Tensor force in doubly odd deformed nuclei

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We study the nucleus ^{176}Lu within the framework of the particle-rotor model. The aim of this study is to obtain information on the residual neutron-proton interaction in doubly odd deformed nuclei. To this end, both zero-range and finite-range interactions are considered with particular attention focused on the role of the tensor force, which is still a controversial issue. A detailed comparison of the calculated results with experimental data provides clear evidence of the importance of the tensor-force effects. [S0556-2813(97)02512-0]

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I. INTRODUCTION

The role of the neutron-proton interaction in the unified model description of doubly odd deformed nuclei has long been a subject of special interest. As is well known, the two most important effects associated with the residual interaction between the unpaired neutron and proton are the Gallagher-Moszkowski (GM) splitting [1] and the odd-even or Newby (N) shift [2]. In the 1970s several efforts [3–5] were made to obtain detailed information on the effective neutron-proton interaction in the rare-earth region from the empirical values of these quantities. In this context, both zero-range and finite-range interactions were considered, the parameters involved being determined from least-squares fits of the calculated matrix elements to the experimental GM splittings and N shifts.

The main conclusions of this early work may be summarized as follows: (i) a zero-range force is generally adequate to reproduce the GM splittings while it fails to describe the N shifts; (ii) a finite-range force including tensor terms satisfactorily reproduces both the splitting energies and the odd-even shifts. Regarding the latter point, the extensive study of Ref. [5] emphasized the importance of the tensor-force effects for the N shifts.

Despite these initial achievements in understanding the role of the residual neutron-proton interaction in odd-odd rotational nuclei, there was little work along these lines in the following years. In fact, in most of the studies [6] carried out over the last two decades a simple spin-dependent δ force, first used by Pyatov [7] in the early 1960s, has been adopted. As a consequence, the role of the tensor force was not further explored. Recently, however, this problem was revived by the work of Ref. [8], where an extensive analysis of all the existing (up to January 1993) GM splittings and N shifts in the rare-earth and actinide regions has been performed. In this study both a delta interaction and a finite-range force with a Gaussian radial shape were considered. The latter is of the same form as that previously employed by Boisson *et al.* [5] which contains both central and tensor terms (see Sec. II). These two studies differ, however, in two main aspects. First, in the work of Ref. [5] only experimental data regarding the rare-earth region were analyzed. Second, while in Ref. [5] a sample of data containing GM as well as N terms was considered, the authors of Ref. [8] performed two kinds of fits using the empirical values of the GM splittings and, separately, the N shifts. It may not be surprising, therefore, that the conclusions reached in Ref. [8] are not in line with the early findings of Boisson *et al.* [5], the main point of disagreement being the role of the tensor force. In fact, Nosek *et al.* [8] find that the most significant contributions to the N shifts are given by the space-exchange and the spin-spin space-exchange forces. In this connection, it may be mentioned that the importance of the space-exchange terms has also been pointed out in Ref. [9]. Concerning the tensor force itself, however, the work of Ref. [8] does not provide much information since, as pointed out by the authors, the empirical tensor parameters are not well determined in their experimental set of N shifts.

On the above grounds it seems fair to say that the role of the tensor force in doubly odd deformed nuclei still remains to be assessed. In this situation, we found it interesting to try to shed light on this problem. To this end, we chose as a test case the nucleus ^{176}Lu which is a prototype of odd-odd well-deformed nuclei. We have performed a complete Coriolis band-mixing calculation within the framework of the particle-rotor model, focusing attention on the lowest $K^{\pi}=0^{-}$ band where the effects of the residual neutron-proton interaction are particularly evident. We have also studied the lowest $K^{\pi}=1^{+}$ band which presents a rather large odd-even staggering [10]. This effect is of great interest since it may give further information on the neutron-proton interaction. In fact, it may be traced to direct Coriolis coupling with N-shifted $K=0^{+}$ bands.

The paper is organized as follows. In Sec. II we first give a bare outline of the model and then describe our calculations. In Sec. III we present the results obtained by making use of a δ force and of a finite-range interaction with and without tensor components. These results are compared with the experimental data. In Sec. IV we draw the conclusions of our study.

II. OUTLINE OF THE MODEL AND CALCULATIONS

We assume that the odd neutron and the odd proton are strongly coupled to an axially symmetric core and interact through an effective interaction. The total Hamiltonian is written as

$$H=H_0+H_{\rm RPC}+H_{\rm ppc}+V_{np}\,. \tag{1}$$

The term H_0 includes the rotational energy of the whole system, the deformed, axially symmetric field for the neutron and proton, and the intrinsic contribution from the rotational degrees of freedom. It is given by

$$H_0=\frac{\hbar^2}{2\mathfrak{I}}(\boldsymbol{I}^2-I_3^2)+H_n+H_p+\frac{\hbar^2}{2\mathfrak{I}}[(\boldsymbol{j}_n^2-j_{n3}^2)+(\boldsymbol{j}_p^2-j_{p3}^2)]. \quad (2)$$

The two terms H_{RPC} and H_{ppc} in Eq. (1) represent the Coriolis coupling and the coupling of particle degrees of freedom through the rotational motion, respectively. They are written as

$$H_{\mathrm{RPC}}=-\frac{\hbar^2}{2\mathfrak{I}}(I^+J^-+I^-J^+), \quad (3)$$

$$H_{\mathrm{ppc}}=\frac{\hbar^2}{2\mathfrak{I}}(j_n^+j_p^-+j_n^-j_p^+). \quad (4)$$

In Eqs. (2)–(4), $\mathfrak{I}$ is the moment of inertia of the core while $\boldsymbol{I}$ and $\boldsymbol{J}=\boldsymbol{j}_p+\boldsymbol{j}_n$ are the total and intrinsic angular momentum operators, respectively. I_3 and J_3 are their projections on the intrinsic symmetry axis.

The residual neutron-proton interaction is written in the general form

$$V_{np}=V(r)[u_0+u_1\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n+u_2P_M+u_3P_M\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n+V_TS_{12}+V_{TM}P_MS_{12}], \quad (5)$$

with standard notation [5]. In our calculations we have used a finite-range force with a radial dependence $V(r)$ of the Gaussian form,

$$V(r)=\exp(-r^2/r_0^2), \quad (6)$$

as well as a zero-range force. In the latter case, V_{np} takes the simple form

$$V_{np}^{\delta}=\delta(r)[v_0+v_1\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n]. \quad (7)$$

As basis states we use the eigenvectors of H_0 properly symmetrized and normalized,

$$|\nu_n\Omega_n\nu_p\Omega_pIMK\rangle=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}[D_{MK}^I|\nu_n\Omega_n\rangle|\nu_p\Omega_p\rangle+(-)^{I+K}D_{M-K}^I|\nu_n\overline{\Omega_n}\rangle|\nu_p\overline{\Omega_p}\rangle], \quad (8)$$

where the state $|\nu\overline{\Omega}\rangle$ is the time-reversal partner of $|\nu\Omega\rangle$.

In our calculations we use the Nilsson potential as defined, for instance, in Ref. [11] to generate the single-particle Hamiltonians H_n and H_p. The parameters μ and k have been fixed by using the mass-dependent formulas of Ref. [12]. The adopted values are $\mu_n=0.41$, $k_n=0.064$, and $\mu_p=0.61$, $k_p=0.063$. As regards the deformation parameter β_2, we have used the value 0.26, according to Ref. [4]. In a more recent work [13] the value 0.24 was taken for β_2. We have verified that such a change does not significantly modify the results of our calculations. For the harmonic oscillator parameter $\nu=m\omega/\hbar$ we have taken the value 0.176 fm^{-2}. The adopted single-particle level schemes for both the neutron and proton are listed in Table I. They are mainly derived from the experimental spectra [14] of the two neighboring odd-mass nuclei ^{175}Yb and ^{175}Lu. As regards the odd proton we have added the configuration $\frac{7}{2}[523]$, while for the odd neutron we have included the configuration $\frac{5}{2}[523]$ and slightly increased the experimental excitation energies of the $\frac{9}{2}[624]$ and $\frac{7}{2}[633]$ states. For the rotational parameter $\hbar^2/2\mathfrak{I}$ the value 14 keV has been adopted. This has been deduced from the $K^\pi=7^-$ ground-state band, which is essentially of pure rotational character.

TABLE I. Energies (in keV) of the proton and neutron single-particle states.

Proton configuration	E	Neutron configuration	E
$\frac{7}{2}[404]$	0	$\frac{7}{2}[514]$	0
$\frac{5}{2}[402]$	343	$\frac{9}{2}[624]$	468
$\frac{1}{2}[541]$	370	$\frac{1}{2}[510]$	557
$\frac{9}{2}[514]$	396	$\frac{5}{2}[512]$	639
$\frac{1}{2}[411]$	626	$\frac{3}{2}[512]$	811
$\frac{7}{2}[523]$	1300	$\frac{1}{2}[521]$	920
		$\frac{5}{2}[523]$	930
		$\frac{7}{2}[633]$	1400

For the neutron-proton interaction, as already mentioned above, we have used both a finite-range force with a Gaussian radial shape and a zero-range interaction. To completely explore the role of the tensor force, we have performed two different calculations with the Gaussian potential, with and without the tensor terms, respectively. In both cases we have not tried to optimize the parameters of the interaction to reproduce the specific properties of ^{176}Lu, but have adopted the values determined by Boisson *et al.* [5]. These are $u_1=-15$ MeV, $u_2=-0.1$ MeV, and $u_3=8.4$ MeV for the central force; $u_1=-15$ MeV, $u_2=-0.1$ MeV, $u_3=8.4$ MeV, $V_T=-55$ MeV, and $V_{TM}=-68$ MeV for the central plus tensor force. The value of the range r_0 is 1.4 fm. The first set of values was obtained by fitting a sample of data containing GM splittings as well as N shifts in the rare-earth region. As regards the second set, the values of the parameters of the central force were not redetermined and V_T and V_{TM} were fixed by fitting the same sample of data. The constant term u_0 in Eq. (5) does not contribute either to the GM splittings or to the N shifts and therefore it was not determined by the fit of Ref. [5]. We have fixed this parameter at -10 MeV.

As already pointed out in the Introduction, a different procedure was used instead by Nosek *et al.* [8] to determine the parameters of the central and central plus tensor force. As a result, for each interaction two different sets of parameters were obtained, one for the fit of the GM splittings and the other for the N shifts. This means that a single interaction was not made available in this work. Open to criticism is also the fact that the samples of experimental data used for the fits by these authors contained information on nuclei of both the rare-earth and the actinide regions. In our opinion, to have the same effective interaction for so vast an ensemble of nuclei is certainly questionable.

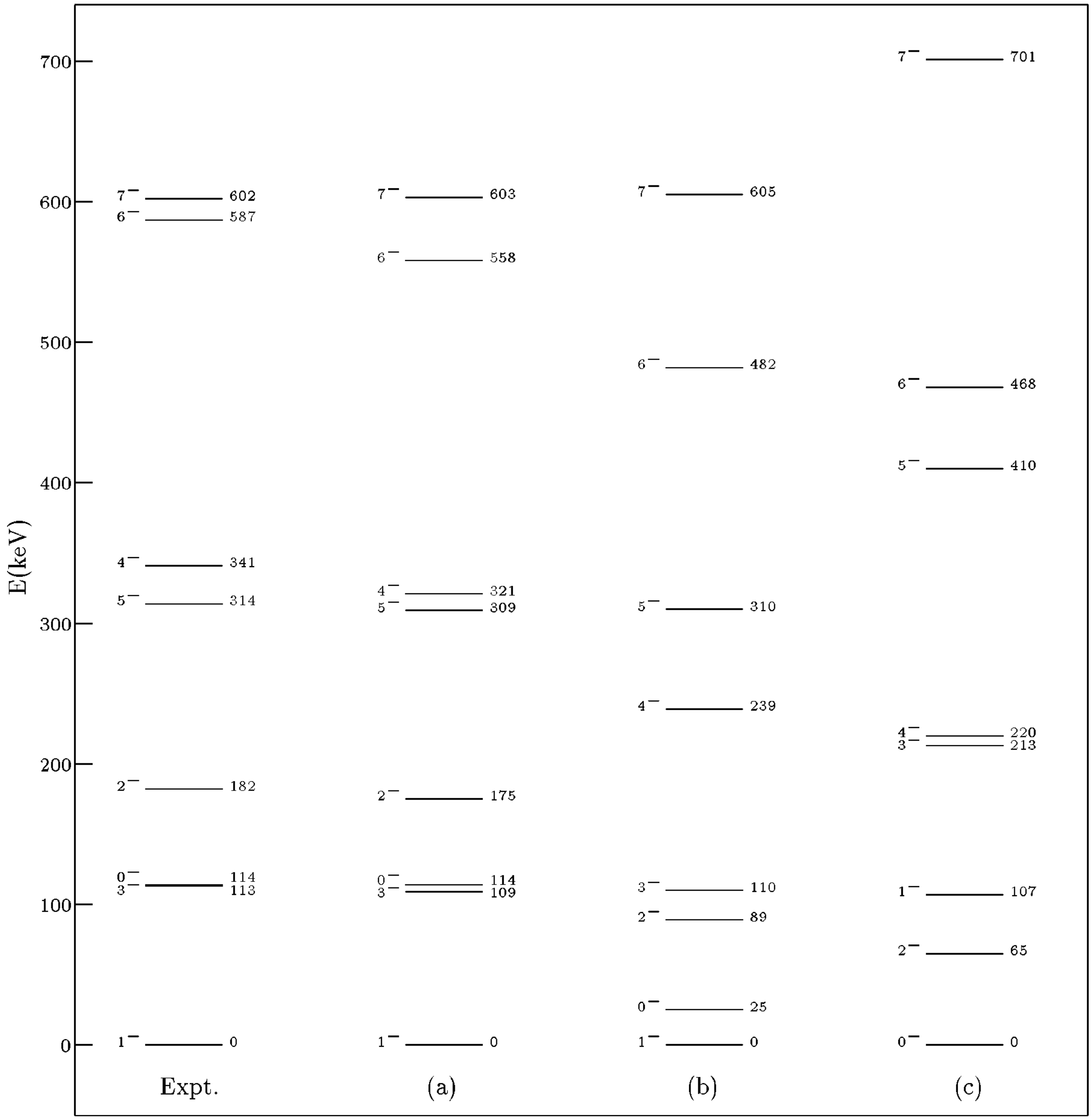

FIG. 1. Experimental and calculated spectra of the lowest $K^{\pi}=0^{-}$ band in ^{176}Lu. The theoretical spectra have been obtained by using (a) a central plus tensor force with a Gaussian radial shape, (b) a Gaussian central force, and (c) a δ force.

While the role of the tensor force in the description of the N shifts is still not assessed, there is a wide consensus, as pointed out in the Introduction, that a zero-range force is inadequate. This interaction, however, is still very popular. In particular it has been used in recent papers [15,16] to explain the so-called signature inversion in odd-odd nuclei. Therefore, for the sake of completeness, we have also made use of a δ interaction. As regards the strength of the spin-spin parameter v_1, we have varied it over a large range, namely, between -0.2 and -0.9 MeV. The GM splittings are well reproduced by the value -0.9 MeV, which is in accordance with the values determined by other authors. No value of v_1 leads to a satisfactory description of the N shifts, the least disagreement occurring for $v_1=-0.2$ MeV. This value comes close to that (-0.191 MeV) determined in Ref. [8] by fitting the experimental N shifts and also to that originally used by Pyatov (-0.24 MeV) [7]. On these grounds we have adopted in the present work the value $v_1=-0.2$ MeV. As in the case of the Gaussian force, we have taken for v_0 the value -10 MeV.

III. RESULTS

The nucleus ^{176}Lu has been extensively studied over the past ten years also because of its great interest for nuclear astrophysics. A very complete level scheme has been presented by Klay *et al.* [17]. In this work a total of 97 energy levels were observed and, in particular, 31 rotational bands, with the corresponding Nilsson configurations, were identified. All the bandheads were found to lie below 1.1 MeV excitation energy. Above this energy the experimental information is very scanty [18]. In our complete Coriolis-mixed calculations we have included, as can be seen from Table I, 96 rotational bands. Of these, 31 correspond to those determined experimentally and all their calculated bandhead energies, except two, turn out to lie below 1.1 MeV. Out of the remaining 65 bandheads only 6 are predicted to lie below 1.1 MeV.

In this paper, we have focused attention on only one of the two established $K=0$ bands. This is the band $K^{\pi}=0^{-}$

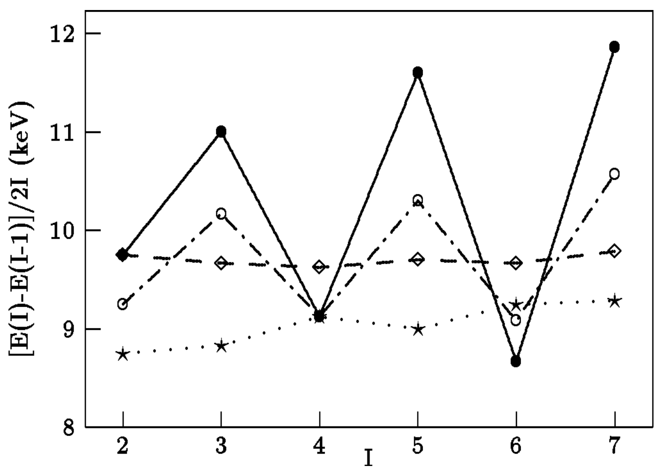

FIG. 2. Experimental and calculated odd-even staggering of the lowest $K^{\pi}=1^{+}$ band in ^{176}Lu. Solid circles correspond to experimental data. The theoretical results are represented by open circles (Gaussian central plus tensor force), diamonds (Gaussian central force), and stars (δ force).

TABLE II. Experimental and calculated GM splittings (keV).

Configuration Proton	Neutron	K^{π}		Expt.	Calc.
$\frac{7}{2}[404]$	$\frac{7}{2}[514]$	0^-	7^-	246	200
$\frac{9}{2}[514]$	$\frac{7}{2}[514]$	1^+	8^+	-219	-154
$\frac{7}{2}[404]$	$\frac{9}{2}[624]$	1^+	8^+	-12	-90
$\frac{5}{2}[402]$	$\frac{7}{2}[514]$	1^-	6^-	-120	-98
$\frac{7}{2}[404]$	$\frac{5}{2}[512]$	1^-	6^-	-66	-121
$\frac{1}{2}[411]$	$\frac{7}{2}[514]$	3^-	4^-	128	137
$\frac{7}{2}[404]$	$\frac{1}{2}[510]$	3^-	4^-	-112	-100
$\frac{1}{2}[541]$	$\frac{7}{2}[514]$	3^+	4^+	103	103
$\frac{5}{2}[402]$	$\frac{9}{2}[624]$	2^+	7^+	186	160
$\frac{7}{2}[404]$	$\frac{1}{2}[521]$	3^-	4^-	65	105
$\frac{9}{2}[514]$	$\frac{1}{2}[510]$	4^+	5^+	224	99
$\frac{7}{2}[404]$	$\frac{3}{2}[512]$	2^-	5^-	229	87

$p\frac{7}{2}[404]n\frac{7}{2}[514]$ starting at 237 keV excitation energy. The description of the other $K=0$ band at 780 keV, with the assigned Nilsson configuration $p\frac{9}{2}[514]n\frac{9}{2}[624]$, is beyond the scope of our calculation. In fact, this band is likely to be characterized by a strong admixture of different intrinsic wave functions, being based on high-j unique-parity shell-model states, the $h_{11/2}$ and $i_{13/2}$ for the proton and neutron Nilsson orbitals, respectively. It should be noted that in our calculations we have included only the $\frac{9}{2}[514]$ and $\frac{7}{2}[523]$ levels originating from $h_{11/2}$ and the $\frac{9}{2}[624]$ and $\frac{7}{2}[633]$ from $i_{13/2}$. In this connection, another point has to be stressed here. Aside from the residual neutron-proton interaction, other mechanisms may be responsible for the odd-even shift. In fact, a contribution to this effect is also given by the signature-dependent parts of some matrix elements of $H_{\rm ppc}$ and $H_{\rm RPC}$. For those bands based on high-j unique-parity shell-model states a complex interplay of the various interactions may occur, thus making it a very difficult task to disentangle the effects of the neutron-proton interaction.

In Fig. 1 the three calculated spectra for the first $K^{\pi}=0^{-}$ band are compared with the experimental one. The theoretical spectra have been obtained making use of the three different interactions mentioned in Sec. II. We see the that the right level order and an excellent agreement with experiment is obtained for the Gaussian force with tensor terms. The experimental N shift (70 keV) is exactly reproduced and the largest discrepancy in the excitation energies is only about 30 keV. This is not the case for the calculations performed with the pure central finite-range interaction and the δ force. From Fig. 1 it appears that the three calculations yield almost the same level spacings for the states with even and odd angular momenta separately. This means that the main difference between the outcomes of these calculations resides in the relative displacement in the energy of the states with odd and even angular momentum, i.e., the N shift. While in case (a) this is in agreement with the experimental value, it becomes too small in case (b), and in case (c) it has even the wrong sign.

Let us now come to the odd-even staggering in the $K^{\pi}=1^{+}$ $p\frac{9}{2}[514]n\frac{7}{2}[514]$ band. In Fig. 2 the experimental ratio $[E(I)-E(I-1)]/2I$ for this band is plotted vs I and compared with the calculated ones. This band, whose experimental bandhead energy is at 197 keV, exhibits, as already mentioned in the Introduction, the greatest magnitude of staggering. We see that the experimental behavior is satisfactorily reproduced by the calculation including the tensor force. When using the pure central Gaussian force the staggering is almost nonexistent and in the case of the δ force not only is its magnitude very small, but it has also the opposite phase. We have already mentioned in the Introduction that the reason for the staggering in the $K>0$ bands is the coupling with $K=0$ bands through the Coriolis interaction. In all of our three calculations we have found that the wave functions of the states of the $K^{\pi}=1^{+}$ band contain significant components (5–10%) of states with $K^{\pi}=0^{+}$ $p\frac{7}{2}[523]n\frac{7}{2}[514]$. The Coriolis interaction is favored, in fact, by the presence of the $\frac{9}{2}[514]$ and $\frac{7}{2}[523]$ single-particle levels arising from the $h_{11/2}$ shell-model state. The fact that only the calculation including the tensor terms gives the right staggering implies that only this force is able to reproduce a sizable N shift for the $K^{\pi}=0^{+}$ $p\frac{7}{2}[523]n\frac{7}{2}[514]$ band. Unfortunately, this band has not been definitely recognized. A tentative assignment of the $I^{\pi}=0^{+}$ through 5^{+} members of this band was made by Dewberry *et al.* [19] but it is not reported in Ref. [18].

We would like to conclude this section by comparing, in Table II, all the experimental GM splittings [17] with the calculated ones. We only report the results obtained with the central plus tensor force. Our purpose is, in fact, to show that this interaction with the parametrization of Ref. [5] is able to give a good description of ^{176}Lu on the whole. All the signs are reproduced and also the quantitative agreement can be considered quite satisfactory. The discrepancy with experiment is larger than 100 keV in two cases only.

IV. CONCLUDING REMARKS

In this paper, we have performed a detailed study of the doubly odd deformed nucleus ^{176}Lu within the framework of the particle-rotor model. Our aim was to assess the role of the effective neutron-proton interaction, with particular attention focused on the tensor force. To this end, we considered the lowest $K^{\pi}=0^{-}$ and $K^{\pi}=1^{+}$ bands where the effects of the neutron-proton interaction are particularly evident. In our calculations we have used a δ interaction and a Gaussian force with and without tensor terms.

The results of our calculations lead to the conclusion that the space-exchange and spin-spin space-exchange forces as well as the tensor force are very relevant for $K=0$ bands and, owing to the Coriolis coupling, also for some $K>0$ bands. As already mentioned in the Introduction, the importance of the exchange forces has also been pointed out in recent works [8,9]. It is the main achievement of our work to have evidenced the role of the tensor force.

Finally, it is worth emphasizing that the central plus tensor force used in our calculations is just that originally proposed by Boisson *et al.* [5]. The results obtained for ^{176}Lu indicate that this force may be profitably used for a systematic study of doubly odd deformed nuclei in the rare-earth region. Work in this direction is in progress.

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