this point, it is convenient to take the 't Hooft form owing to the transparent relation between the instanton coordinate and that of the skyrmion. Furthermore, our configuration based on the 't Hooft form is still valid in the region where the individual skyrmions are identified.

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