Meson self-energies calculated by the relativistic particle-hole-antiparticle representation

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A new formulation of meson self-energies is introduced for $\sigma,\omega,\pi,\rho,\delta$, and η mesons on the basis of the particle-hole-antiparticle representation. We have studied the difference between the meson self-energy (MSE) of this representation and the MSE of the traditional density-Feynman (DF) representation. It is shown that the new formulation describes exactly the physical processes such as particle-hole excitations or particle-antiparticle excitations, and that, on the other hand, the meson self-energy based on the DF representation includes unphysical components. By numerical calculations, the meson self-energies describing the particle-hole excitations are shown to be close to each other for most of the meson self-energy in low momentum ($R<500$ MeV) and low energy ($R_0<200$ MeV). This fact implies that former calculations using the low momentum and low-energy part do not change greatly. The density part of the density-Feynman representation has been shown to have a resonant structure around the energy of particle-antiparticle excitation, which causes a large difference between the two representations in the meson spectrum calculations. Our investigation concludes that the former calculations based on the density-Feynman representation are not invalidated in many cases, but the particle-hole-antiparticle representation is more appropriate to treat exactly the physical processes. [S0556-2813(97)03809-0]

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I. INTRODUCTION

In the relativistic many-body theory, the meson self-energy (MSE) is an essential quantity whose importance in nuclear physics has been shown by many authors [1–3]. The MSE represents medium effects from particle-hole excitations or particle-antiparticle excitations while the meson propagates in nuclear matter, and has been often used in various studies; for example, the energy contribution from ring diagrams [4–9], the instability of the random-phase approximation (RPA) [10–12], the meson mass in nuclear matter [13,14], and the nuclear interactions in nuclear matter [14]. In these papers, the MSE has been calculated on the basis of the density-Feynman (DF) representation of the nucleon propagator.

In this representation, the nucleon propagator is separated into two parts; one is the Feynman part (G^F), which is similar to the free nucleon propagator, except that the free mass is replaced by the effective nucleon mass in nuclear matter, and the other is the density part (G^D), which represents additional effects arising from the existence of the matter. Even though this representation has been successful and widely used, the MSE of the DF representation does not represent exactly physical processes, such as particle-hole excitations or particle-antiparticle excitations. In fact, recently it has been pointed out that the MSE of the DF representation includes unphysical components [15,16]. Instead of the traditional density-Feynman representation, we propose the particle-hole-antiparticle (PHA) representation to treat the physical processes exactly, which facilitates the investigation of the validity of the traditional MSE calculations. Despite the importance of the problem, up until now there have been no systematic investigations on this point.

In this paper, we first show a new formulation of the MSE of the PHA representation and point out explicitly that the DF representation includes unphysical components. Next, from various sides we investigate the approximation of the MSE of the DF representation, and make clear the effectiveness of the DF representation from the point of numerical calculations.

II. FORMALISM

A. Lagrangian

In order to investigate the difference between two representations, we adopt the often used Lagrangian density of Yukawa coupling, which is constructed from the degrees of freedom associated with two isoscalar mesons (σ and ω) and two isovector ones (π and ρ):

$$L_{\mathrm{INT}}=\sum_{\alpha}\overline{\psi}\Gamma^{\alpha}\phi^{\alpha}\psi,\quad\text{for}\quad\alpha=\sigma,\ \omega,\ \pi,\ \rho,\tag{1}$$

where Γ symbolizes the spin and isospin structure of the coupling and is given as

$$\Gamma^{\sigma}=g^{\sigma},\tag{2}$$

$$\Gamma^{\omega}_{\mu}=ig^{\omega}\gamma_{\mu},\tag{3}$$

$$\Gamma^{\pi} = i\ \frac{f_{\pi}}{m_{\pi}}\ \gamma_{\mu}\gamma_5 \boldsymbol{\tau}\partial_{\mu}, \tag{4}$$

$$\Gamma^{\rho}_{\mu} = i g^{\rho}\left(\gamma_{\mu}\boldsymbol{\tau} - \frac{\kappa^{\rho}}{2M}\ \sigma_{\mu\nu}\boldsymbol{\tau}\partial_{\nu}\right). \tag{5}$$

The operator ψ is the nucleon field, φ are the meson fields, M and m_{π} are the rest masses of the nucleon and pion, respectively, and g^{σ}, g^{ω}, f^{π}, g^{ρ}, and κ^{ρ} are the coupling constants. The notation in this paper is the same as Ref. [8].

The π-NN interaction can be written in two possible forms, pseudoscalar (PS) and pseudovector (PV) coupling. We chose the PV coupling in this work, because from general arguments regarding chiral symmetry, PV coupling has to be used to obtain reasonable results in the one-pion exchange approximation. For the ρ meson, the tensor couplings are included along with the vector coupling. Thus the Lagrangian includes four types of interactions, scalar, vector, pseudovector, and tensor couplings. These couplings are also applied to other mesons such as δ meson of a scalar coupling and η meson of a pseudovector coupling, which results are included in this paper.

B. Nucleon propagator in the density-Feynman (DF) representation

The nucleon propagator G has been traditionally written as the sum of the density-dependent part (G^D) and Feynman part (G^F):

$$G(q) = G^D(q) + G^F(q), \tag{6}$$

$$G^D(q) = [-i\gamma_{\mu}q^*_{\mu} + M^*]\frac{\pi i}{E_q}\,\delta(q^*_0 - E_q)\,\theta(k_f - |q|), \tag{7}$$

$$G^F(q) = -\frac{-i\gamma_{\mu}q_{\mu} + M^*}{q^2 + M^{*2} - i\varepsilon}. \tag{8}$$

One merit of this density-Feynman (DF) representation is that the density part goes to zero and only the Feynman part remains when the baryon density of nuclear matter becomes zero, resulting in the nucleon propagator smoothly changing to that of the elementary particle at zero density. Moreover, its form is convenient to use because the density part G^D includes explicitly the δ function. However, since the form of G^D is made artificially by the sum of two pole-parts of the denominators of the nucleon propagator, it does not have an exact physical meaning.

C. Particle-hole-antiparticle (PHA) representation

In order to represent the physical processes, alternatively we should use the particle-hole-antiparticle (PHA) representation where the propagation of particles, holes and antiparticles are separately described as follows:

$$G(q) = G^p(q) + G^h(q) + G^a(q), \tag{9}$$

$$G^p(q) = -\frac{-i\gamma_{\mu}q_{\mu} + M^*}{2E_q}\,\frac{\theta(|q| - k_f)}{E_q - q_0 - i\varepsilon}, \tag{10}$$

$$G^h(q) = -\frac{-i\gamma_{\mu}q_{\mu} + M^*}{2E_q}\,\frac{\theta(k_f - |q|)}{E_q - q_0 + i\varepsilon}, \tag{11}$$

$$G^a(q) = -\frac{-i\gamma_{\mu}q_{\mu} + M^*}{2E_q}\,\frac{1}{E_q + q_0 - i\varepsilon}, \tag{12}$$

where, in our notation, $i\gamma_{\mu}q_{\mu} = i\gamma_i q_i - \gamma_0 E_0$ for the particle and hole, $i\gamma_{\mu}q_{\mu} = i\gamma_i q_i + \gamma_0 E_0$ for the antiparticle propagator, and $E_q = \sqrt{M^{*2} + q^2}$; the effective nucleon mass M^* is given by using the nucleon self-energy (Σ_s) as $M^* = M + \Sigma_s$. (See Sec. III on the expression of Σ_s.)

D. Meson self-energy in the PHA representation

The meson self-energy (MSE) usually used is given under random phase approximation as

$$\Pi^{\alpha}(R) = (-i)\int \frac{d^4q}{(2\pi)^4}\ \mathrm{Tr}[G(q)\Gamma^{\alpha}G(q+R)\Gamma^{\alpha}], \tag{13}$$

where the trace includes also the summation of isospin, so that the factor λ (=2 for the isoscalar mesons) appears after the summation.

First, we show the exact expression of the MSE by using the PHA representation. By inserting Eq. (9) into Eq. (13), five unphysical terms are dropped after integration of q_0 on the complex plane because of the boundary condition ($i\varepsilon$) and only four physical terms are left:

$$\Pi^{\alpha}(R) = \Pi^{\alpha}_{\mathrm{ph}}(R) + \Pi^{\alpha}_{\mathrm{pa}}(R), \tag{14}$$

$$\Pi^{\alpha}_{\mathrm{ph}}(R) = -\lambda\int \frac{d^3q}{(2\pi)^3}\,\frac{n_q(1-n_k)}{4E_qE_k}\left[\frac{\mathrm{Tr}(\vec{q},\vec{q}+\vec{R},E_q,E_k)}{E_k - E_q - R_0 - i\varepsilon} + \frac{\mathrm{Tr}(-\vec{q},-\vec{q}-\vec{R},E_q,E_k)}{E_k - E_q + R_0 - i\varepsilon}\right], \tag{15}$$

$$\Pi^{\alpha}_{\mathrm{pa}}(R) = \lambda\int \frac{d^3q}{(2\pi)^3}\,\frac{1-n_q}{4E_qE_k}\left[\frac{\mathrm{Tr}(\vec{q},\vec{q}+\vec{R},E_q,-E_k)}{E_k + E_q + R_0 - i\varepsilon} + \frac{\mathrm{Tr}(-\vec{q},-\vec{q}-\vec{R},E_q,-E_k)}{E_k + E_q - R_0 - i\varepsilon}\right], \tag{16}$$

where n_q stands for a step function which is 1 for a hole state and 0 for a particle state, λ is the isospin degeneracy, and Tr means the trace part of Tr $[G(q)\Gamma^{\alpha}G(q+k)\Gamma^{\alpha}]$, and $k = |\vec{q} + \vec{R}|$. We rewrote the second terms in Eqs. (15) and (16) by replacing $\vec{q} \to -\vec{q}$. The meson self-energy in Eq. (15) contains a factor $n_q(1-n_k)$, which represents the contribution from particle-hole excitations. This particle-hole part Π_{ph} is finite because of the factor n_q. The MSE in Eq.(16) contains the factor $1-n_q$; it describes the contribution from particle-antiparticle excitations. The particle-antiparticle part Π_{pa} is a nonzero-density version of vacuum polarization. It should be stressed that the expression of the particle-hole part, although it is completely relativistic, is similar to that of the nonrelativistic polarization function, except for the trace part. This point is one of the merits of the PHA representation. Since there are few expressions of the MSE using the PHA representation, it is worthwhile to write down their explicit forms for various mesons. Calculating the trace part in Eq. (15) for various interaction types, we obtained the following expressions for the particle-hole part of the MSE:

$$\Pi_{\rm ph}^{\sigma}(R)=\lambda(g^{\sigma 2})\int\frac{d^3q}{(2\pi)^3}\frac{n_q(1-n_k)}{E_qE_k}\Bigg[(E_k-E_q)-\left(2M^{*2}+\frac{R_\mu^2}{2}\right)(f_{\rm ph+}+f_{\rm ph-})\Bigg], \tag{17}$$

$$\Pi_{\rm ph}^{\omega_l}(R)=\lambda(g^{\omega})^2\int\frac{d^3q}{(2\pi)^3}\frac{n_q(1-n_k)}{E_kE_q}[2(E_k-E_q)-(R_\mu^2+q_3k_3-E_qE_k)(f_{\rm ph+}+f_{\rm ph-})], \tag{18}$$

$$\Pi_{\rm ph}^{\omega_t}(R)=\lambda(g^{\omega})^2\int\frac{d^3q}{(2\pi)^3}\frac{n_q(1-n_k)}{E_kE_q}\times\Bigg[(E_k-E_q)-\left(\frac{R_\mu^2}{2}+q_1k_1+q_2k_2\right)(f_{\rm ph+}+f_{\rm ph-})\Bigg] \tag{19}$$

$$\Pi_{\rm ph}^{\sigma\omega{\rm mix}}(R)=\lambda(g^{\sigma}g^{\omega})\int\frac{d^3q}{(2\pi)^3}\frac{n_q(1-n_k)}{E_qE_k}\times[-M^*(E_k+E_q)(f_{\rm ph+}+f_{\rm ph-})], \tag{20}$$

$$\Pi_{\rm ph}^{\pi}(R)=\lambda\left(\frac{f_\pi}{m_\pi}\right)^2\int\frac{d^3q}{(2\pi)^3}\frac{n_q(1-n_k)}{E_kE_q}[-(E_k-E_q)\times((E_k+E_q)^2-\vec{R}^{\,2})+2M^{*2}R_\mu^2(f_{\rm ph+}+f_{\rm ph-})], \tag{21}$$

$$\Pi_{\rm ph}^{\rho,T_l}(R)=\lambda\left(g^{\rho}\frac{\kappa}{M}\right)^2\int\frac{d^3q}{(2\pi)^3}\frac{n_q(1-n_k)}{E_kE_q}R_\mu^2\Bigg[(E_k-E_q)+\left(2M^{*2}+\frac{R_\mu^2}{2}+2(q_3k_3-E_qE_k)\right)\times(f_{ph+}+f_{ph-})\Bigg], \tag{22}$$

$$\Pi_{\rm ph}^{\rho,T_t}(R)=\lambda\left(g^{\rho}\frac{\kappa}{M}\right)^2\int\frac{d^3q}{(2\pi)^3}\frac{n_q(1-n_k)}{E_kE_q}(-)[2(E_k-E_q)\times((E_k+E_q)^2-\vec{R}^{\,2})+R_\mu^2\{(2M^{*2}+q_1k_1+q_2k_2)\times(f_{\rm ph+}+f_{\rm ph-})\}], \tag{23}$$

where $f_{\rm ph\pm}=1/(E_k-E_q\pm R_0-i\varepsilon)$, the suffix ph of the coefficient represents the particle-hole excitation energy of E_k-E_q, and the sign + or – means the sign before the meson energy R_0. Note that $R_\mu^2=R^2-R_0^2$ in our notation. As for the ρ meson, the MSE of the vector interaction part is the same as that of the ω meson, except for the coupling constants, thus only its tensor part is given here. [See Appendix I of Ref. [16] for the derivation of Eqs. (22) and (23) and for the current conservation of the tensor part of ρ meson self-energy.] The MSE of δ meson and η meson have the same form as those of σ (scalar coupling) and π (pseudovector coupling) mesons respectively, except for their coupling constants.

E. Meson self-energy in the DF representation

Next we consider the MSE of the density-Feynman (DF) representation. By using Eqs. (6)–(8), the meson self-energy is given by

$$\Pi(R)=\Pi_D(R)+\Pi_F(R)+\Pi_{\rm imag}(R), \tag{24}$$

$$\Pi_D(R)=-\int\frac{d^4q}{(2\pi)^3}\frac{n_q}{2E_q}\delta(q_0-E_q)\times\Bigg[\frac{{\rm Tr}(q,q+R)}{(q+R)^2+M^{*2}-i\varepsilon}+\frac{{\rm Tr}(q,q-R)}{(q-R)^2+M^{*2}-i\varepsilon}\Bigg], \tag{25}$$

$$\Pi_F(R)=+(-i)\int\frac{d^4q}{(2\pi)^4}\times\frac{{\rm Tr}(q,q+R)}{(q^2+M^{*2}-i\varepsilon)[(q+R)^2+M^{*2}-i\varepsilon]}, \tag{26}$$

$$\Pi_{\rm imag}(R)=-(-i)\int\frac{d^4q}{(2\pi)^2}\frac{n_qn_k}{4E_qE_k}{\rm Tr}(q,q+R)\times\delta(q_0-E_q)\delta(q_0+R_0-E_k). \tag{27}$$

The second term Π_F comes from $G^FG^F$and is called the Feynman part, the first term Π_D comes from $G^DG^F+G^FG^D$ and is called the density part, and the third term $\Pi_{\rm imag}$ comes from G^DG^D,which is pure imaginary. The Feynman part is divergent and a special treatment is needed to render it finite. Since the density part is finite and it is considered to represent approximately the particle-hole excitation effects, it has been often used in many studies. Actually most of the studies mentioned in the Introduction use the density part Π_D. Therefore, in this paper we focus mainly on the difference between the density part Π_D of the DF representation and the particle-hole part $\Pi_{\rm ph}$ of the PHA representation. As for a treatment of the divergent part, we have shown in Ref. [16] one possible way to obtain meaningful results based on the cutoff field theory.

F. Difference between the PHA and DF representations

In this subsection, the difference between two representations based on the equations in Secs. II D and II E is discussed. In order to show explicitly that the density-part (Π_D) of the MSE in the DF representation does not represent a proper physical process, we rewrote the density part of Eq. (25) and obtained

$$\Pi_D(R)=-\lambda\int\frac{d^3q}{(2\pi)^3}\frac{n_q}{4E_qE_k}\left[\frac{\mathrm{Tr}(\vec{q},\vec{q}+\vec{R},E_q,E_q+R_0)}{E_k-E_q-R_0-i\varepsilon}+\frac{\mathrm{Tr}(-\vec{q},-\vec{q}-\vec{R},E_q,E_q-R_0)}{E_k-E_q+R_0-i\varepsilon}+\frac{\mathrm{Tr}(\vec{q},\vec{q}+\vec{R},E_q,E_q+R_0)}{E_k+E_q+R_0-i\varepsilon}+\frac{\mathrm{Tr}(-\vec{q},-\vec{q}-\vec{R},E_q,E_q-R_0)}{E_k+E_q-R_0-i\varepsilon}\right]. \tag{28}$$

The density part Π_D of Eq. (28) includes the term of $f_{\mathrm{ph}+}+f_{\mathrm{ph}-}$, whose denominator is the same as Π_{ph} of Eq. (15). Exactly speaking, however, the density part does not represent particle-hole excitations since the factor n_q in Eq. (28) is different from the particle-hole factor $n_q(1-n_k)$ in Eq. (15). The factor n_q represents the integration in the hole states so that the factor constrains one particle in a hole state, however, the other particle is not constrained. Thus the MSE in the DF representation includes, in principle, unphysical components such as hole-hole excitations besides the particle-hole excitations. One more important difference is that the density part Π_D includes a part of the particle-antiparticle excitation and it is not fully included because the factor n_k is also different from the factor of the particle, i.e., $(1-n_k)$. This partial inclusion of the particle-antiparticle excitations is undesirable.

Furthermore the trace part in the DF representation is different from the PHA representation because of the different on-shell conditions in the two representations. This is clearly shown in Eqs. (15) and (28). Taking the σ and ω meson cases as examples, one obtains the following expressions of Π_D:

$$\Pi_D^{\sigma}(R)=\lambda(g^{\sigma})^2\int\frac{d^3q}{(2\pi)^3}\frac{n_q}{E_kE_q}\left[2E_k-\left(2M^{*2}+\frac{R_\mu^2}{2}\right)\times(f_{\mathrm{ph}+}+f_{\mathrm{ph}-}+f_{\mathrm{pa}+}+f_{\mathrm{pa}-})\right], \tag{29}$$

$$\Pi_D^{\omega_l}(R)=\lambda(g^{\omega})^2\int\frac{d^3q}{(2\pi)^3}\frac{n_q}{E_kE_q}[4E_k-[R_\mu^2+2q_3k_3-2E_q(E_q+R_0)](f_{\mathrm{ph}-}+f_{\mathrm{pa}+})-[R_\mu^2+2q_3k_3-2E_q(E_q-R_0)](f_{\mathrm{ph}+}+f_{\mathrm{pa}-})], \tag{30}$$

$$\Pi_D^{\omega_t}(R)=\lambda(g^{\omega})^2\int\frac{d^3q}{(2\pi)^3}\frac{n_q}{E_kE_q}\left[2E_k-\left(\frac{R_\mu^2}{2}+q_1k_1+q_2k_2\right)(f_{\mathrm{ph}+}+f_{\mathrm{ph}-}+f_{\mathrm{pa}+}+f_{\mathrm{pa}-})\right] \tag{31}$$

The first terms in the above expressions, which come from the trace part in Eq. (28), are different from those in Eqs. (17)–(19). (As is shown in the following subsection, this difference in the first terms gives an appreciable deviation in the transverse component of the MSE of the vector interaction.) The density part in the DF representation does not represent proper particle-hole excitations nor particle-antiparticle excitations.

We can show, however, that by adding the Feynman part and the pure imaginary part to the density part Π_D, one can obtain the same contribution as the total of the particle-hole part and the particle-antiparticle part. The relation of two representations is explicitly shown for the σ meson case as an example. The Feynman and the pure imaginary parts of the σ meson in the DF representation are obtained by calculating the trace part in Eqs. (26) and (27):

$$\Pi_F(R)=\lambda g^2\int\frac{d^3q}{(2\pi)^3}\frac{-1}{E_kE_q}\left[E_k+E_q-\left(2M^{*2}+\frac{R_\mu^2}{2}\right)(f_{\mathrm{pa}+}+f_{\mathrm{pa}-})\right], \tag{32}$$

$$\Pi_{\mathrm{imag}}(R)=\lambda g^2\int\frac{d^3q}{(2\pi)^3}\frac{-n_qn_k}{E_kE_q}\left[E_k-E_q-\left(2M^{*2}+\frac{R_\mu^2}{2}\right)(f_{\mathrm{ph}+}+f_{\mathrm{ph}-})\right]. \tag{33}$$

By adding the density part (29) and the pure imaginary part (33) together, one obtains the correct particle-hole part and the n_q part of the particle-antiparticle part; the latter is so-called Pauli blocking term. The Pauli blocking term makes the correct particle-antiparticle part by adding further the Feynman part (32). In a similar way, we can verify that the total sums in the two representations are formally the same as for the other mesons. The verification of the equivalence for the other mesons is not so straight forward, as it depends on the type of the couplings.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we investigate more concretely the difference between the DF and PHA representations from numerical calculations. For the calculations of the MSE, we adopt the relativistic Hartree approximation, which includes the contribution to the nucleon self-energy arising from the occupied Fermi sea as well as from the full Dirac sea. In this approximation, the nucleon self-energy Σ_s is expressed by the self-consistent equation

$$\Sigma_s=M^*-M=-\left(\frac{g^{\sigma}}{m_\sigma}\right)^2\frac{2}{(2\pi)^3}\int_0^{k_F}d^3qM^*(\vec{q}^{\,2}+M^2)^{1/2}+\left(\frac{g^{\sigma}}{m_\sigma}\right)^2\frac{2}{(2\pi)^2}\left[M^{*3}\ln(M^*/M)+M^2(M-M^*)-\frac{5}{2}M(M-M^*)^2+\frac{11}{6}(M-M^*)^3\right]. \tag{34}$$

In this approximation, the coupling constants g^{σ} and g^{ω} are 7.93 and 8.93, respectively; these values reproduce the binding energy ($E_b=-15.8$ MeV) of the nuclear matter at

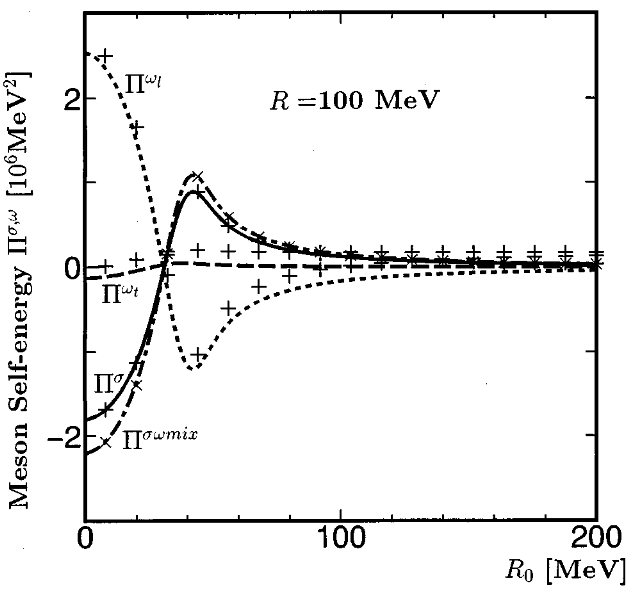

FIG. 1. Comparison of the energy (R_0) dependence in the σ and ω meson self-energies given by the particle-hole-antiparticle (PHA) representation and the density-Feynman (DF) representation. The momentum is fixed at $R=100$ MeV. The four lines represent the σ meson self-energy (solid line), the longitudinal component of the ω meson (dotted line), the transverse component of the ω meson (dashed line), and the σ-ω meson mixing self-energy (dash-dotted line). The lines show the particle-hole part (Π_{ph}) of the PHA representation; the density part (Π_D) of the DF representation is given by the symbols $+$ and $\times$. As for the couplings, see text.

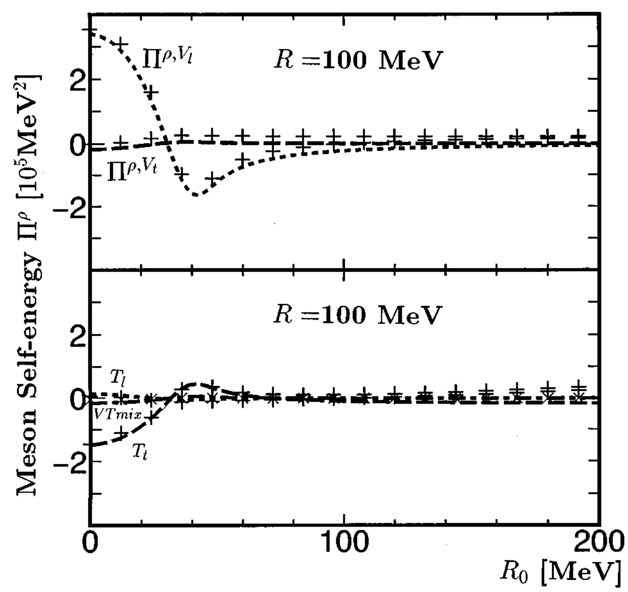

FIG. 2. Comparison of the energy (R_0) dependence in the ρ meson self-energies given by the particle-hole-antiparticle (PHA) representation and the density-Feynman (DF) representation. The momentum is fixed at $R=100$ MeV. The two lines in the upper figure represent the longitudinal component of the vector interaction of the ρ meson (dotted line) and the transverse component of the vector interaction of the ρ meson (dashed line). The three lines in the lower figure represent the longitudinal component of the tensor interaction of the ρ meson (dotted line), the transverse component of the tensor interaction of the ρ meson (dashed line), and the vector-tensor mixing self-energy (dash-dotted line). The lines show the particle-hole part (Π_{ph}) of the PHA representation; the density part (Π_D) of the DF representation is given by the symbols $+$ and $\times$.

the normal density $\rho_B=0.193$ fm^{-3} ($k_F=1.42$ fm^{-1}). For the ρ meson, we adopt $g^{\rho}=2.72$ and $\kappa^{\rho}=6.0$ which are obtained from the N-N forward dispersion relation [17], and for the π meson, $f^{\pi}=0.98$ [18]. The couplings for δ and η mesons are $g^{\delta}=2.36$, $g^{\eta}=8.12$ taken from Ref. [19].

First in Figs. 1–3 we show the comparison of the R_0 dependence of meson self-energies using the two representations. Figure 1 shows the MSE of σ and ω, where Π^{ω_t} is transverse, Π^{ω_l} is a longitudinal component of ω meson self-energy, and $\Pi^{\sigma\omega\mathrm{mix}}$ is the σ-ω meson mixing self-energy. (See Ref. [8] on the definition of $\Pi^{\sigma\omega\mathrm{mix}}$.) Figure 2 shows the comparison of the MSE of the ρ meson. Five lines and symbols represent the longitudinal and transverse components of vector and tensor interactions, and the vector-tensor mixing component. Figure 3 shows the comparison of the MSE of the π, δ, and η mesons, and the δ-ρ mixing. In these figures, the MSE of the PHA representation is shown with lines, while the DF representation is shown with $+$ or $\times$ symbols. We find that most of the MSE of the DF representation well reproduce those of the PHA representation. However, the MSE of the transverse component of vector interaction has an appreciable deviation.

It is easy to understand the reason why the DF representation is good, except for the transverse component of vector interaction Π^{ω_t}. Taking σ meson case as an example, the second term has a dominant contribution , then the difference of the first term [(E_k-E_q) in Eq. (17) and $2E_k$ in Eq. (29)] is negligible. Among four denominators of the second term, only one term with the denominator $f_{\mathrm{ph}-}$ is dominant. Compared with this dominant term, the unphysical particle-antiparticle terms with denominators $f_{\mathrm{pa}-}$ and $f_{\mathrm{pa}+}$ are small for a small R_0. Therefore both representations give similar values. However, the situation is different for the transverse component of vector interaction Π^{ω_t}. In this case, the numerator of the second term $R_{\mu}^2/2+q_1k_1+q_2k_2$ in Eqs. (31) and (19) is small compared with the first term $2E_k$ for small R_0 and R. Therefore, the difference in the first term becomes important for the transverse component of vector interaction, as was mentioned in the previous subsection.

Next we discuss the momentum R dependence of the meson self-energies. The comparison of R dependence of the MSE is shown in Figs. 4–6. As in Figs. 1–3, the σ and ω meson self-energies are shown in Fig. 4, ρ in Fig. 5, and π,δ,η in Fig. 6, where lines represent PHA results and the symbols ($+$ and $\times$) show the DF results. The two representations give similar results, especially for the scalar and vector interactions. One may notice the following two different points. One is the difference of the ω meson self-energy in very small momentum $R\sim0$, and the difference in the tensor interaction of ρ meson. The difference of $R\sim0$ does not have an important effect on the results because the integration has the weight of momentum R^2, i.e., because the phase volume of the $R\sim0$ is very small. When we discuss the limit value at $R=0$ of the MSE, the difference may be a serious problem. The limit value of the MSE of the DF representation at $R=0$ is not zero. However, it should decrease to zero

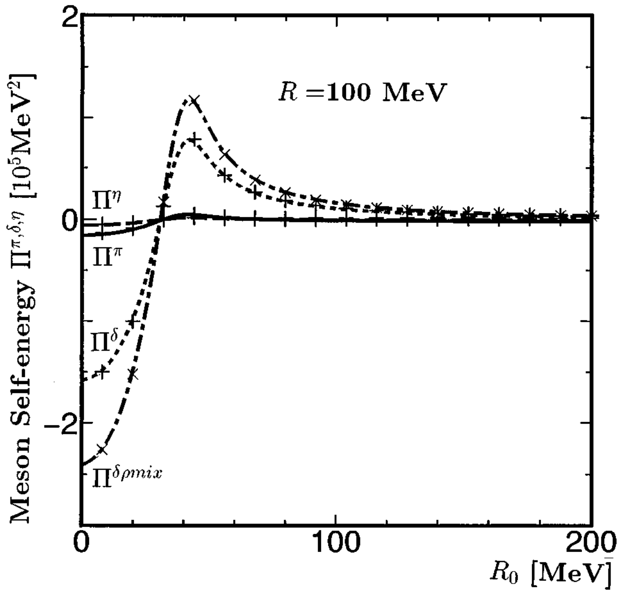

FIG. 3. Comparison of the energy (R_0) dependence in the π, δ, and η meson self-energies given by the particle-hole-antiparticle (PHA) representation and the density-Feynman (DF) representation. The momentum is fixed at $R=100$ MeV. The four lines represent the π meson self-energy (solid line), the δ meson (dotted line), the η meson (dashed line), and the δ-ρ meson mixing self-energy (dash-dotted line). The lines show the particle-hole part ($\Pi_{\rm ph}$) of the PHA representation; the density part (Π_D) of the DF representation is given by the symbols + and $\times$.

as shown in the PHA representation because a particle-hole pair can not be excited by the zero momentum transfer. This is ensured by the factor $n_q(1-n_k)$ in the PHA representation.

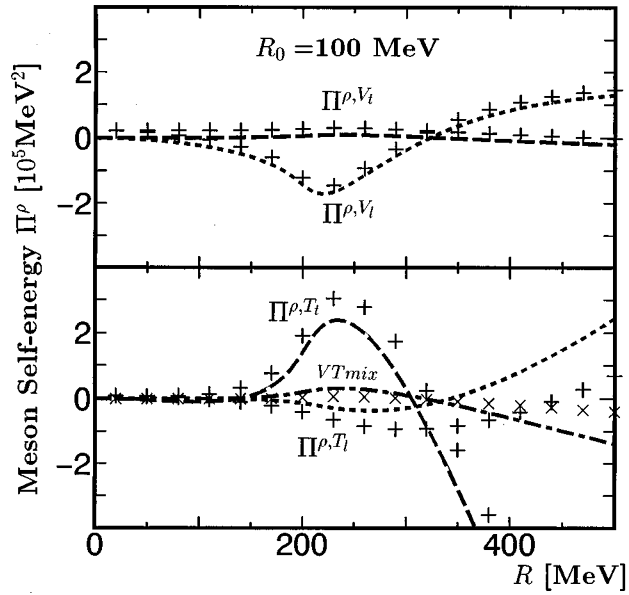

FIG. 5. Comparison of the momentum (R) dependence in the ρ meson self-energies given by the PHA representation and DF representation. The energy is fixed at $R_0=100$ MeV. The meaning of the lines and symbols is the same as in Fig. 2.

The difference in tensor interaction comes from the calculation of the trace part. The tensor interaction includes the term $(Rk)(Rq)$. For the calculation of this term, the definition of k_μ, i.e., $k_0=q_0+R_0$, is used in the DF representation, but on the other hand, the on-shell condition $k_0=E_k=\sqrt{M^{*2}+k^2}$ is used for the PHA representation. Because of this difference, the numerators of the second term are also different from each other in the ρ meson case. This difference becomes large as the momentum R increases,

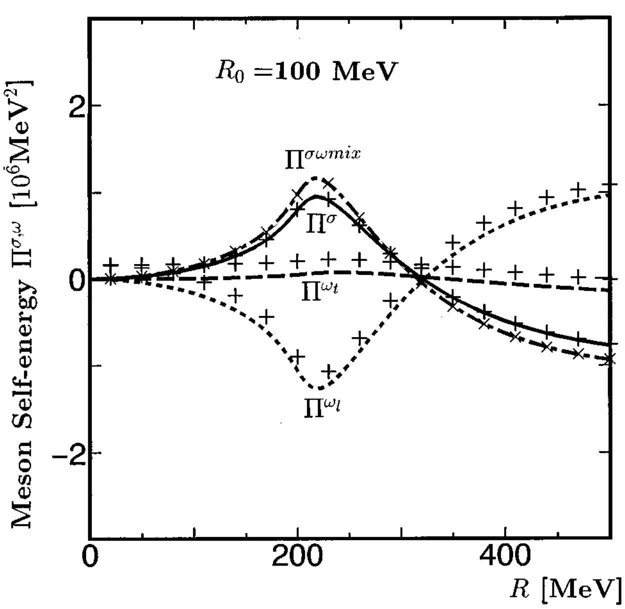

FIG. 4. Comparison of the momentum (R) dependence in the σ and ω meson self-energies given by the PHA and DF representations. The energy is fixed at $R_0=100$ MeV. The meaning of the lines and symbols is the same as in Fig. 1.

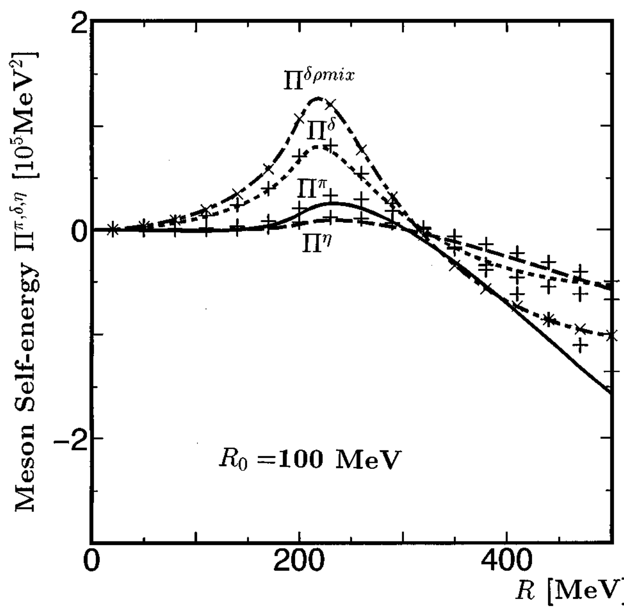

FIG. 6. Comparison of the momentum (R) dependence in the π, δ, and η meson self-energies given by the PHA and DF representations. The energy is fixed at $R_0=100$ MeV. The meaning of the lines and symbols is the same as in Fig. 3.

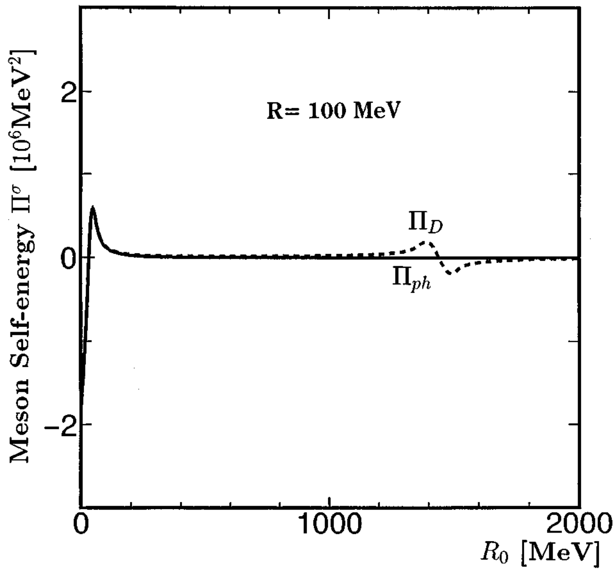

FIG. 7. Comparison of large energy R_0 dependence of σ meson self-energy Π^{σ} at fixed $R=100$ MeV. The solid line represents the particle-hole part ($\Pi_{\rm ph}$) of the particle-hole-antiparticle (PHA) representation, while the dotted line shows the density part (Π_D) of the density-Feynman (DF) representation.

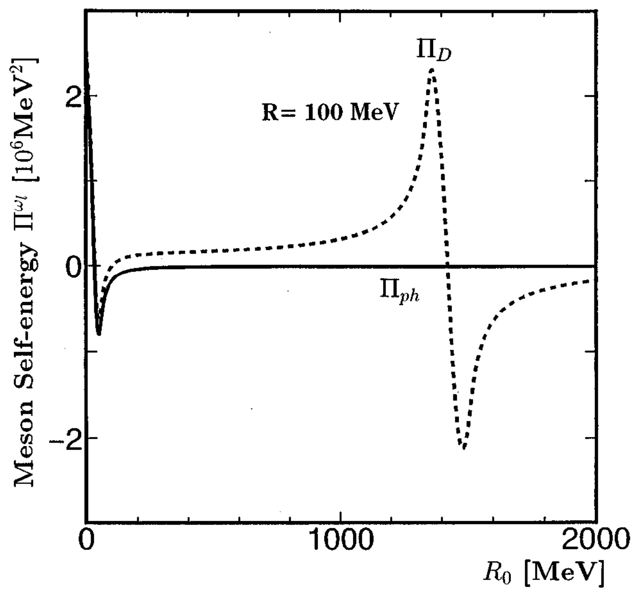

FIG. 8. Comparison of large energy R_0 dependence of the longitudinal component of the ω meson self-energy Π^{ω_l} at fixed $R=100$ MeV. The solid line represents the particle-hole part ($\Pi_{\rm ph}$) of the particle-hole-antiparticle (PHA) representation, while the dotted line shows the density part (Π_D) of the density-Feynman (DF) representation.

which implies a serious difference at high-density, i.e., a large Fermi momentum system.

The MSE in this momentum region is important for the ring energy calculations. In many studies, the meson self-energy in such momentum regions as $R<2K_F\sim500$ MeV and $R_0<50$ MeV is integrated to calculate the ring energy contribution to the binding energy. Therefore, the above fact implies that the new calculations on the PHA representation does not change greatly the previous results, when the low momentum transfer region is used.

A large difference occurs in a region of high-energy transfer such as around 1300 MeV. The comparison of two representations in the large R_0 region is shown in Fig. 7 for the σ meson and in Fig. 8 for the ω meson. These figures clearly show that the MSE of the DF representation has a resonant structure around the particle-antiparticle excitation energy. On the other hand, the MSE of the PHA representation has no structure. The resonant structure causes a large difference in the lower energy region below the resonance. The MSE has a large positive enhancement from the tail of the resonance, especially in the ω meson. This spurious enhancement makes the meson spectrum calculations doubtful.

In Fig. 9, we show the ω meson spectrum, where the dotted lines are the meson spectrum using the DF representation, and the full lines are those using the PHA representation. There are three meson spectra; one longitudinal spectrum (the upper line) and two transverse spectra (the upper and lower lines). The lower line represents the so-called zero-mode solution and it coincides with each other in the two representations. On the other hand, the upper spectrum given by the DF representation lies higher than that of the PHA representation. As was explained, this is because the MSE of the DF representation has the spurious enhancement from the resonance tail. The meson spectrum of the PHA representation starts from the free meson mass because the

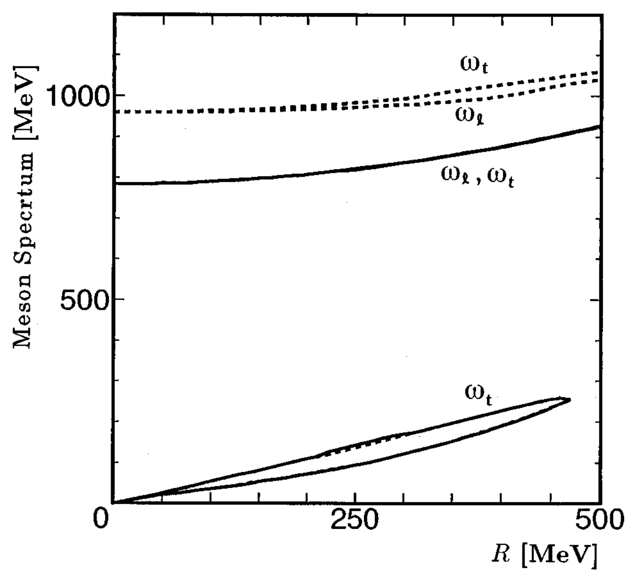

FIG. 9. The ω meson spectrums calculated by using two representations. The solid lines represent the spectrums using the particle-hole part ($\Pi_{\rm ph}$) of the particle-hole-antiparticle (PHA) representation, while the dotted lines show the spectrums using the density part (Π_D) of the density-Feynman (DF) representation. Here the lower spectrums, so-called zero modes, coincide with each other. On the other hand, the upper spectrums calculated by using the Π_D run over the one by $\Pi_{\rm ph}$. The longitudinal and transverse components give the same meson spectrum in the PHA representation; on the other hand, they separate little from each other in the DF representation.

particle-hole excitation contributes nothing to the meson mass at $R=0$. This is also ensured by the factor $n_q(1-n_k)$. Note that the meson mass shift in nuclear matter comes from the particle-antiparticle part of the MSE. As for a treatment of the divergent term of this part, see Ref. [16].

IV. CONCLUSIONS

We have derived new expressions of meson self-energies for the $\sigma,\omega,\rho,$ and π (also δ and η) mesons and shown the difference between the meson self-energy in the traditional density-Feynman (DF) representation and in the particle-hole-antiparticle (PHA) representation. In the discussion on the relation of the two representations, we have shown that, as for the real part, the density part in the DF representation includes correct particle-hole excitations and a part of particle-antiparticle excitations. Since the particle-antiparticle part makes small contributions in the low-energy region, one can expect that the DF representation causes a similar result to the PHA representation. Actually, we have shown by the numerical calculations that the deviation is not so large in the region of the low momentum ($R<500$ MeV) and low energy ($R_0<200$ MeV) transfer, although the deviations become appreciable in the transverse component of the vector interaction and the tensor interaction. This fact implies that among former calculations, applications using the MSE of the low momentum and low-energy regions are justified. Such examples may be the ring energy calculations, binding energy calculations, and the zero mode calculations. We should investigate these problems further.

The DF representation, however, also includes a part (not full) of the particle-antiparticle excitations, which is not desirable. This brings about a difference between the two representations. The resonant part $f_{\mathrm{pa}-}$ in the MSE of the DF representation has the maximum peak around $R_0=1300$ MeV. We have shown that the resonance has a significant effect also on the off-resonant region. Because of this resonance tail, the meson spectrum of the DF representation lies higher than that of the PHA representation. Therefore, one should pay particular attention to previous results on the upper spectrum using the DF representation. In order to avoid the spurious enhancement of the DF representation, we should use the PHA representation.

Several merits for using the PHA representation are summarized in the following sentences. First, it is not difficult to calculate the MSE in the PHA representation, even if compared with those in the DF representation. In the PHA representation, the nucleon propagator is written on the energy-shell expression, which easily corresponds to the classical picture, i.e., particle-hole image, and the expression is still fully relativistic. In addition, it can be verified that the ω meson self-energy $\Pi_{ph_{\mu\nu}}$ in the PHA representation satisfies the current conservation, while on the other hand, it can not be satisfied only in the density part $\Pi_{D_{\mu\nu}}$ in the DF representation. Furthermore, it is also evident that the real and imaginary part of the MSE in the PHA representation satisfies the dispersion relation. On the contrary it is not so straightforward in the DF representation because the imaginary part is given by the sum of the density part and the pure-imaginary part to make the particle-hole factor $n_q(1-n_k)$ while the real part Π_D has a different factor n_q, which can not satisfy the dispersion relation.

Finally, since the PHA representation describes exactly the physical process, it is easy to describe the isovector components such as the proton particle and neutron hole. The PHA representation can serve as a powerful tool to describe isovector meson self-energies with a nonzero isospin. Such components, however, can not be described by the DF representation since the G^D in the DF representation does not represent the particle nor hole propagation.

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