Noncentral interactions in elastic scattering with arbitrary spins: The case of $N\Delta\rightarrow N\Delta$

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The role of noncentral forces is brought out in elastic scattering involving particles with arbitrary spins using a formalism employing projection operator techniques. The particular case of $N\Delta\rightarrow N\Delta$ is considered explicitly. [S0556-2813(97)03411-0]

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The role of noncentral forces in the scattering of spin-s particles on spin-0 targets was discussed by Johnson [1] by expressing the transition amplitude in terms of irreducible tensor operators $\tau^k_q(\mathbf{S})$ of rank $k=0,1,\ldots 2,s$ constructed out of the spin operators $\mathbf{S}$ of the spin-s particle. It is desirable to extend the formalism to elastic scattering of particles of spin s_1 on targets with spin s_2. We develop the formalism here, employing techniques of projection operators.

Let us first of all define irreducible tensor operators

$$\tau_q^{(k_1k_2)k}(\mathbf{S}_1,\mathbf{S}_2)\equiv(\tau^{k_1}(\mathbf{S}_1)\otimes\tau^{k_2}(\mathbf{S}_2))^k_q \tag{1}$$

of rank k, where $\tau^k_q(\mathbf{S})$ are normalized as in [1], $\mathbf{S}_1$ and $\mathbf{S}_2$ denote, respectively, the spin operators of the particles with spins s_1 and s_2, and we use the shorthand notation

$$(A^k\otimes B^{k'})^K_Q=\sum_q C(kk'K,qq'Q)A^k_qB^{k'}_{q'} \tag{2}$$

to denote an irreducible tensor of rank K constructed out of two irreducible tensors A^k_q of rank k and $B^{k'}_{q'}$ of rank k'. The rest of the notations follow [2]. Clearly,

$$\begin{aligned}\mathrm{Tr}[\tau_q^{(k_1k_2)k}(\mathbf{S}_1,\mathbf{S}_2)\tau_{q'}^{(k_1'k_2')k'}(\mathbf{S}_1,\mathbf{S}_2)^\dagger]\\ =[s_1]^2[s_2]^2\delta_{k_1k_1'}\delta_{k_2k_2'}\delta_{kk'}\delta_{qq'},\end{aligned} \tag{3}$$

where $[s]=\sqrt{2s+1}$. It is known [3] that the projection operator

$$\begin{aligned}\mathcal{P}^s(s_1,s_2)=\sum_\kappa [\kappa][s]^2[s_1]^{-1}[s_2]^{-1}W(s_1\kappa ss_2;s_1s_2)\\ \times\tau_0^{(\kappa\kappa)0}(\mathbf{S}_1,\mathbf{S}_2)\end{aligned} \tag{4}$$

can be used to project channel spin s. Following [4], we define the projection cum orbital flip irreducible tensor operator

$$\begin{aligned}S^\lambda_\mu(l_2,l_1)=\sum_{m_2}(-1)^{l_1-m_1}[l_2]\\ \times C(l_2l_1\lambda;m_2-m_1\mu)|l_2m_2\rangle\langle l_1m_1|\end{aligned} \tag{5}$$

of rank λ. Using Eq. (5), we may now define a projection cum spin-orbit flip operator,

$$\begin{aligned}S_{s_1,s_2}(l's';j;ls)=\sum_{k_1=0}^{2s_1}\sum_{k_2=0}^{2s_2}\sum_k G_{k_1k_2k}(l's';j;ls)\\ \times[S^k(l',l)\cdot\tau^{(k_1k_2)k}(\mathbf{S}_1,\mathbf{S}_2)],\end{aligned} \tag{6}$$

where the geometrical factors are explicitly given by

$$\begin{aligned}G_{k_1k_2k}(l's';j;ls)=(-1)^{l'-s'-j}[j]^2[s][s']\\ \times[l']^{-1}[s_1]^{-1}[s_2]^{-1}W(ll'ss',kj)\\ \times(-1)^{k_1+k_2+k}[k_1][k_2]\begin{Bmatrix} s_1 & s_2 & s\\ s_1 & s_2 & s'\\ k_1 & k_2 & k\end{Bmatrix}\end{aligned} \tag{7}$$

and j denotes total angular momentum. The above operator selects states $|(ls)jm\rangle$ and flips them to $|(l's')jm\rangle$.

Defining the matrix $\mathcal{M}$ in spin space for elastic scattering of spin s_1 particles on spin s_2 targets in c.m. frame through $\mathcal{M}=E_1E_2T/2\pi E$, where T denotes the on energy shell transition matrix, E_1 and E_2 the energies of the two particles, and $E=E_1+E_2$ the c.m. energy, we may now express $\mathcal{M}$ using Eq. (6) in the form

$$\mathcal{M}=\sum_{l,'s',j,l,s}\mathcal{M}^j_{l's';ls}(E)S_{s_1,s_2}(l's';j;ls), \tag{8}$$

where $\mathcal{M}^j_{l's';ls}(E)$ are as defined in [4] and are expressible, in the case of isospin-conserving processes, as

$$\mathcal{M}^j_{l's';ls}(E)=\sum_I \mathcal{M}^{Ij}_{l's';ls}(E)\mathcal{P}^I(I_1,I_2) \tag{9}$$

in terms of the partial wave scattering amplitudes $\mathcal{M}^{Ij}_{l's';ls}(E)$ in isospin channels I which are allowed for the system of two particles with isospins I_1 and I_2. It may readily be seen, on evaluating Eq. (8) between initial and final states with c.m. momenta $\mathbf{p},\mathbf{p}'$, respectively, that the channel spin matrix elements assume the well-known [1,4,5] form

$$\begin{aligned}\langle s'\mu';\mathbf{p}'|\mathcal{M}|\mathbf{p};s\mu\rangle=\sum_{l',l,j,\lambda}(-1)^{l+s+l'-j}(-1)^\nu\\ \times\mathcal{M}^j_{l's';ls}(E)[j]^2[\lambda][s']^{-1}\end{aligned}$$

$$\times W(sls'l';j\lambda)$$
$$\times C(s\lambda s';\mu\nu\mu')$$
$$\times (Y_{l'}(\hat{\mathbf{p}}')\otimes Y_l(\hat{\mathbf{p}}))^{\lambda}_{-\nu}. \quad (10)$$

More importantly, the form of $\mathcal{M}$ as given by Eq. (8) helps us, on taking the Fourier transform, to identify the types of noncentral interactions that could contribute to elastic scattering of a spin-s_1 particle on spin-s_2 targets. Clearly the nonlocal effective interaction must be of the form

$$\langle \mathbf{r}'|V_{\text{eff}}|\mathbf{r}\rangle = \sum_{k_1,k_2,k} [\tau^{(k_1k_2)k}(\mathbf{S}_1,\mathbf{S}_2)\cdot\langle \mathbf{r}'|V^{(k_1k_2)k}|\mathbf{r}\rangle], \quad (11)$$

where

$$\langle \mathbf{r}'|V_q^{(k_1k_2)k}|\mathbf{r}\rangle = \sum_{l',s',j,l,s} G_{k_1k_2k}(l's';j;ls) \times \langle \mathbf{r}'|V^j_{l's';ls}S^k_q(l',l)|\mathbf{r}\rangle. \quad (12)$$

The angular dependence in Eq. (12) may be expressed either as

$$\langle \hat{\mathbf{r}}'|S^k_q(l',l)|\hat{\mathbf{r}}\rangle = (-1)^l[l'](Y_{l'}(\hat{\mathbf{r}}')\otimes Y_l(\hat{\mathbf{r}}))^k_q \quad (13)$$

for diagonal as well as off-diagonal terms in l or by identifying

$$S^k_q(ll) = \tau^k_q(\mathbf{L}) \quad (14)$$

for diagonal terms in l. Further, $V^j_{l's';ls}$ must be related to $\mathcal{M}^j_{l's';ls}(E)$ through

$$\langle r'|V^j_{l's';ls}|r\rangle = \frac{2}{\pi}(i)^{l'-l}\int p^2 dp\, j_{l'}(pr') \times \mathcal{M}^j_{l's';ls}(E)j_l(pr), \quad (15)$$

where j_l are spherical Bessel functions and the c.m. momentum p is determined by E.

Invariance under parity implies that the summation over l',l must be restricted only to terms where $l+l'$ are even; moreover, symmetry properties restrict the choice (13) to even values of k. On the other hand, parity invariance permits the choice (14) for even as well as odd values of k. These considerations become more transparent in the case of local interactions, $\mathbf{r}=\mathbf{r}'$ when

$$(Y_{l'}(\hat{\mathbf{r}})\otimes Y_l(\hat{\mathbf{r}}))^k_q = \frac{[l'][l]}{\sqrt{4\pi}[k]}C(l'lk;000)Y_{kq}(\mathbf{r}). \quad (16)$$

For example, the ‘‘tensor force’’ between nucleons is realized for $k_1=k_2=1$, $k=2$ if $\mathbf{r}=\mathbf{r}'$ and choice (13) is made. On the other hand, if we add the $k=1$ terms with $k_1=1$, $k_2=0$ and $k_1=0$, $k_2=1$, the familiar spin-orbit form $\mathbf{L}\cdot\mathbf{S}$ is realized, where $\mathbf{S}=\mathbf{S}_1+\mathbf{S}_2$. Higher-order spin-orbit terms are realized as we increase k. For example, with $k=2$, $k_1=k_2=1$, we have the form $3(\mathbf{L}\cdot\mathbf{S}_1)(\mathbf{L}\cdot\mathbf{S}_2)-l(l+1)(\mathbf{S}_1\cdot\mathbf{S}_2)$.

In particular, if we consider the case of an $N\Delta$ interaction which is of topical interest, it is clear that Eq. (11) contains as many as 14 different spin tensors of the form (1), since k_1 takes values 0,1 and k_2 can take values 0,1,2,3. Of these, it may be seen that two are scalars, four are vectors, four are second rank tensors, three are third rank tensors, and one is a fourth rank tensor with k taking values 0,1,2,3,4, respectively. Clearly the tensors with odd rank k admit only the choice (14), while the second and fourth rank tensors admit both the choices (13) and (14). The two scalar terms may be combined and represented in terms of the triplet and pentuplet projection operators $\mathcal{P}^1(\frac{1}{2},\frac{3}{2})$ and $\mathcal{P}^2(\frac{1}{2},\frac{3}{2})$ which can be derived from Eq. (4). Thus

$$V^{N-\Delta}_{\text{eff}} = \sum_{s=1}^{2} V_s\mathcal{P}^s(\tfrac{1}{2},\tfrac{3}{2}) + \sum_{k=1}^{4}\sum_{k_1=0}^{1}\sum_{k_2=0}^{3} V^{(k_1k_2)k}[\tau^{(k_1k_2)k}(\mathbf{S}_N,\mathbf{S}_\Delta)\cdot\tau^k(\mathbf{L})] + \sum_{k=2,4}\sum_{k_1=0}^{1}\sum_{k_2=1}^{3}[\tau^{(k_1k_2)k}(\mathbf{S}_N,\mathbf{S}_\Delta)\cdot U^{(k_1k_2)k}], \quad (17)$$

where

$$\langle r'|V_s|r\rangle = \sum_{l,j}\sum_{k=0}^{1} G_{kk0}(ls;j;ls)\langle r'|V^j_{ls,ls}|r\rangle, \quad (18)$$

$$\langle r'|V^{(k_1k_2)k}|r\rangle = \sum_{l,s,s',j} G_{k_1k_2k}(ls';j;ls)\langle r'|V^j_{ls';ls}|r\rangle, \quad (19)$$

$$\langle \mathbf{r}'|U_q^{(k_1k_2)k}|\mathbf{r}\rangle = \sum_{l',s',l,s,j} G_{k_1k_2k}(l's';j;ls)\langle r'|V^j_{l's';ls}|r\rangle \times(-1)^l[l'](Y_{l'}(\hat{\mathbf{r}}')\otimes Y_l(\hat{\mathbf{r}}))^k_q \quad (20)$$

represent, respectively, the central (spin-independent and spin-spin), spin-orbit, and tensor interactions. It is interesting to note that the $V^{N-\Delta}$ interaction [6], consisting of longitudinal and transverse components, derived on the basis of π and ρ exchange diagrams contain only spin tensors $\tau_q^{(11)k=0,2}(\mathbf{S}_N,\mathbf{S}_\Delta)$, the spin-orbit interaction being entirely absent.

In view of the above, the general form (17) of the $N\Delta$ interaction can be used with advantage in discussing Δ excitation and propagation in nuclear processes at intermediate energies on which considerable amount of experimental effort is currently underway.

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