Different approach to calculating average angular distributions of elastically scattered neutrons in the resonance region

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A relatively simple formalism for calculating the *average* neutron elastic angular distribution $\overline{d\sigma_{\rm el}}/d\Omega$ in the resonance region below several hundred keV is presented. The expression for $\overline{d\sigma_{\rm el}}/d\Omega$ depends mainly on the R-matrix parameters S_0, R', S_1, and R_1^∞. Comparisons between calculated and experimental angular distributions are presented for ^{103}Rh, ^{139}La, ^{232}Th, and ^{238}U. A fit to ^{238}U data at 75 keV led to a value of the p-wave strength function of $S_1=1.81\pm0.35\times10^{-4}$. Except for measuring a complete set of individual $l=1$ resonances, determining the p-wave strength function by fitting low-energy angular distributions is probably more reliable than, or competitive with, other techniques which are available. An analysis of elastic angular distributions as a function of neutron energy is also well suited to a search for intermediate structure in the s- or p-wave strength function. [S0556-2813(97)04011-9]

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INTRODUCTION

When a neutron with an energy $\lesssim$ several hundred keV interacts with a heavy nucleus (excluding fissile nuclei), elastic scattering, neutron capture, and inelastic scattering to low-lying states of the target nucleus, are possible outcomes. However, in this energy regime, elastic scattering is usually the dominant process. The interaction between a neutron and a nucleus can be viewed as being partitioned between ''hard sphere'' elastic scattering and elastic scattering, capture, and inelastic scattering which proceeds through the formation of compound nuclear states. In heavier nuclei, the narrow resonances observed in high-resolution neutron total cross-section measurements persists far above the incident neutron energies where experimental resolution is capable of observing this structure. Over the years, the analysis of data collected in high-resolution experiments in the keV energy range have led to the determination of the $l=0$ elastic scattering length R', the s- and p-wave strength functions S_0, S_1, the average capture width Γ_γ^l, and the average $l=0$ resonance spacing D_0 for many nuclei throughout the periodic table [1].

R', S_0, and S_1, have played an important role in determining the parameters of the low-energy optical potential and in addition, they provide for a simple parametrization of cross sections. For example, averaging over many s- and p-wave resonances, the average neutron total cross section in the low-energy limit can be expressed as

$$\overline{\sigma_T}=4\pi(R')^2+2\pi^2\lambda\!\!\!^-{}^2\sqrt{E}S_0+3(2\pi^2\lambda\!\!\!^-{}^2)\sqrt{E}\left(\frac{x^2}{x^2+1}\right)S_1, \tag{1}$$

where $\lambda\!\!\!^-$, E are the neutron wavelength and incident neutron energy and $x=kR$ where k, $R=1.35A^{1/3}$ are the neutron wave number and effective nuclear radius. In a low-energy optical model calculation [2], the average total neutron cross section can be expressed as $\sigma_T=\sigma_{\rm se}+\sigma_c$, where $\sigma_{\rm se}$, the shape elastic cross section, is associated with the ''hard sphere'' scattering $4\pi(R')^2$, and σ_c, the compound nucleus formation cross section, represents the remaining terms of Eq. (1). The optical model successfully describes the *average* behavior of R', S_0, and S_1 as a function of mass number, but for an individual nucleus the predictions are less reliable. This is evidenced by the fluctuations of the experimental values of R', S_0, and S_1 from the predicted average values. The angular distribution of the ''hard sphere'' elastically scattered neutrons is given by $d\sigma_{\rm se}/d\Omega$, but this does not include the contribution of neutrons scattered elastically via the compound nuclear states. Thus, an additional calculation must be carried out to more accurately predict elastic angular distributions [2].

Presented here is an approach to calculating average neutron elastic angular distributions in the resonance region, *below* several hundred keV, which anchors itself more directly to measured quantities such as S_0, R', S_1, and R_1^∞. This method, which automatically includes both the shape elastic and compound elastic scattering, should better reflect the individual properties of a particular nucleus. In addition, since the expression for $\overline{d\sigma_{\rm el}}/d\Omega$ depends on strength functions, by fitting angular distribution data it is possible to determine, e.g., the p-wave strength function. This important quantity is not easily extracted from experimental data.

CALCULATION

For neutron energies less than several hundred keV, the neutron interacting with the nucleus is most likely s or p ($l=0,1$) wave and of much less importance, d wave ($l=2$). The low-energy resonances associated with the compound nuclear states, analyzed using the R-matrix formalism [3] are characterized by a neutron width Γ_n^{lJ}, a reaction width Γ_R^{lJ} (which at a few keV is basically the capture width Γ_γ^{lJ}), and a resonance energy E^{lJ}. The width Γ_n^{lJ} has an energy l and J (total angular momentum) dependence. Another important R-matrix parameter for a given l (and also in principle J) is R_l^∞; it represents the effects of far away levels not included in the local analysis of individual resonances. At low energies the $l=0$ scattering length R' is related to R_0^∞

through the relationship $R'=R(1-R_0^\infty)$. Information about R_1^∞ for a number of nuclei in the mass range $75\leqslant A\leqslant 232$ is available from average transmission measurements [4].

The basic idea is to express, for a given nucleus, the *average* angular distribution in terms of as many experimentally measurable quantities as possible. Because the low-energy neutron resonances have been analyzed using an R-matrix theory parametrization, this would seem to require that this theory be used to calculate angular distributions.

The expression for the angular distribution of a spin-$\frac{1}{2}$ particle incident upon a spin-0 nucleus (no polarizations) which includes the effects of spin-orbit coupling is

$$\frac{d\sigma}{d\Omega}=\frac{\lambda\!\!\!^{-2}}{4}\left|\sum_{l=0}[(l+1)(1-U_l^+)+l(1-U_l^-)]P_l\right|^2+\frac{\lambda\!\!\!^{-2}}{4}\left|\sum_{l=1}(U_l^- -U_l^+)P_l^1\right|^2, \tag{2}$$

where $\pm$ refer to $J=l\pm\frac{1}{2}$, and P_l, P_l^1 are Legendre and associated Legendre polynomials. In an optical model calculation, an *average* $U_l^\pm$ is determined by a matching of the logarithmic derivative of the wave function at a distance where the complex nuclear potential is insignificant. Here, an expression for $U_l^\pm$ in terms of resonance parameters is employed which is suitable when the average spacing between resonances is much greater than the widths of the resonances [3]. The expression for $U_l^\pm$ has the form

$$U_l^\pm\approx e^{-2i\theta_l}\left(1+\sum_{j=1}^{N_l^\pm}\frac{i\Gamma_{nj}^{l\pm}}{E_j^{l\pm}-\Delta_{lj}-E-i(\Gamma_j^{l\pm}/2)}\right),$$

$$\Gamma_j^{l\pm}=\Gamma_{nj}^{l\pm}+\Gamma_{Rj}^{l\pm}, \tag{3}$$

where $\Gamma_{Rj}^{l\pm}=\Gamma_{\gamma j}^{l\pm}+\Gamma_{nn'j}^{l\pm}$ is a total reaction width, expressed in terms of a capture and an inelastic scattering width. $U_l^\pm$ includes a sum over $N_l^\pm$ levels within an energy interval ΔE. The effect of levels not included in the sum, i.e., R_l^∞, appears through θ_l. This expression for $U_l^\pm$ is substituted into Eq. (2) and the expression for the differential cross section is *averaged* over an energy interval ΔE containing many resonances [5]. After some straight forward calculations, which are outlined in the Appendix, a relatively simple expression for the average angular distribution was found. Importantly, it depends mainly on measurable average parameters of resonances. Thus, existing data [1] can be used to predict angular distributions or an analysis of angular distribution data can determine the average parameters. The expression given below includes only $l=0,1$ contributions to the cross section. The additional terms required for the d-wave contribution, which is of minor importance, as well as other approximations and assumptions made are given in the Appendix.

The average angular distribution of elastically scattered s- and p-wave neutrons was found to be

$$\overline{\frac{d\sigma_{\rm el}}{d\Omega}}=\frac{\lambda\!\!\!^{-2}}{4}[|s|^2(P_0)^2+|p|^2(P_1)^2+2RP(sp^*)P_0P_1+|p'|^2(P_1^1)^2], \tag{4a}$$

$$|s|^2=4\sin^2\theta_0+2\pi\cos(2\theta_0)\sqrt{E}S_0+2\pi\left[\frac{1}{\Delta E}\sum_{j=1}^{N_0}\frac{\Gamma_{nj}^{0\,2}}{\Gamma_j^0}-\sqrt{E}S_0\right], \tag{4b}$$

$$|p|^2=36\sin^2\theta_1+18\pi\cos(2\theta_1)\mathcal{P}_1S_1+\pi\frac{1}{\Delta E}\left[\sum_{j=1}^{N_1^+}\frac{(2\Gamma_{nj}^{1+})^2}{\Gamma_j^{1+}}+\sum_{j=1}^{N_1^-}\frac{(\Gamma_{nj}^{1-})^2}{\Gamma_j^{1-}}\right]-18\pi\mathcal{P}_1S_1, \tag{4c}$$

$$2RP(sp^*)=6[2\sin^2\theta_0+2\sin^2\theta_1-2\sin^2(\theta_0-\theta_1)]+\pi[\cos(2\theta_0)\sqrt{E}S_0+\cos(2\theta_1)\mathcal{P}_1S_1]-\pi\cos2(\theta_0-\theta_1)(\sqrt{E}S_0+\mathcal{P}_1S_1)], \tag{4d}$$

$$|p'|^2=2\pi\frac{1}{\Delta E}\left[\sum_{j=1}^{N_1^+}\frac{(\Gamma_{nj}^{1+})^2}{\Gamma_j^{1+}}+\sum_{j=1}^{N_1^-}\frac{(\Gamma_{nj}^{1-})^2}{\Gamma_j^{1-}}\right]. \tag{4e}$$

S_0, S_1 are the $l=0,1$ strength functions, E is the incident neutron energy, and the R-matrix definition of the remaining parameters defined in Lane and Thomas [3] are

$$\theta_l=\varphi_l+\gamma_l,\quad \tan\varphi_l=-\frac{j_l(x)}{\eta_l(x)},\quad \tan\gamma_l=\frac{-R_l^\infty\mathcal{P}_l(x)}{1-R_l^\infty\mathrm{Sh}_l^0(x)},$$

$$\mathrm{Sh}_l^0(x)=\mathrm{Sh}_l(x)+l,\quad L_l^0(x)=\mathrm{Sh}_l^0(x)+i\mathrm{Pe}_l(x),$$

$$\Gamma_{nj}^{l\pm}=\mathcal{P}_l(x)(\gamma_{nj}^{l\pm})^2,\quad \mathcal{P}_l(x)=\frac{\mathrm{Pe}_l(x)}{|1-R_l^\infty L_l^0(x)|^2}. \tag{5a}$$

$\mathrm{Pe}_l(x)$ and $\mathrm{Sh}_l(x)$ are the penetrability and shift functions and the energy dependence of these quantities enters through $x=kR$, where k is the neutron wave number. $(\gamma_{nj}^{l\pm})^2$ is an intrinsic width with an absorbed factor of 2. The strength functions have their usual definitions, namely,

$$S_0=\frac{1}{\Delta E}\sum_{j=1}^{N_0}(\gamma_{nj}^0)^2,$$

$$S_1=\frac{1}{3\Delta E}\left[\sum_{j=1}^{N_1^+}2(\gamma_{nj}^{1\pm})^2+\sum_{j=1}^{N_1^-}(\gamma_{nj}^{1-})^2\right]. \tag{5b}$$

Except for sums of the form $\Sigma_j\Gamma_{nj}^2/\Gamma_j$, Eq. (4a) depends only on S_0, R_0^∞, S_1, and R_1^∞. Through Monte Carlo techniques, these sums can be evaluated by estimates of average widths, a knowledge of the $l=0$ level spacing D_0, and by assuming that the intrinsic neutron widths $(\gamma_{nj}^{l\pm})^2$ obey the Porter-Thomas distribution [6]. The details are described in the Appendix.

The total elastic cross section $\overline{\sigma_{\rm el}}$ is determined by integrating Eq. (4a) over 4π radians and in the low-energy limit as $E\rightarrow 0$, $\overline{\sigma_{\rm el}}$ reduces to Eq. (1) *minus* the reaction or capture cross section. An examination of Eq. (4a) indicates that except for the interference term between s and p waves, the angular distribution is symmetric about 90°; thus any asymmetry arises from s-p interference.

COMPARISON WITH EXPERIMENTAL DATA

The amount of elastic neutron angular distribution data below $\sim$250 keV is sparse. In addition, the angular distribution measurements examined may be correct with respect to shape but not in absolute value, and as a result, it is essential that the data be properly normalized. Consequently, comparisons between calculated angular distributions and experimental data were only carried out for nuclei where total, capture, and inelastic cross-section data were available. The integrated experimental angular distributions were normalized to insure that $\sigma_{el} = \sigma_T - \sigma_{n\gamma} - \sigma_{nn'}$. The major uncertainty of normalizing the data arises from the uncertainty of the total cross section. Typically, the angular distribution data was collected in ''poor resolution'' measurements in which the spread in the neutron beam energy was $\sim$10 keV. Thus, the measured angular distribution is an average over many resonances.

The calculations were carried out as follows. The experimentally determined values of D_0, S_0, R', S_1, and R_1^∞, for the nucleus of interest, were used as input. An effective reaction width Γ_R was found which yielded the experimental value of $\sigma_R = \sigma_{n\gamma} + \sigma_{nn'}$ to within a few %. The sums appearing in Eq. (4a) and in the expression for σ_R given in the Appendix [Eq. (A8)] were calculated with randomly generating widths obeying the Porter-Thomas distribution [6]. Estimates of the average widths needed for this calculation were determined using the procedure outlined in the Appendix. For the energies of interest here, the very small contribution of d waves is mostly apparent at the very large and small angles. In all the calculations it was assumed that $S_2 = S_0$ and $R_2^\infty = R_0^\infty$.

Since the measured quantities D_0, S_0, R', S_1, and R_1^∞ have uncertainties, it would be unrealistic to always expect satisfactory agreement between the calculated and experimental angular distributions. However, to within the uncertainties of these parameters, acceptable agreement should be obtained in most cases. This assumes there is no significant modulation of the values of the s- or p-wave strength functions due to the presence of isolated doorway states or due to any intrinsic energy dependence. Except for the ^{238}U data at 75 keV, no fitting to the experimental angular distribution data was carried out. Therefore, to within the uncertainty of the measured parameters, the calculations presented below represent predicted $\overline{d\sigma_{el}}/d\Omega$. All calculated angular distributions were transformed to the laboratory frame of reference. Remarks about each nucleus examined follow.

^{103}R$_h$. Barnard and Reitman [7] have measured the elastic neutron angular distribution of ^{103}R$_h$ at 200 keV. The total elastic cross section was determined by integrating, over 4π radians, a best fit polynomial to the data. This procedure yielded $\sigma_{el} = 12.05$ b, which is significantly larger than the total cross section. At this energy $\sigma_{n\gamma}, \sigma_{nn'}$ are 0.340, 0.057 b, respectively [8,9], and σ_T lies between 8.0 and 8.4 b [8]. Consequently, the data plotted in Fig. 1 was multiplied by the two normalizing factors 0.63 and 0.66 (filled and open triangles) which reflect the minimum and maximum values of σ_T. The dot-dashed line of Fig. 1 is the predicted $\overline{d\sigma_{el}}/d\Omega$ using the experimentally determined parameters ($D_0 = 27.0$ eV, $S_0 = 0.54 \times 10^{-4}$) [10], ($R' = 6.2\ f$, $S_1 = 5.5 \times 10^{-4}$, $R_1^\infty = -0.1$) [4]. An effective reaction width of $\Gamma_R = 0.13$ eV reproduced the reaction cross section σ_R of 0.40 b. Although the general behavior is predicted by the dot-dashed curve, the calculated cross section is somewhat more forward peaked then the experimental data (the shape of which is assumed to be ''perfect''). As noted previously, the measured parameters have uncertainties and a p-wave strength function of $S_1 = 5.0 \times 10^{-4}$ is well within the quoted uncertainty [4] of $S_1 = 5.5 \pm 0.9 \times 10^{-4}$. This small reduction in S_1 (all other parameters the same) leads to the solid curve of Fig. 1, which gives a reasonable representation of the data.

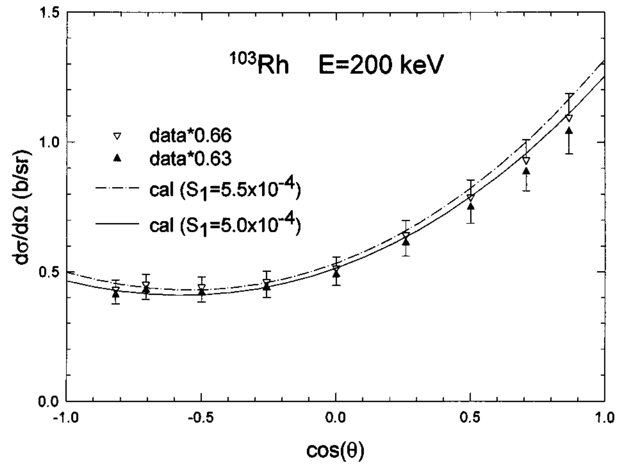

FIG. 1. Elastic angular distribution of ^{103}Rh as a function of cos(θ) in the lab frame at the incident neutron energy of 200 keV. The filled and open triangles represent estimated minimum and maximum values of the experimental elastic angular distribution as described in the text. The dot-dashed curves are the predicted elastic angular distribution using values of the parameters S_0, R', S_1, and R_1^∞ as reported in the literature. The solid lines are the predicted angular distribution when one of parameters was varied (within the quoted uncertainty) to achieve improved agreement with the data.

^{139}La. Malan *et al.* [11] have measured the elastic angular distribution of neutrons scattered from ^{139}La at 232 keV. In addition these authors measured the $\sigma_{nn'}$ cross section at higher energies from which an estimate (0.21 b) could be made at 232 keV. The total cross section data from Ref. [8] suggests that σ_T lies between 4.9 and 5.1 b, and that the capture cross section is 0.01 b. Thus, the measured elastic cross section should lie between 4.68 and 4.88 b. A polynomial fit to the angular distribution data yielded $\sigma_{el} = 5.00$ b, and consequently, the normalization of the data should lie between 0.94 and 0.98. Figure 2 shows a plot of the experimental data at 232 keV with both normalizations (filled and open triangles). The dot-dashed curve was calculated with ($D_0 = 208$ eV, $S_0 = 0.78 \times 10^{-4})^1$ and ($R' = 5.3f$, $S_1 = 0.5 \times 10^{-4}$, $R_1^\infty = -0.25$) [4] where the 0.22 b reaction cross section required $\Gamma_R = 0.65$ eV. By changing R_1^∞ to -0.30, which is within the ± 0.1 uncertainty of this parameter, the solid curve results. To within the uncertainty of the normalizations both curves are, except for the most forward angle, in reasonable agreement with the data.

^{232}Th. The ^{232}Th 144 keV angular distribution data of Fujita *et al.* [12], when integrated over 4π radians, yielded $\sigma_{el} = 11.0$ b. The minimum and maximum σ_T values of 11.20 and 11.60 b [8] and $\sigma_{n\gamma}$, $\sigma_{nn'}$, cross sections of 0.18 and 0.74 b [8,12], led to the data plotted in Fig. 3 being multi-

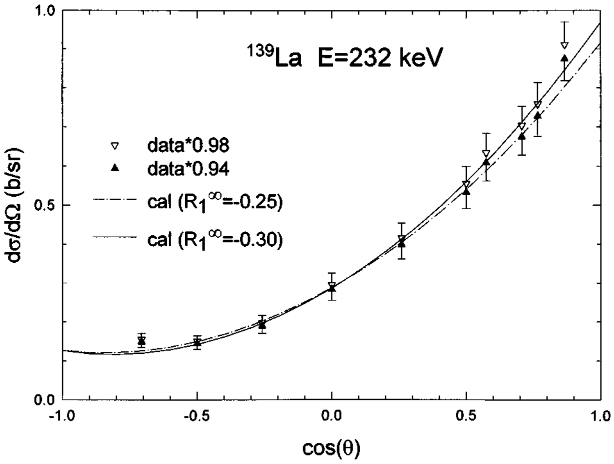

FIG. 2. Elastic angular distribution of ^{139}La as a function of cos(θ) in the lab frame at the incident neutron energy of 232 keV. The symbols, as well as the dot-dashed and solid lines, have the same meaning as in Fig. 1.

plied by the factors 0.94 and 0.97. The predicted behavior of $\overline{d\sigma_{\rm el}/d\Omega}$, represented by the dot-dashed curve, was calculated with ($D_0=16.7$ eV, $S_0=0.84\times10^{-4}$) [13], and ($R'=9.72\,f$, $S_1=1.5\times10^{-4}$, and $R_1^\infty=0.1$) [4]. An effective value of $\Gamma_R=0.39$ eV reproduced the 0.92 b reaction cross section. Although this curve gives a reasonable representation of the data, by changing R_1^∞ to 0.15 (within the ±0.1 uncertainty of this parameter) improved agreement represented by the solid line results.

^{238}U. For ^{238}U angular distribution data at 75 and 157 keV is available from the work of Barnard *et al.* [14]. Polynomial fits to these data yielded integrated elastic cross sections of 12.87 and 10.56 b, respectively. The s-wave strength function $S_0=1.08\pm0.1\times10^{-4}$, determined from resonance parameters [13], has a small uncertainty and σ_T, $\sigma_{n\gamma}$, $\sigma_{nn'}$ are available at both energies [8,15]. Measurements of the p-wave strength function span a range of values, they are 1.4 [13], 1.8 [16], 1.9 [17], and 2.4×10^{-4} [18]. Thus in this case, a fit to the data at 75 keV was carried out in order to

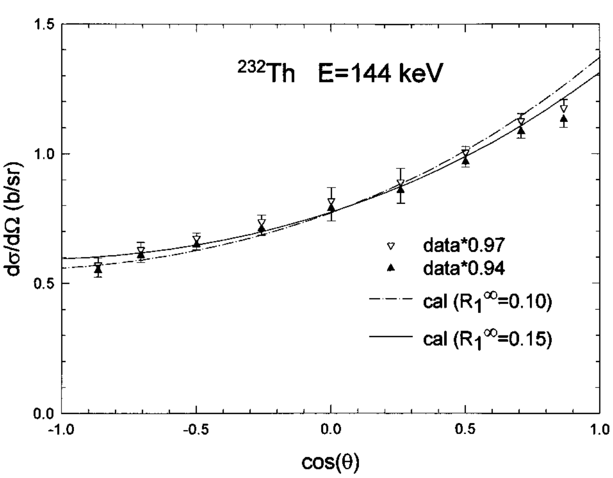

FIG. 3. Elastic angular distribution of ^{232}Th as a function of cos(θ) in the lab frame at the incident neutron energy of 144 keV. The symbols, as well as the dot-dashed and solid lines, have the same meaning as in Fig. 1.

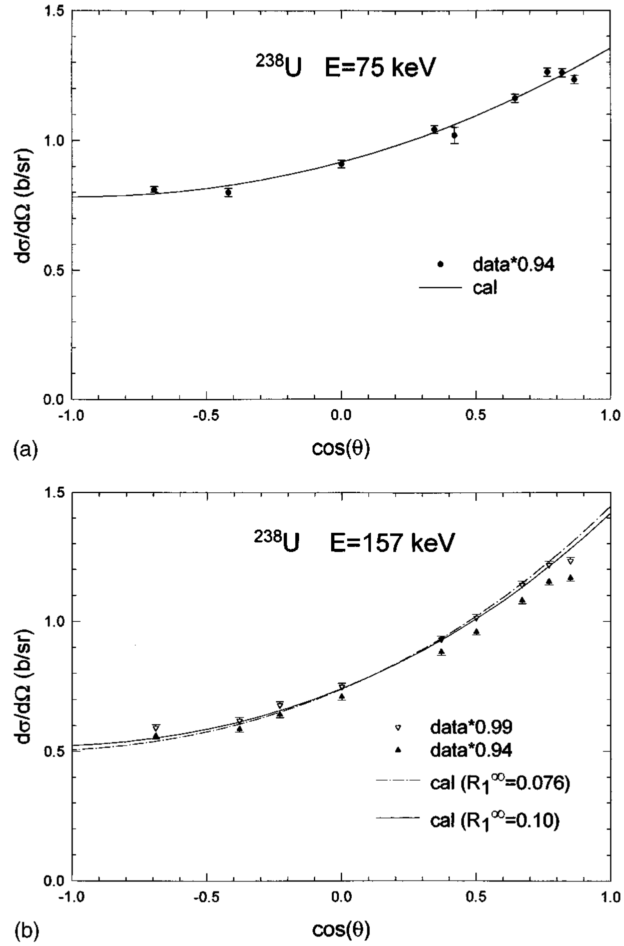

FIG. 4. (a) The smooth curve is the result of a least squares fit to the 75 keV experimental elastic angular distribution of ^{238}U represented by the solid circles. The fit was carried out by holding D_0, S_0 fixed to the values 20.8 eV, 1.08×10^{-4} and letting R', S_1, and R_1^∞ vary. The value of the p-wave strength function determined by the fit is $1.81\pm.35\times10^{-4}$. (b) The experimental elastic angular distribution data of ^{238}U at 157 keV is depicted by the filled and open triangles which represent the minimum and maximum values determined by the normalization procedure described in the text. The dot-dashed curve was calculated with parameters determined by the fit to the ^{238}U data at 75 keV. The solid line was calculated with a change in R_1^∞ which was within the uncertainty of this parameter.

determine the parameters R', S_1, and R_1^∞; D_0, S_0 assumed the values (20.8 eV, 1.08×10^{-4}) [13]. To avoid having the additional parameter Γ_R to vary, the angular distribution data fitted was normalized to $\sigma_T-\sigma_{n\gamma}=12.46$, where σ_T, $\sigma_{n\gamma}$ are 12.7 and 0.245 b [8]. Γ_R was chosen to be 0.023 eV, the capture width determined by examining individual resonances [13]. The range of parameters which gave acceptable fits yielded average values as well as their uncertainties. The values of R', S_1, and R_1^∞ found at 75 keV are $9.53\pm0.20\,f$, $1.81\pm.35\times10^{-4}$, 0.075 ± 0.026 [19]. The value of the p-wave strength function found here is consistent with the recommended value [1] of $1.7\pm0.3\times10^{-4}$. The final fit to the data at 75 keV is shown in Fig. 4(a) with the values of R', S_1, and R_1^∞ found and with Γ_R increased to 0.085 eV to give a σ_R of 0.55 b. The data shown in Fig. 4(a) was multiplied by the normalizing factor 0.94. As expected, the curve gives a good representation of the data.

For the ^{238}U data at 157 keV, σ_T was estimated to lie between 11.00 and 11.53 b [8] and with $\sigma_{n\gamma}$, $\sigma_{nn'}=0.145$ b [8], 0.955 b [14], normalizing factors of 0.94 and 0.99 were applied to the data. The dot-dashed curve of Fig. 4(b) was calculated with the ^{238}U parameters determined at 75 keV, and with $\Gamma_R=0.63$ eV to yield $\sigma_R=1.1$ b. The predicted angular distribution is somewhat larger at smaller angles than observed experimentally. Slightly improved agreement with the data normalized by the factor 0.99 can be obtained by increasing R_1^∞ to 0.10 as is shown by the solid curve of Fig. 4(b).

CONCLUSION

A relatively straightforward method for calculating the angular distribution of elastically scattered neutrons in the resonance region below several hundred keV incident neutron energy has been presented. The expression for $\overline{d\sigma_{\rm el}/d\Omega}$ is expressed directly in terms of measurable quantities, e.g., S_0, R', S_1, and R_1^∞. Comparisons of predicted $\overline{d\sigma_{\rm el}/d\Omega}$ with angular distribution data gave favorable agreement with experiment to within the uncertainties of the parameters and the experimental data. The formalism presented above, as shown in the case of ^{238}U, can be used to extract p-wave strength functions as well as other parameters.

The presence of doorway states can modulate the value of either the s- or p-wave strength functions as a function of neutron energy. Because the expression obtained for $\overline{d\sigma_{\rm el}/d\Omega}$ is expressed directly in terms of strength functions, it is suggested that an analysis of angular distribution data taken at different energies is well suited to a search for intermediate structure.

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APPENDIX

Described here are the equations and assumptions leading to an expression for $\overline{d\sigma_{\rm el}/d\Omega}$ which is mainly a function of experimentally determined average resonance parameters. The expression for the differential cross section, Eq. (2), can be rewritten as

$$\frac{d\sigma_{\rm el}}{d\Omega}=\frac{\lambda\!\!\!^{-2}}{4}\,|\mathrm{I}|^2+\frac{\lambda\!\!\!^{-2}}{4}\,|\mathrm{II}|^2. \tag{A1}$$

If only contributions from $l=0$, 1,2, i.e., s, p, and d waves are included, I and II will have the form

$$\begin{aligned}\mathrm{I}=&(1-U_0)P_0+[2(1-U_1^+)+(1-U_1^-)]P_1\\&+[3(1-U_2^+)+2(1-U_2^-)]P_2,\end{aligned} \tag{A2}$$

$$\mathrm{II}=(U_1^- - U_1^+)P_1^1+(U_2^- - U_2^+)P_2^1, \tag{A3}$$

where all quantities have the same meaning as noted for Eq. (2). Substituting the R-matrix resonance expression for $U_l^\pm$ [Eq. (3)] into Eqs. (A2) and (A3) and averaging Eq. (A1) over an energy interval ΔE containing many resonances, the average cross section is defined as

$$\overline{\frac{d\sigma_{\rm el}}{d\Omega}}=\frac{1}{\Delta E}\int_{E_1}^{E_2}\frac{d\sigma_{\rm el}}{d\Omega}\,dE. \tag{A4}$$

$\overline{d\sigma_{\rm el}/d\Omega}$ can be given an explicit form by the following two basic relationships with which $|\mathrm{I}|^2$ and $|\mathrm{II}|^2$ can be determined:

$$\begin{aligned}\mathrm{RP}\,&\frac{1}{\Delta E}\int_{E_1}^{E_2}(1-U_l^a)(1-U_{l'}^b)^*dE\\&=2\sin^2\theta_l+2\sin^2\theta_{l'}-2\sin^2(\theta_l-\theta_{l'})+\pi\cos2\theta_l\mathcal{P}_lS_l^a\\&\quad+\pi\cos2\theta_{l'}\mathcal{P}_{l'}S_{l'}^b-\pi\cos[2(\theta_l-\theta_{l'})]\\&\quad\times\left(\mathcal{P}_lS_l^a+\mathcal{P}_{l'}S_{l'}^b-\frac{1}{\Delta E}\,\mathrm{sum}_{ll'}[a,b]\right),\end{aligned} \tag{A5a}$$

$$\begin{aligned}\mathrm{RP}\,&\frac{1}{\Delta E}\int_{E_1}^{E_2}U_l^a(U_{l'}^b)^*dE\\&=\cos[2(\theta_l-\theta_{l'})]\Bigg(1-\pi\Bigg[\mathcal{P}_lS_l^a+\mathcal{P}_{l'}S_{l'}^b\\&\quad-\frac{1}{\Delta E}\,\mathrm{sum}_{ll'}[a,b]\Bigg]\Bigg),\end{aligned} \tag{A5b}$$

where RP means, the real part of, and the asterisk is complex conjugation. a, b are either $+$ or $-$ representing the $l+\frac{1}{2}$ and $l-\frac{1}{2}$ compound nuclear states. S_l^a, S_l^b are "strength functions" for the $+$ and $-$ compound nuclear states:

$$S_l^+=\frac{1}{\Delta E}\sum_{j=1}^{N_l^+}(\gamma_{nj}^{l+})^2,\quad S_l^-=\frac{1}{\Delta E}\sum_{j=1}^{N_l^-}(\gamma_{nj}^{l-})^2. \tag{A5c}$$

The remaining parameters have been previously defined. In the expression for $\overline{d\sigma_{\rm el}/d\Omega}$ the quantities S_l^a, S_l^b always appear in the combinations $(l+1)S_l^+ + lS_l^-$ so that the final expression depends on experimentally measured strength functions.

The term $\mathrm{sum}_{ll'}[a,b]$ in Eqs. (A5a) and (A5b) has the form

$$\begin{aligned}\mathrm{sum}_{ll'}[a,b]&=\sum_{j,j'=1}^{N_l^a,N_{l'}^b}\frac{\Gamma_{nj}^{la}\Gamma_{nj'}^{l'b}(\Gamma_j^{la}+\Gamma_{j'}^{l'b})}{(E_j^{la}-E_{j'}^{l'b})^2+(\Gamma_j^{la}+\Gamma_{j'}^{l'b})^2/4};\\ \Gamma_j^{la,l'b}&=\Gamma_{nj}^{la,l'b}+\Gamma_{\gamma j}^{la,l'b}+\Gamma_{nn'j}^{la,l'b}.\end{aligned} \tag{A6a}$$

If $a=b$ and $l=l'$, then Eq. (A6a) can be written as the sum of diagonal and nondiagonal terms:

$$\begin{aligned}\mathrm{sum}_{ll}[a,a]=&\sum_{j=1}^{N_l^a}\frac{(\Gamma_{nj}^{la})^2}{\Gamma_j^{la}}\\&+\sum_{j\neq j'=1}^{N_l^a,N_l^a}\frac{\Gamma_{nj}^{la}\Gamma_{nj'}^{la}(\Gamma_j^{la}+\Gamma_{j'}^{la})}{(E_j^{la}-E_{j'}^{la})^2+(\Gamma_j^{la}+\Gamma_{j'}^{la})^2/4}.\end{aligned} \tag{A6b}$$

For the nuclei and energies at which the angular distributions were calculated for comparison with experiment, the nondiagonal sums, for which $E_j^{la}-E_{j'}^{la}\neq 0$, had a negligible effect on the cross sections and were neglected. At higher energies where the nondiagonal sums becomes important (or when the sums with $a\neq b$ or $l\neq l'$ become important), the assumptions underlying Eq. (3) start to break down. Thus it is a signal that the approach presented here is not on firm ground [20].

The diagonal sums $\Sigma_j(\Gamma_{nj}^{la})^2/\Gamma_j^{la}$ were calculated by generating neutron widths obeying the Porter-Thomas distribution and by assuming that the reaction width $\Gamma_{Rj}^{l\pm}=\Gamma_{\gamma j}^{l\pm}+\Gamma_{nn'j}^{l\pm}$ had a constant value for all resonances. As noted below the value of Γ_R was chosen to reproduce the reaction cross section $\sigma_R=\sigma_{n\gamma}+\sigma_{nn'}$ at the energy of interest. For this calculation it is also necessary to estimate the average neutron widths for the different l^J compound nuclear states. This was done through the relationships

$$\overline{\Gamma_n^0}=\sqrt{E}D_0S_0,$$

$$\overline{\Gamma_n^{l-}}=\mathcal{P}_l\,\frac{(2l+1)}{l}\,\frac{D_l^-S_l}{(C_l+1)},$$

$$\overline{\Gamma_n^{l+}}=\mathcal{P}_l\,\frac{(2l+1)}{(l+1)}\,\frac{C_l}{(C_l+1)}\,D_l^+S_l, \qquad \text{(A7)}$$

where D_0, D_l^-, and D_l^+ are the s wave and $l\neq 0$ $+,-$ average level spacings. C_l is the ratio of the average reaction cross section, for a given l value, of the $+$ and $-$ compound nuclear states, i.e., $C_l=\overline{\sigma_c^{l+}}/\overline{\sigma_c^{l-}}$. This ratio, which was calculated using an optical model, varies with the mass number A of the nucleus and for $l=1$ is not sensitive to the parameters of the optical potential.

Typically the sums were evaluated by assuming that 200 $l=0$ levels were within an energy interval ΔE. All penetrabilities, shift functions, etc., were evaluated at the energy of the measured angular distribution. Assuming a $2J+1$ level density dependence, then in the same interval ΔE, there are 200, 200, 400, 400, 600 $l=0$, 1^-, 1^+, 2^-, and 2^+ levels, respectively. The calculation was repeated until the statistical uncertainty of the average sum was negligible.

Finally, given the assumptions above, the average reaction cross section takes the form

$$\overline{\sigma_R}=2\pi^2\lambda\!\!\!^{-2}\sum_{l=0}\left[(l+1)\,\frac{1}{\Delta E}\sum_{j=1}^{N_l'}\frac{\Gamma_{nj}^{l+}}{\Gamma_j^{l+}}\,\Gamma_{Rj}^{l+}+l\,\frac{1}{\Delta E}\sum_{j=1}^{N_l^-}\Gamma_{Rj}^{l-}\right], \qquad \text{(A8)}$$

where, $\Gamma_{Rj}^{l\pm}=\Gamma_{\gamma j}^{l\pm}+\Gamma_{nn'j}^{l\pm}$, assumed to be constant from resonance to resonance, was varied to reproduce the experimental reaction cross section $\sigma_R=\sigma_{n\gamma}+\sigma_{nn'}$. The terms $\Sigma_j(\Gamma_{nj}^{l\pm}/\Gamma_j^{l\pm})$ were evaluated, as described above, by randomly generating widths obeying the Porter-Thomas distribution.

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[20] For the sums in which $E_j^{la}-E_j^{l'b}\neq 0$, the positions of the energy levels were simulated by randomly generating sequences of energy levels which obeyed the Wigner distribution of adjacent levels spacings.