Medium effect on charge symmetry breaking

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We examine the nuclear medium effect on charge symmetry breaking (CSB) caused by isospin mixing of two neutral vector mesons interacting with nucleons in the nuclear medium. Isospin mixing is assumed to occur through the transition between isoscalar and isovector mesons. We use a quantum hadrodynamic nuclear model in the mean-field approximation for the meson fields involved. We find that (i) charge symmetry is gradually restored in nuclear matter in β equilibrium as the nucleon density increases; (ii) when the system departs from β equilibrium, CSB is much enhanced because the isospin mixing depends strongly on the nucleon isovector density; (iii) this leads to the symmetry energy coefficient of 32 MeV, of which more than 50 percent arises from the mesonic mean fields; (iv) the Nolen-Schiffer anomaly regarding the masses of neighboring mirror nuclei can be resolved by considering these aspects of CSB in nuclear medium. [S0556-2813(97)02012-8]

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I. INTRODUCTION

In our previous work [1], we examined the effect of charge symmetry breaking (CSB) due to isospin mixing of the isoscalar and isovector mesons on nuclear matter properties. Identifying the mesons, each of which has a small component of ''wrong'' isospin with the physical ω and ρ^0 mesons, we found that a delicate cancellation takes place between two large CSB contributions from these physical mesons and results in a small yet significant isovector asymmetry in nuclear matter. This isovector asymmetry can give neighboring mirror nuclei a mass difference which is in the right direction to account for the Nolen-Schiffer anomaly [2].

The extent of the isospin mixing is usually parametrized in terms of a parameter called the mixing angle, which can depend on the nucleon density of the medium. In Ref. [1] we took the mixing angle as a free parameter and varied it within a certain reasonable range. We evaluated the mass difference between neighboring mirror nuclei for various values of the mixing angle in that range. The result was that the mixing angle determined to fit the difference between the nn and Coulomb corrected pp scattering lengths yielded a mass difference insufficient to resolve the Nolen-Schiffer anomaly fully.

The purpose of this paper is to examine the nucleon density dependence of the mixing angle in nuclear matter, and thereby, the nucleon density dependence of CSB. A typical manifestation of CSB in nuclear systems is seen in the small difference between the nn and pp scattering lengths that remains after the Coulomb correction is made [3]. The mass difference between ^{3}H and ^{3}He is another example that could be understood in terms of CSB due to the isospin mixing [4–6]. It has been suggested that the isospin mixing of vector mesons can account for most of the difference between the nn and Coulomb corrected pp scattering lengths [6–8]. We propose that a significant CSB effect in nuclear medium will be due to the isospin mixing of vector mesons also.

The isospin mixing can be dictated by the transition between two vector mesons in different isospin eigenstates through a baryon loop. The basic idea was initially discussed by Piekarewicz and Williams [9]. We extend this to incorporate the medium effect in the following way. A vector meson in the isosinglet or isotriplet state dissociates virtually into a pair of baryons. The pair can be a nucleon and an antinucleon or a nucleon and a hole in the Fermi sea. The process is followed by recombination of the pair leading to another vector meson in an isospin eigenstate different from the one before the dissociation. Owing to the Pauli principle for the nucleon appearing in the intermediate state of the transition, the magnitude of the isospin mixing depends on the nucleon density. When the transition takes place for the vector meson exchanged between two nucleons, the third component of the isospin matrices which is attached at the vertex associated with the isovector meson and nucleon gives opposite signs to the emerging nuclear forces between two protons and between two neutrons. This results in CSB in the nucleon-nucleon interaction and the extent of this CSB depends on the nucleon density. We will see that this process can take place in nuclear medium even when the masses of the proton and neutron are equal, provided that the proton and neutron densities are different.

The effect of CSB due to the isospin mixing on nuclear matter properties is evaluated with the help of a quantum hadrodynamic nuclear model which can explain the bulk properties of nuclear matter. The nuclear model which we use is the same one proposed by Zimanyi and Moszkowski (ZM) apart from the additional isovector degree of freedom [10,11]. The model by ZM is a modified version of the standard σ-ω model (SSOM). The SSOM reproduces the correct saturation density and binding energy per nucleon of nuclear matter, but tends to give too large a value of the compression modulus [12,13]. This difficulty was resolved in ZM's modified σ-ω model by introducing a derivative coupling of σ with the nucleon. We utilize ZM's model with the scheme of the mean-field approximation (MFA) in this work.

In Sec. II we propose a version of the σ-ω model in which an interaction is incorporated such that transitions between two vector mesons in different isospin eigenstates take place. The study of nuclear matter with broken charge symmetry is formulated by means of ZM's model in the MFA in Sec. III. The matrix element of the transition between the vector mesons with different isospins is given in Sec. IV, and an approximation is made to simplify it. Section V is devoted to a numerical calculation and a discussion of the medium effect on CSB. There, also, we justify a few simplifying assumptions which we make in the calculations.

II. σ-ω MODEL WITH BROKEN ISOSPIN SYMMETRY

We start with the Lagrangian density

$$\mathcal{L}=\overline{\psi}[\gamma^\mu(i\partial_\mu-G_0V^{(0)}_\mu-G_1\tau_3V^{(1)}_\mu)-\hat{M}^*(\phi)]\psi+\frac{1}{2}(\partial_\mu\phi\partial^\mu\phi-m_\sigma^2\phi^2)+\sum_I\left(-\frac{1}{4}F^{(I)}_{\mu\nu}F^{(I)\mu\nu}+\frac{1}{2}m_I^2V^{(I)}_\mu V^{(I)\mu}\right), \tag{1}$$

where ψ is the nucleon field, ϕ the neutral scalar (σ) field, suffix I refers to the isospin which is 0 or 1, $V^{(I)}_\mu$ is the field describing the vector meson with isospin I, $F^{(I)}_{\mu\nu}=\partial_\mu V^{(I)}_\nu-\partial_\nu V^{(I)}_\mu$, m_I, and G_I are the mass and the coupling constant of the vector meson with isospin I, respectively, and τ_3 is the third component of the Pauli isospin matrices.[1] We follow the metric convention of Bjorken and Drell [14]. The nucleon field is a spinor in isospin space and explicitly written as

$$\psi(x)=\begin{pmatrix}\psi_p(x)\\ \psi_n(x)\end{pmatrix}, \tag{2}$$

where the upper component is for the proton and the lower for the neutron. The ϕ-dependent "mass" matrix $\hat{M}^*(\phi)$ is defined by

$$\hat{M}^*(\phi)=\begin{pmatrix}\alpha_p(\phi)M_p & 0\\ 0 & \alpha_n(\phi)M_n\end{pmatrix}, \tag{3}$$

[1] We assume that the isosinglet vector meson has components which belong to the SU(3) octet and singlet.

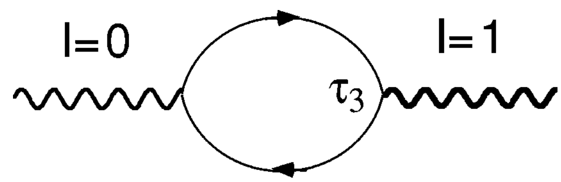

FIG. 1. Illustration of the transition between the isoscalar and isovector mesons. The thin and thick wavy lines represent the vector mesons with $I=0$ and 1, respectively. The third component of the Pauli isospin matrices τ_3 is indicated at the isovector meson-nucleon vertex explicitly. The loop represents a nucleon and antinucleon pair or a nucleon and hole pair in the Fermi sea.

with

$$\alpha_b(\phi)=\left(1+f\frac{\phi}{M_b}\right)^{-1}, \tag{4}$$

where $b=p$ or n, M_b is the mass of the free nucleon b, and f the coupling constant of σ with the nucleon field [10]. We ignore the Coulomb potential between protons, which yields a divergent contribution to the energy density of nuclear matter because of its long-range nature. Hence, the nuclear system described by the Lagrangian of Eq. (1) breaks charge symmetry only when $M_p\neq M_n$. We will comment on the effect of the Coulomb potential on CSB in finite nuclei in the last section.

In the nuclear system described by the Lagrangian of Eq. (1), the transition between the isoscalar and isovector mesons is allowed. The S-matrix element for the transition illustrated in Fig. 1 is proportional to

$$\mathcal{M}^{\mu\nu}(k)=iG_0G_1\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu G(p)\gamma^\nu\tau_3G(p-k)] \tag{5}$$

in the lowest order of the meson-nucleon couplings, where k is a four-momentum of the vector mesons and $G(p)$ is the propagator of the nucleon with four momentum p in nuclear medium. The trace is taken over the isospin space as well as the spin space. We will give a detailed expression for $\mathcal{M}^{\mu\nu}$, particularly for $\mathcal{M}^{00}$ which we need in the present study, in Sec. IV. Note that the transition does *not* take place in the nucleon vacuum unless $\delta M_N\equiv M_p-M_n\neq 0$. When the vector meson undergoes the transition to change its isospin in the exchange process between two nucleons, the emerging nuclear force in the one-boson-exchange picture is

$$V^{\mu\nu}_{\mathrm{CSB}}(k)=G_0f^{(0)}(k^2)\gamma^\mu D^{(0)}_{\mu\lambda}(k)\mathcal{M}^{\lambda\rho}(k)D^{(1)}_{\rho\nu}(k)\tau_3\gamma^\nu f^{(1)}(k^2)G_1, \tag{6}$$

where $D^{(I)}_{\mu\nu}(k)$ is the usual propagator of the vector meson with isospin I and $f^{(I)}(k^2)$ is a phenomenological vertex function. This breaks charge symmetry due to the presence of τ_3. It should be emphasized that $V^{\mu\nu}_{\mathrm{CSB}}$ depends on the nucleon density through the nucleon propagators in $\mathcal{M}^{\mu\nu}$ [see Eq. (32) for detail], and is different from CSB potentials for free nucleons.

The effect of CSB on nuclear properties appears remarkably at low energies; the effect is brought about through a large number of nucleon-nucleon collisions. In the MFA, which is used in the next section, only processes with zero four-momentum transfer between the nucleon and meson are allowed. Since $\mathcal{M}^{\mu\nu}(k)$ vanishes for $k=0$, the MFA cannot describe the effect of CSB due to the isospin mixing of vector mesons. In order to incorporate CSB into the scheme of MFA properly, we take advantage of the fact that the momentum transfer between two nucleons in each meson exchange process in multiple scatterings is distributed over a limited range specified by the form factor at the meson-nucleon vertices. In light of this, we propose to replace $\mathcal{M}^{\mu\nu}$ in $V_{\rm CSB}^{\mu\nu}$ by the average with respect to the momentum transfer, taking the form factors as the weight function. In the static limit, thus, $\mathcal{M}^{\mu\nu}$ can be replaced with

$$\overline{\mathcal{M}}^{\mu\nu}(n_p,n_n)=\frac{\int d^3k f^{(0)}(k_0=0,\mathbf{k})\mathcal{M}^{\mu\nu}(k_0=0,\mathbf{k}^2)f^{(1)}(k_0=0,\mathbf{k})}{\int d^3k f^{(0)}(k_0=0,\mathbf{k})f^{(1)}(k_0=0,\mathbf{k})}, \tag{7}$$

where we explicitly showed the dependence of $\overline{\mathcal{M}}^{\mu\nu}$ on the proton and neutron densities denoted with n_p and n_n, respectively. The CSB effect on the low energy scattering parameters can be studied in terms of the OBEP given by Eq. (6) with $\overline{\mathcal{M}}^{\mu\nu}$ substituted for $\mathcal{M}^{\mu\nu}$ in a good approximation. The same OBEP can be obtained in the lowest order of the meson-nucleon coupling by the use of the effective Lagrangian

$$\mathcal{L}_{\rm eff}=\mathcal{L}+V_\mu^{(0)}(x)\epsilon^{\mu\nu}(n_p,n_n)V_\nu^{(1)}(x) \tag{8}$$

with

$$\epsilon^{\mu\nu}(n_p,n_n)=\epsilon_0^{\mu\nu}\frac{\overline{\mathcal{M}}^{\mu\nu}(n_p,n_n)}{\overline{\mathcal{M}}_0^{\mu\nu}}, \tag{9}$$

where $\overline{\mathcal{M}}_0^{\mu\nu}$ stands for $\overline{\mathcal{M}}^{\mu\nu}$ with vanishing nucleon density ($n_B\equiv n_p+n_n\to 0$), and $\epsilon_0^{\mu\nu}$ is fixed such that the discrepancy between the pp and nn scattering lengths is reproduced. In Eq. (9), we take into account all factors leading to CSB for $n_B=0$ by introducing $\epsilon_0^{\mu\nu}$ phenomenologically and thus separate out the medium effect on CSB. We emphasize that the approximation leading to momentum-independent $\epsilon^{\mu\nu}$ allows us to make this simplification in a benign way.

III. MEAN-FIELD APPROXIMATION

We apply the MFA to the field equations which are obtained from Eq. (8). Many authors have discussed the validity of the MFA in the σ-ω model and found it to be a good approximation to study nuclear matter or heavy nuclei in the relativistic scheme [12,13]. For nuclear matter, the nucleon density n_B is fixed at a given value and its partition into proton and neutron densities is determined such that the system has the lowest energy. Spatial isotropy is assumed throughout. In the MFA, ϕ, $V_\mu^{(0)}$, and $V_\mu^{(1)}$ are replaced by static and uniform classical fields, which we denote with ϕ_c, $\delta_{\mu 0}V_c^{(0)}$, and $\delta_{\mu 0}V_c^{(1)}$, respectively. The Lagrangian in the MFA is, thus, given by

$$\begin{aligned}\mathcal{L}_{\rm MFA}=&\bar{\psi}[i\gamma^\mu\partial_\mu-G_0\gamma^0V_c^{(0)}-\gamma^0G_1\tau_3V_c^{(1)}-\hat{M}^*(\phi_c)]\psi\\&-\frac{1}{2}(m_\sigma^2\phi_c^2-m_0^2V_c^{(0)2}-m_1^2V_c^{(1)2})\\&+\epsilon(n_p,n_n)V_c^{(0)}V_c^{(1)}\ .\end{aligned} \tag{10}$$

We suppress the tensor indices of ϵ^{00}. This Lagrangian is similar to the one which we have used in our previous work except for the nucleon density dependence of the isospin mixing interaction.

Introducing the vector meson fields defined by

$$\omega_c=V_c^{(0)}\cos\theta-V_c^{(1)}\sin\theta, \tag{11}$$

$$\rho_c=V_c^{(0)}\sin\theta+V_c^{(1)}\cos\theta, \tag{12}$$

we eliminate the isospin mixing term. The mean-field Lagrangian reduces to

$$\begin{aligned}\mathcal{L}_{\rm MFA}=&\bar{\psi}[i\gamma^\mu\partial_\mu-g\gamma^0\omega_c-\gamma^0g'\tau_3\rho_c-\hat{M}^*(\phi_c)]\psi\\&-\frac{1}{2}(m_\sigma^2\phi_c^2-m_\omega^2\omega_c^2-m_\rho^2\rho_c^2).\end{aligned} \tag{13}$$

The angle θ, which represents the degree of the isospin mixing, is determined by

$$\tan 2\theta=\frac{2\epsilon(n_p,n_n)}{m_1^2-m_0^2}\ . \tag{14}$$

Remember that θ depends on the proton and neutron densities through $\epsilon(n_p,n_n)$. We identify ω_c and ρ_c with the classical fields which represent the *physical* ω and ρ^0 mesons. The m_ω and m_ρ are defined by

$$m_\omega^2=\frac{m_0^2+m_1^2}{2}+\frac{m_0^2-m_1^2}{2\cos 2\theta}, \tag{15}$$

$$m_\rho^2=\frac{m_0^2+m_1^2}{2}-\frac{m_0^2-m_1^2}{2\cos 2\theta}, \tag{16}$$

and regarded as the observed masses of ω and ρ^0 for $n_B=0$, namely, $m_\omega=782$ MeV and $m_\rho=770$ MeV for $n_B=0$. Each of ω_c and ρ_c has a small component of ‘‘wrong’’ isospin when $\epsilon \neq 0$. The coupling constants of ω and ρ^0 with the nucleon can be written in a matrix form in the isospin space:

$$g=\begin{pmatrix} g_p & 0 \\ 0 & g_n \end{pmatrix}=\begin{pmatrix} G_0\cos\theta - G_1\sin\theta & 0 \\ 0 & G_0\cos\theta + G_1\sin\theta \end{pmatrix}, \tag{17}$$

$$g'=\begin{pmatrix} g'_p & 0 \\ 0 & g'_n \end{pmatrix}=\begin{pmatrix} G_1\cos\theta + G_0\sin\theta & 0 \\ 0 & G_1\cos\theta - G_0\sin\theta \end{pmatrix}. \tag{18}$$

Since m_0 and m_1 are independent of n_B, Eq. (14) can be expressed in terms of θ and θ_0, which is the θ for $n_B=0$, to be

$$\tan 2\theta = \frac{\overline{\mathcal{M}}^{00}(n_p,n_n)}{\overline{\mathcal{M}}_0^{00}} \tan 2\theta_0 \quad . \tag{19}$$

We have distinguished the proton and neutron in their masses so far, so that CSB could be induced for free nucleons through the mutual transition of the vector mesons in different isospin eigenstates. In other words, the nonvanishing θ_0 originates in the nonvanishing δM_N. While δM_N plays a crucial role in arriving at $\epsilon(n_p,n_n)$, its effect on the bulk properties of nuclear matter is insignificant. Therefore we shall maintain the nonvanishing δM_N only when we determine $\epsilon(n_p,n_n)$, otherwise ignore it by equating M_p and M_n to their average M. We will comment on this approximation in the last section in connection with the exclusion of the Coulomb potential between protons.

The equation for the nucleon field is derived from $\mathcal{L}_{\mathrm{MFA}}$ to be

$$[i\gamma^\mu \partial_\mu - M^*(\phi_c)]\psi = (g\gamma^0\omega_c + g'\tau_3\gamma^0\rho_c)\psi \tag{20}$$

with

$$\phi_c = \frac{f^*(\phi_c)}{m_\sigma^2} n_s, \tag{21}$$

$$\omega_c = \frac{g_p}{m_\omega^2} n_p + \frac{g_n}{m_\omega^2} n_n, \tag{22}$$

$$\rho_c = \frac{g'_p}{m_\rho^2} n_p - \frac{g'_n}{m_\rho^2} n_n, \tag{23}$$

where n_s, the scalar density, and n_b ($b=p$ or n) are evaluated with

$$n_s = \langle \bar{\psi}\psi \rangle, \tag{24}$$

$$n_b = \langle \psi_b^\dagger \psi_b \rangle, \tag{25}$$

$\langle A \rangle$ is the expectation value of quantity A in the ground state, and

$$f^*(\phi_c) = \left(1 + f\frac{\phi_c}{M}\right)^{-2} f, \tag{26}$$

$$M^*(\phi_c) = \left(1 + f\frac{\phi_c}{M}\right)^{-1} M. \tag{27}$$

One can solve the wave equation corresponding to Eq. (20) to determine the eigenenergies of the nucleon and antinucleon:

$$W_b^{(\pm)}(\mathbf{k}) = \sqrt{M^{*2}+\mathbf{k}^2} \pm \sum_{b'} \left(\frac{g_b g_{b'}}{m_\omega^2} + \eta_b \eta_{b'} \frac{g'_b g'_{b'}}{m_\rho^2} \right) n_{b'} \tag{28}$$

where $W_b^{(+)}$ and $W_b^{(-)}$ are the energies of nucleon and antinucleon, respectively, the summation with b' runs over p and n, and $\eta_b = 1$ for proton and -1 for neutron.

For nuclear matter the ground state is formed by accommodating protons and neutrons in the particle states such that their energies are below appropriate Fermi energies. Corresponding Fermi momenta, which we denote with K_b ($b=p,n$), are determined by minimizing the energy density with respect to either n_p or n_n under a given nucleon density $n_B = n_p + n_n$ with the relation $K_b = (3\pi^2 n_b)^{1/3}$. The energy density of the ground state is

$$\mathcal{E} = \frac{1}{8\pi^2}\sum_b \left[2K_b E_{bF}^{*3} - K_b E_{bF}^* M^{*2} - M^{*4} \ln\left(\frac{K_b + E_{bF}^*}{M^*} \right) \right] + \frac{m_\sigma^2}{2f^2}\left(\frac{M}{M^*}\right)^2 (M-M^*)^2 + \frac{(g_p n_p + g_n n_n)^2}{2m_\omega^2} + \frac{(g'_p n_p - g'_n n_n)^2}{2m_\rho^2}, \tag{29}$$

where

$$E_{bF}^* = \sqrt{M^{*2}+K_b^2} \quad . \tag{30}$$

Equations (21) and (26) can be combined to create a self-consistent equation to determine the scalar mean-field ϕ_c. Hence, with the help of Eq. (27), we derive the equation for the effective mass M^*

$$M^* = M - \frac{f^2 M}{2\pi^2 m_\sigma^2}\left(\frac{M^*}{M}\right)^4 \sum_b \times \left[K_b E_{bF}^* - M^{*2} \ln\left(\frac{K_b + E_{bF}^*}{M^*} \right) \right] \quad . \tag{31}$$

This will be obtained likewise by minimizing $\mathcal{E}$ with respect to M^*.

IV. TRANSITION MATRIX OF VECTOR MESONS

Here we give the explicit expression for $\mathcal{M}^{00}(k_0=0,\mathbf{k})$ which is substituted into Eq. (7) to determine θ through Eq. (19). In nuclear matter the nucleon propagator is given by [15]

$$G_b(p)=\frac{\gamma P_b+M_b^*}{P_b^2-M_b^{*2}}+i\pi\frac{\gamma P_b+M_b^*}{\widetilde{E}_b^*(\mathbf{p})}\delta[P_{b0}-\widetilde{E}_b^*(\mathbf{p})]\theta(K_b-|\mathbf{p}|), \tag{32}$$

where

$$\widetilde{E}_b^*(\mathbf{p})=\sqrt{M_b^{*2}+\mathbf{p}^2}, \tag{33}$$

M_b^* is the effective mass of nucleon b ($b=p$ or n) appearing in the intermediate state of the transition process, and P_b^μ is the four-momentum $(P_{b0},\mathbf{p})$ in which the time component is defined in terms of the time component p_0 of p^μ by

$$P_{b0}=p_0-\sum_{b'}\left(\frac{g_b g_{b'}}{m_\omega^2}+\eta_b\eta_{b'}\frac{g_b' g_{b'}'}{m_\rho^2}\right)n_{b'} \quad . \tag{34}$$

After a little algebra we obtain

$$\mathcal{M}^{00}(k_0=0,\mathbf{k};n_p,n_n)=\frac{k^2}{\pi^2}\int_0^1 dz z(1-z)\ln\left|\frac{M_p^{*2}+z(1-z)k^2}{M_n^{*2}+z(1-z)k^2}\right|-\frac{1}{2\pi^2 k}\sum_b \eta_b\int_0^{K_b}dp\,p\,\frac{4\widetilde{E}_b^{*2}(p)-k^2}{\widetilde{E}_b^*(p)}\ln\left|\frac{2p+k}{2p-k}\right|-\frac{1}{\pi^2}\sum_b \eta_b\left\{K_b\widetilde{E}_{bF}^*-M_b^{*2}\ln\left(\frac{K_b+\widetilde{E}_{bF}^*}{M_b^*}\right)\right\}, \tag{35}$$

where

$$\widetilde{E}_{bF}^*=\sqrt{M_b^{*2}+K_b^2} \quad , \tag{36}$$

and p and k on the right-hand side represent $|\mathbf{p}|$ and $|\mathbf{k}|$, respectively. Note that the term arising from the interaction with vector fields in Eq. (34) produces no influence on $\mathcal{M}^{00}$ after the p_0 integration in Eq. (5) is performed. Needless to say, $\mathcal{M}^{00}$ vanishes when $M_p=M_n$ and $K_p=K_n$. In order to determine M_b^*, one needs coupled self-consistent equations for M_p^* and M_n^* which would follow if we maintained δM_N in the MFA. Instead of doing so, however, we multiply M_b by (M^*/M) to obtain M_b^* approximately. We will see the validity of this approximation in the next section.

Now we take the average of $\mathcal{M}^{00}$ according to Eq. (7) with the form factors $f^{(I)}$ which are the same as those used in the Bonn potential calculation [16]:

$$f^{(I)}(k^2)=\left(\frac{\Lambda_I^2-m_I^2}{\Lambda_I^2-k^2}\right)^2, \tag{37}$$

where the cutoff masses Λ_I are taken to be 1850 MeV for both of $I=0$ and 1. A lengthy but straightforward calculation gives

$$\overline{\mathcal{M}}^{00}(n_p,n_n)=\frac{\Lambda^2}{3\pi^2}\ln\left(\frac{M_p^*}{M_n^*}\right)-\frac{\Lambda^2}{3\pi^2}\sum_b \eta_b\left[\frac{1}{\Lambda^2}\left\{6K_b\widetilde{E}_{bF}^*-\Lambda^2\ln\left(\frac{K_b+\widetilde{E}_{bF}^*}{M_b^*}\right)\right\}-\frac{4K_b\widetilde{E}_{bF}^*}{\Lambda^2+4K_b^2}\times\left\{1-\frac{4M_b^{*2}\Lambda^2}{(\Lambda^2-4M_b^{*2})(\Lambda^2+4K_b^2)}-\frac{4M_b^{*2}(\Lambda^2+2M_b^{*2})}{(\Lambda^2-4M_b^{*2})^2}\right\}-\frac{8M_b^{*2}(\Lambda^2+2M_b^{*2})-\Lambda^2(\Lambda^2-10M_b^{*2})W_b}{2(\Lambda^2-4M_b^{*2})^2}\right] \tag{38}$$

where we denote both of Λ_I for I=0 and 1 with Λ.

with

$$W_b=\frac{2\Lambda}{\sqrt{4M_b^{*2}-\Lambda^2}}\left[\tan^{-1}\left(\frac{K_b}{\Lambda\widetilde{E}_{bF}^*}\sqrt{4M_b^{*2}-\Lambda^2}\right)-\sin^{-1}\left(\frac{\sqrt{4M_b^{*2}-\Lambda^2}}{2M_b^*}\right)\right] \quad \text{when} \quad 2M_b^*>\Lambda, \tag{39}$$

$$=\frac{\Lambda}{\sqrt{\Lambda^2-4M_b^{*2}}}\left[\ln\left|\frac{K_b\sqrt{\Lambda^2-4M_b^{*2}}+\Lambda\widetilde{E}_{bF}^*}{K_b\sqrt{\Lambda^2-4M_b^{*2}}-\Lambda\widetilde{E}_{bF}^*}\right|-\ln\left|\frac{\Lambda+\sqrt{\Lambda^2-4M_b^{*2}}}{\Lambda-\sqrt{\Lambda^2-4M_b^{*2}}}\right|\right] \quad \text{when} \quad 2M_b^*<\Lambda, \tag{40}$$

V. SELF-CONSISTENT CALCULATION AND DISCUSSION

We have three adjustable parameters in our model: f, G_0, and G_1. For G_0 and G_1, we use the SU(6) ratio $G_0/G_1=3$ [17], so that only two parameters are free.[2] We fix the mixing angle θ_0 in Eq. (19) such that the discrepancy between the pp and nn scattering lengths is reproduced [3], assuming that the isospin mixing alone yields its amount. The Bonn potential produces $\theta_0=4°$ which we have used in our previous work [1,18].

The self-consistent calculation to determine the param-

[2]In giving this ratio, we have assumed the ideal mixing of the isoscalar mesons which belong to the SU(3) octet and singlet.

eters is as follows. First, we choose an arbitrary set of values for two free parameters (f and either G_0 or G_1). Then we share out a given n_B to the proton and neutron densities such that $n_B = n_p + n_n$ is satisfied. We can then solve Eq. (31) numerically to obtain M^*. We evaluate Eqs. (38) and (19) to determine θ for the set of n_p and n_n. All coupling constants of the physical vector mesons to the nucleon are calculated from Eqs. (17) and (18) in terms of θ. The partition of n_B into n_p and n_n is fixed such that the energy of the system is minimized, that is, the system reaches ''β equilibrium'' under a given n_B. Finally the above procedure gives the average nucleon energy which we define to be

$$\mathcal{T} = \frac{\mathcal{E}}{n_B} - M. \tag{41}$$

Two free parameters are determined such that $\mathcal{T}$ is saturated to yield -15.75 MeV for $n_B = 0.17\ \mathrm{fm}^{-3}$, the normal density of nuclear matter. According to this procedure we obtain

$$(M/m_\sigma)f = 12.6317, \quad G_0 = 6.1613, \quad G_1 = 2.0538. \tag{42}$$

The compression modulus is obtained to be $K = 223$ MeV; this is almost the same as the one in ZM's model and receives a negligible CSB effect.

We are interested in (1) the density dependence of the isospin mixing angle θ, (2) the symmetry energy coefficient defined by

$$a_S = \frac{n_B^2}{2}\left(\frac{\partial^2 \mathcal{T}}{\partial n_3^2}\right)_0, \tag{43}$$

where n_3 is the isovector density defined by $n_3 = n_p - n_n$ and the subscript 0 denotes the differentiation taken at equilibrium, and (3) to extract the CSB contribution to the mass difference of neighboring mirror nuclei from our nuclear matter results.

We show θ in Fig. 2. It is seen that CSB due to the isospin mixing of the vector mesons is gradually diminished in nuclear medium, and θ is decreased to 2.21°, to 55% of θ_0, for the normal density of nuclear matter. As n_B is increased further, θ keeps decreasing. This suggests that charge symmetry will be perfectly restored for some higher density. Note that we underestimate the medium effect on θ by ignoring the Coulomb potential, as we will confirm shortly. Therefore perfect restoration of charge symmetry should occur for n_B lower than the trend predicted in Fig. 2.

When β equilibrium is destroyed by varying either n_p or n_n with $n_B = n_p + n_n$ fixed, the energy increases. From this we can evaluate a_S according to Eq. (43). The density dependence of a_S is exhibited with the solid line in Fig. 3. At the normal density of nuclear matter, we obtain

$$a_S = 32.0 \text{ MeV}, \tag{44}$$

which is in good agreement with recent empirical values [19].

We emphasize the significant roles of θ, m_ρ, and m_ω in determining a_S. As suggested in Fig. 2, the CSB effect on various quantities is small in magnitude, as long as the system stays at β equilibrium. The deviations of m_ω and m_ρ from their values for $n_B = 0$ are also negligible. This may mislead us to conclude that the density dependence of the CSB effect is also irrelevant. Suppose, however, we keep θ at its β equilibrium value and change either n_p or n_n with a fixed n_B. The excitation energy which accompanies this change comes mostly from the nucleonic part, the first term on the right-hand side of Eq. (29). We have 15.7 and 3.2 MeV as the nucleonic and mesonic contributions to a_S for the normal density of nuclear matter, respectively, obtaining $a_S = 18.9$ MeV [1]. The density dependence of a_S is shown with the dotted line in Fig. 3. As soon as the system

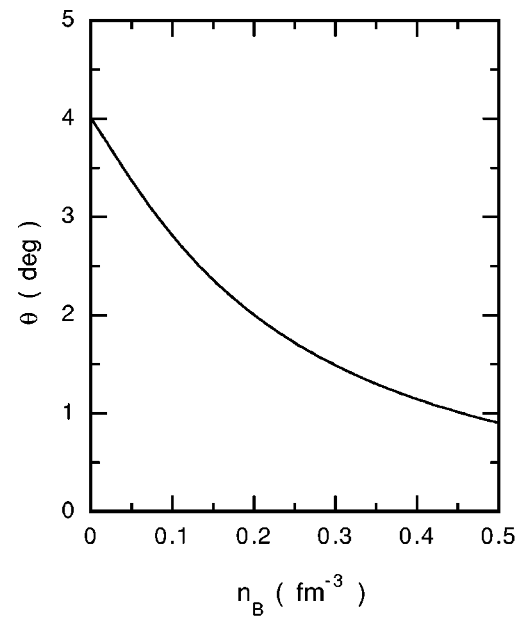

FIG. 2. The isospin mixing angle. The value $\theta = 4°$ for $n_B = 0$ is determined such that the experimental value of $(a_{pp} - a_{nn})/(a_{pp} + a_{nn})$ is fitted.

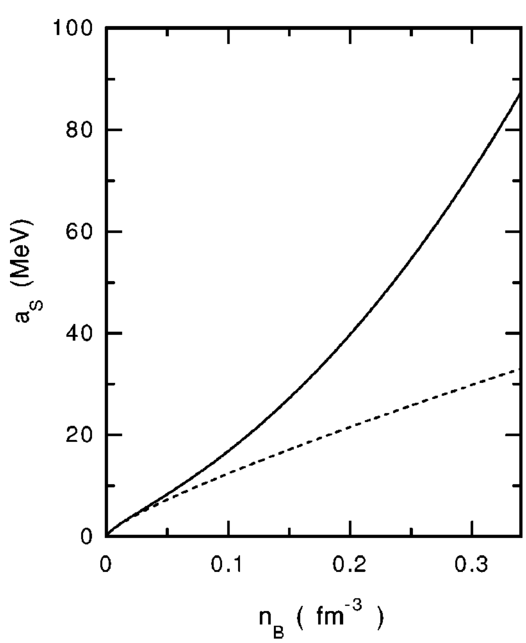

FIG. 3. The symmetry energy coefficient a_S. The solid line shows a_S as obtained from our self-consistent calculation, while the dotted line shows the results which we obtain by fixing θ to the value at β equilibrium.

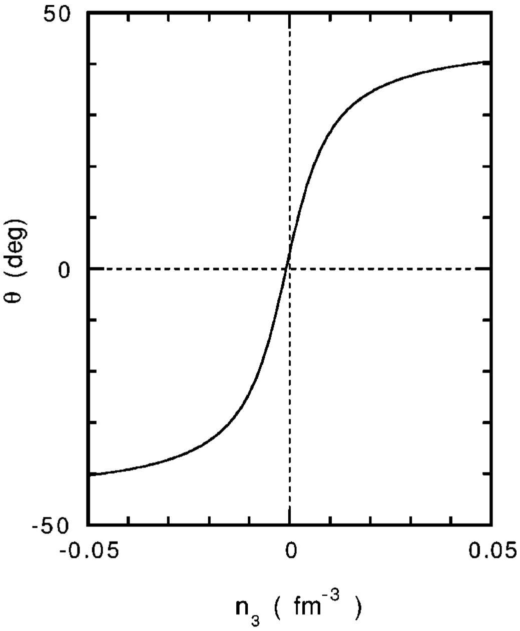

FIG. 4. Behavior of θ around β equilibrium for $n_B=0.17\ \mathrm{fm}^{-3}$, the normal density of nuclear matter. At equilibrium, $\theta=2.21°$.

leaves β equilibrium, however, θ also departs from its equilibrium value rapidly as shown in Fig. 4. The enhancement of the isospin mixing as a result of this large change of θ gives rise to sizable modulations of m_ω and m_ρ away from their equilibrium values, as shown in Fig. 5. Thus the change in the mesonic field energies through large variations of θ, m_ω, and m_ρ becomes as important as that in nucleonic energy. Actually the mesonic contribution to a_S turns out to be 16.3 MeV for the normal density of nuclear matter, while the nucleonic one stays at 15.7 MeV. This is how $a_S=32.0$ MeV arises.

With a_S thus evaluated, let us extract the CSB contribution to the mass difference between mirror nuclei with a large number of nucleons. Since intricate many-body physics is involved in creating the mass differences between heavier

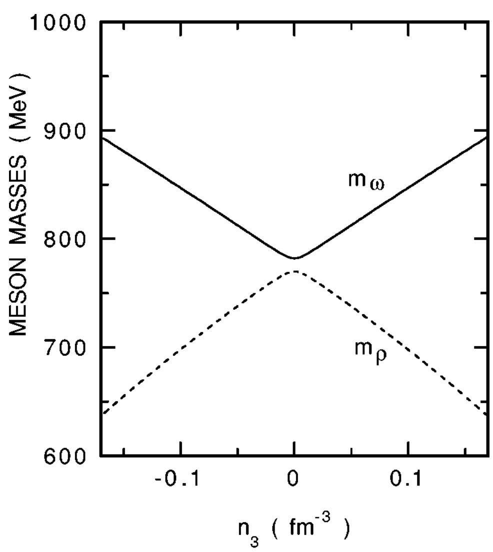

FIG. 5. The vector meson masses as a function of the isovector density for $n_B=0.17\ \mathrm{fm}^{-3}$. The solid and dotted lines represent the masses of ω and ρ mesons.

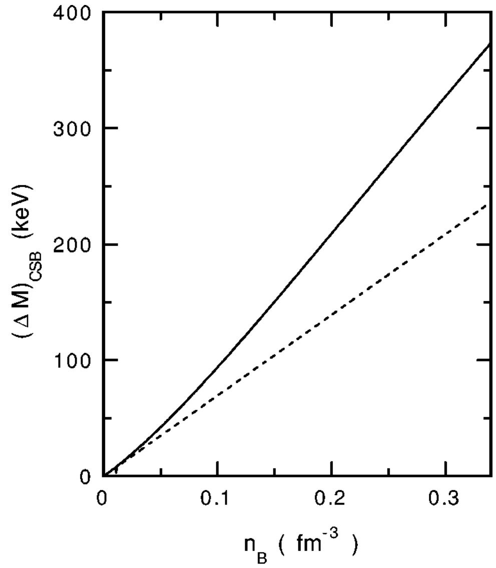

FIG. 6. The mass difference of mirror nuclei due to CSB, $(\Delta M)_{\mathrm{CSB}}$, which was extracted from our nuclear matter calculation. The solid line shows $(\Delta M)_{\mathrm{CSB}}$ with our medium dependent effect included. For the dotted line, θ is fixed to the free space θ_0 throughout.

mirror nuclei, these differences have not been successfully described with the notion of CSB in contrast with the mirror pair of ^{3}H and ^{3}He. Thus, the Nolen-Schiffer anomaly remains in need of further investigation. Here we have in mind a pair of mirror nuclei, one of which has one proton and the other a neutron outside a common spherical core. We define $\Delta M = M_> - M_<$, where $M_>$ and $M_<$ are the masses of such a pair of neighboring mirror nuclei with $(Z+1)$ and Z protons, respectively. Then the CSB contribution to ΔM is given by

$$(\Delta M)_{\mathrm{CSB}} = -4a_S \frac{n_3^{(0)}}{n_B}\ , \tag{45}$$

where $n_3^{(0)}$ is the isovector density at β equilibrium. The result of numerical calculation is shown in Fig. 6. We observe a gradual increase of $(\Delta M)_{\mathrm{CSB}}$ as n_B is increased. This agrees with the general tendency that the mass difference of mirror nuclei becomes large as the mass number is increased [2]. The increase of $(\Delta M)_{\mathrm{CSB}}$ appears contradictory to the restoration of charge symmetry for high density. Actually this arises from a competition between two factors, a_S and $|n_3^{(0)}/n_B|$ in Eq. (45). We have seen in Fig. 3 that a_S increases as n_B is increased. In contrast, $|n_3^{(0)}/n_B|$ decreases as n_B is increased, reflecting the restoration of charge symmetry. Since the increase of a_S dominates over the decrease of $|n_3^{(0)}/n_B|$ for high density, the competition of these two factors results in the gradual increase of $(\Delta M)_{\mathrm{CSB}}$.

The values of $(\Delta M)_{\mathrm{CSB}}$ calculated from Eq. (45) are to be compared with the quantities implied by the Nolen-Schiffer anomaly. Two types of ''empirical'' values as the anomalies have been reported, depending on nonrelativistic or relativistic kinematics applied to a calculation of the binding energies of mirror nuclei. We quote 190 and 298 keV for ^{17}F-^{17}O and ^{41}Sc-^{41}Ca, respectively, as their typical estimates in the nonrelativistic scheme, and 92 and 57 keV, respectively, as those in the relativistic scheme [20]. Let us make an estimate of the interior density of a nucleus with the help of the Woods-Saxon distribution

$$\rho(r)=\frac{\rho_0}{1+\exp[(r-R)/a]}\ , \tag{46}$$

which fits the electron-nucleus scattering data. Here, $R=r_0A^{1/3}$ with $r_0=1.10$ fm, $a=0.53$ fm, and ρ_0 is determined such that $\rho(r)$ is normalized to give the mass number of the nucleus [21]. We identify the central value of $\rho(r)$ with the ''density'' of the nucleus, understanding that we will overestimate the actual average density a little. The densities are 0.12 fm^{-3} for ^{17}F-^{17}O and 0.14 fm^{-3} for ^{41}Sc-^{41}Ca. Our estimates of $(\Delta M)_{\rm CSB}$ based on Eq. (45) are 121 and 143 keV for ^{17}F-^{17}O and ^{41}Sc-^{41}Ca, respectively. They fall into the intervals of the values quoted as the anomalies for these pairs of nuclei in nonrelativistic and relativistic schemes.

The magnitude of $(\Delta M)_{\rm CSB}$ is essentially determined by a_S. As we have mentioned, CSB is much enhanced in nuclear matter when the system leaves β equilibrium. This consideration has made $a_S=32.0$ MeV through the self-consistent procedure, otherwise it would be 18.9 MeV. Therefore, unless we take into account the enhancement of CSB due to the departure of the system from β equilibrium, we underestimate $(\Delta M)_{\rm CSB}$ as we show with the dotted line in Fig. 6. This will also happen in obtaining $(\Delta M)_{\rm CSB}$ for actual nuclei, if we apply a nucleon-nucleon CSB potential which is determined to fit the two-nucleon scattering data and ignore the medium effect on it [22,23]. Actually, if we use θ_0 to evaluate $(\Delta M)_{\rm CSB}$ throughout, for example, we obtain 86 and 100 keV for ^{17}F-^{17}O and ^{41}Sc-^{41}Ca, respectively.

To end our study of the medium effect on CSB, we briefly discuss some points which we have left without discussing in detail. We have ignored δM_N except when we have evaluated θ, and also the Coulomb potential between protons. These are two major and explicit sources of CSB in a nuclear system other than the one we have studied. The contribution from the Coulomb potential to the average nucleon energy increases as $n_p^2A^{2/3}$ with the mass number A, while that from δM_N does so as $-\delta M_N n_3$. The former, which diverges as $A^{2/3}$ in the limit of $A\to\infty$, works to accommodate more neutrons than protons for equilibrium, while the latter opposes this trend. The contribution from the Coulomb potential overwhelms that from δM_N even for light nuclei. Therefore a simultaneous exclusion of the Coulomb potential and δM_N leads to an underestimate of $|n_3|$ and, hence, an underestimate of the nuclear medium effect on the isospin mixing as understood from Eq. (35). In other words, the Coulomb potential will enhance the medium effect on CSB, reducing θ at β equilibrium further.

In order to confirm the above speculation, we made the following calculation. The Coulomb potential originates in the electromagnetic vector field which couples to the proton. In the MFA, a vector field modifies the single nucleon energy to add a term arising from the corresponding classical field. Although the term will diverge in nuclear matter limit for the long range nature of the electromagnetic interaction, it remains finite for a finite nucleus to give a certain contribution to $\mathcal{T}$. Let us simulate the influence of the electromagnetic field on our results by adding a constant to $W_p^{(+)}$ or subtracting it from $W_p^{(-)}$ in Eq. (28), hence, by adding a term proportional to n_p to $\mathcal{E}$. This modification affects K_p and K_n through minimization of $\mathcal{E}$, which in turn affects $\epsilon(n_p,n_n)$. The standard Bethe-Weizsäcker mass formula predicts an approximate Coulomb energy of 3 MeV for a single proton in a typical nucleus in the Pb region [21]. We use this value as the constant arising from the classical electromagnetic field. We repeated the calculation with the same self-consistent procedure and obtained $\theta=2.03°$ for $n_B=0.17$ fm^{-3}. Thus the inclusion of Coulomb potential reduces θ for the normal density of nuclear matter by roughly 10% down from the value which we have obtained without Coulomb potential.

We have replaced M_p^* and M_n^* in $\mathcal{M}^{00}$ of Eq. (35) by the quantities M_p and M_n multiplied by the factor (M^*/M), respectively. Look at the validity of this approximation. We concentrate on a calculation of effective masses, safely ignoring the effect of CSB. Suppose we start with the Lagrangian given by Eq. (1), but maintain a nonvanishing δM_N throughout. Then, in the MFA, the coupled self-consistent equations for M_p^* and M_n^* are of the form

$$M_b^*=M_b-\frac{f^2}{2\pi^2m_\sigma^2}\left(\frac{M_b^*}{M_b}\right)^2\sum_{b'}M_{b'}\left(\frac{M_{b'}^*}{M_{b'}}\right)^2\times\left[K_{b'}\widetilde{E}_{b'F}^*-M_{b'}^{*2}\ln\left(\frac{K_{b'}+\widetilde{E}_{b'F}^*}{M_{b'}^*}\right)\right]. \tag{47}$$

They are solved with f fixed in Eq. (42). We obtain $M_p^*/M_p=0.8500$ and $M_n^*/M_n=0.8501$ for $n_B=0.17$ fm^{-3}, while Eq. (31) gives $M^*/M=0.8501$. This confirms the validity of our approximation made for M_p^* and M_n^*.

To conclude, we have found that (i) charge symmetry is gradually restored for nuclear matter in β equilibrium as the nucleon density is increased, (ii) the transition of the vector mesons between different isospin eigenstates is much enhanced when nuclear state leaves β equilibrium and contributes strong CSB to nuclear matter off β equilibrium, (iii) the symmetry energy coefficient is obtained to be 32 MeV of which more than 50% arises through the density dependence of mesonic mean fields, and (iv) the extracted mass differences due to CSB between neighboring mirror nuclei are in good agreement with the quantities implied by the Nolen-Schiffer anomaly.

We reemphasize that the CSB effects under scrutiny here is induced whenever imbalance between proton and neutron densities occurs in nuclear medium. It should have important implications for analyses of neutron-rich unstable nuclei produced in heavy-ion collisions and of neutron star formation models in astrophysics [24].

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