Large-basis shell-model calculation of the $^{10}C \rightarrow ^{10}B$ Fermi matrix element

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We use a $4\hbar\Omega$ shell-model calculation with a two-body effective interaction derived microscopically from the Reid93 potential to calculate the isospin-mixing correction for the $^{10}C \rightarrow ^{10}B$ superallowed Fermi transition. The effective interaction takes into account the Coulomb potential as well as the charge dependence of $T=1$ partial waves. Our results suggest the isospin-mixing correction $\delta_C \approx 0.1\%$, which is compatible with previous calculations. The correction obtained in those calculations, performed in a $0\hbar\Omega$ space, was dominated by deviation from unity of the radial overlap between the converted proton and the corresponding neutron. In the present calculation this effect is accommodated by the large model space. The obtained δ_C correction is about a factor of 4 too small to obtain unitarity of the Cabibbo-Kobayashi-Maskawa matrix with the present experimental data. [S0556-2813(97)02011-6]

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I. INTRODUCTION

Superallowed Fermi β transitions in nuclei, $(J^\pi=0^+, T=1) \rightarrow (J^\pi=0^+, T=1)$, provide an excellent laboratory for precise tests of the properties of the electroweak interaction and have been the subject of intense study for several decades (cf. Refs. [1–13]). According to the conserved-vector-current (CVC) hypothesis, for pure Fermi transitions the product of the partial half-life t and the statistical phase-space factor f should be nucleus independent and given by

$$ft = \frac{K}{G_V^2 |M_F|^2}, \tag{1}$$

where $K/(\hbar c)^6 = 2\pi^3 \ln 2 \hbar/(m_e c^2)^5 = 8.120\,270(12) \times 10^{-7}$ GeV^{-4} s, G_V is the vector coupling constant for nuclear β decay, and M_F is the Fermi matrix element $M_F = \langle \psi_f | T_\pm | \psi_i \rangle$. By comparing the decay rates for muon and nuclear Fermi β decay, the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix element [6] between u and d quarks (v_{ud}) can be determined and a precise test of the unitarity condition of the CKM matrix under the assumption of the three-generation standard model is possible [5,6].

For tests of the standard model, two nucleus-dependent corrections must be applied to experimental ft values. The first is a series of radiative corrections to the statistical phase-space factor embodied in the factors δ_R and Δ_R, giving [7–9]

$$f_R = f(1+\delta_R+\Delta_R), \tag{2}$$

where δ_R is due to standard, electromagnetic (''inner'') radiative corrections (cf. p. 45 in Ref. [7]) and Δ_R is what has been referred to as the ''outer'' radiative correction (cf. p. 47 of Ref. [7]) and includes axial-vector interference terms [9,10]. The second correction, which is the subject of this work, arises because of the presence of isospin-nonconserving (INC) forces (predominantly Coulomb) in nuclei that lead to a renormalization of the Fermi matrix element. This correction is denoted by δ_C [2,3,12] and modifies the Fermi matrix element by $|M_F|^2 = |M_{F0}|^2(1-\delta_C)$, where $M_{F0} = [T(T+1) - T_{Z_i} T_{Z_f}]^{1/2}$ is the value of the matrix element under the assumption of pure isospin symmetry.

With the corrections δ_R, Δ_R, and δ_C, a ''nucleus-independent'' $\mathcal{F}t$ can be defined by

$$\mathcal{F}t = ft(1+\delta_R+\Delta_R)(1-\delta_C) \tag{3}$$

and the CKM matrix element v_{ud} is given by [10]

$$|v_{ud}|^2 = \frac{\pi^3 \ln 2}{\mathcal{F}t} \frac{\hbar^7}{G_F^2 m_e^5 c^4} = \frac{2984.38(6)\ \text{s}}{\mathcal{F}t}, \tag{4}$$

where the Fermi coupling constant G_F is obtained from muon β decay and includes radiative corrections. Currently, ft values for nine superallowed transitions have been measured with an experimental precision of 0.2% or better [4,14]. With these precise measurements and reliable estimates for the corrections, the CVC hypothesis can be confirmed by checking the constancy of the $\mathcal{F}t$ values for each nucleus, while the unitarity condition of the CKM matrix is tested by comparing the average value of v_{ud} with the values determined for $v_{us} = 0.2199(17)$ [10] and $v_{ub} < 0.0075$ (90% confidence level) [15], i.e., $v^2 = v_{ud}^2 + v_{us}^2 + v_{ub}^2 = 1$.

In the past, the nuclear structure correction δ_C has been computed within the framework of the nuclear shell model [2,3,11–13]. In general, the isospin-nonconserving components of the nuclear Hamiltonian are small and can be treated perturbatively. Due to computational limitations and uncertainties associated with determining an effective Hamiltonian, almost all calculations for nuclei with $A \geqslant 10$ have been performed within a single major oscillator shell, e.g.,

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for ^{10}C the model space spanned by the $0p_{3/2}$ and $0p_{1/2}$ orbitals (p shell). Within this context, two types of isospin mixing must be accounted for. The first is due to the mixing between states that lie within the shell-model configuration space. For example, for $A=10$, there are two, seven, and one p-shell configurations leading to $J^\pi=0^+$ and $T=0$, 1, and 2, respectively. Because of its two-body nature, the INC interaction is composed of isospin operators of rank zero (isoscalar), one (isovector), and two (isotensor) and in the case of $A=10$ it is capable of mixing together all $J^\pi=0^+$ states. Traditionally, the configuration mixing correction is denoted as δ_{IM} and in Ref. [11] it was shown that the best estimates for δ_{IM} are obtained using an INC interaction that correctly describes the Coulomb energy splittings of the binding energies between members of the isospin multiplet, e.g., the $J^\pi=0^+$, $T=1$ states in ^{10}C, ^{10}B, and ^{10}Be. Of the two types of mixing, δ_{IM} is the smallest with a magnitude of approximately 0.04–0.1 %.

In addition to the mixing between states contained within the shell-model configuration space, mixing with states that lie outside the model space must also be accounted for. In particular, the Coulomb interaction can strongly mix one-particle–one-hole $(1p-1h)$ $2\hbar\Omega$ excitations, e.g., $0p_{3/1}\rightarrow 1p_{3/2}$, into the ground state. In previous works, excitations of this type were accounted for by examining differences in the single-particle radial wave functions. Indeed, for closed-shell configurations, mixing with $1p$-$1h$ states is properly accounted for at the Hartree-Fock level. Hence the second correction to the Fermi matrix element, denoted by δ_{RO}, was estimated by evaluating the mismatch in the radial overlap between the single-particle wave functions of the converted proton and the corresponding neutron. The explicit details for the calculation of δ_{RO}, which involve a sum over intermediate $A-1$ parent states that then determine the proton and neutron separation energy for the radial wave function, are given in Refs. [2,11]. For the most part, δ_{RO} is found to be the larger of the two components (with $\delta_C=\delta_{RO}+\delta_{IM}$) and has a magnitude of the order 0.1–0.8 %.

At present, two methods for evaluating δ_{RO} are espoused. The first, the Towner-Hardy-Harvey (THH) method [2], uses Woods-Saxon radial wave functions, while in the second, the Ormand-Brown (OB) method [3,11,13], Hartree-Fock (HF) wave functions are employed. Generally speaking, the two methods yield approximately the same dependence on nucleon number A, but the HF values are systematically smaller by 0.1% for the magnitude of the correction. The reason for the difference lies in the HF mean field. The principal effect of the Coulomb interaction is to push the proton wave functions out relative to the neutrons, hence providing a mismatch in the radial overlap. In Hartree-Fock method, however, the proton and neutron mean fields are coupled and the Coulomb interaction actually induces an attractive isovector mean field between the protons and neutrons. In effect, the Coulomb interaction pushes the protons out, but because of the strong interaction, the protons pull the neutrons out with them, hence reducing the magnitude of the radial overlap mismatch.

When all *known* corrections, i.e., δ_R, Δ_R, δ_{RO}, and δ_{IM}, are applied to the nine experimental data [4,14], it is found that the $\mathcal{F}t$ values are essentially constant within the limits of uncertainty but the unitarity limit is violated at the level of approximately 0.4(1)% or 0.3(1)% for the OB and THH corrections, respectively. In addition, preliminary data from a recent experiment for ^{10}C [16] leads to an $\mathcal{F}t$ value that is significantly smaller than that of Ref. [14], and has been interpreted as possible evidence for an as yet unaccounted for correction that might lead to satisfying the unitarity condition of the CKM matrix. In addition, it must be admitted that the present separation between the configuration mixing and radial overlap contributions to δ_C is somewhat unsatisfying. A much better approach would be to perform a shell-model calculation that includes several $\hbar\Omega$ excitations, so that both corrections would be evaluated on the same footing and simultaneously. Because of recent improvements in computational capabilities and the ability to determine an effective model-space Hamiltonian based on realistic nucleon-nucleon interactions, it is now possible to perform such a calculation for the lightest of the nine accurately measured transitions. We report here the results of large-basis shell-model calculations that include excitations up to $4\hbar\Omega$ for $A=10$ nuclides, with an emphasis on evaluating the isospin-mixing corrections to the matrix element for the Fermi decay of ^{10}C.

The organization of the paper is as follows. First, in Sec. II we discuss the shell-model Hamiltonian with a bound center of mass, the method used to derive the starting-energy-independent effective interaction, and the renormalization of the transfer operator. Results of the Fermi matrix-element calculations are presented in Sec. III and concluding remarks are given in Sec. IV.

II. SHELL-MODEL HAMILTONIAN AND THE EFFECTIVE INTERACTION

In our calculation we use the one- plus two-body Hamiltonian for the A-nucleon system, i.e.,

$$H=\sum_{i=1}^{A}\frac{\vec{p}_i^{\,2}}{2m}+\sum_{i<j}^{A}V_N(\vec{r}_i-\vec{r}_j), \tag{5}$$

where m is the nucleon mass and $V_N(\vec{r}_i-\vec{r}_j)$ the nucleon-nucleon interaction, modified by adding the center-of-mass harmonic-oscillator potential $\frac{1}{2}Am\Omega^2\vec{R}^2$, $\vec{R}=(1/A)\Sigma_{i=1}^{A}\vec{r}_i$. This potential does not influence intrinsic properties of the many-body system. It provides, however, a mean field ''felt'' by each nucleon and allows us to work with a convenient harmonic-oscillator basis. The modified Hamiltonian, depending on the harmonic-oscillator frequency Ω, may be cast into the form

$$H^\Omega=\sum_{i=1}^{A}\left[\frac{\vec{p}_i^{\,2}}{2m}+\frac{1}{2}m\Omega^2\vec{r}_i^{\,2}\right]+\sum_{i<j}^{A}\left[V_N(\vec{r}_i-\vec{r}_j)-\frac{m\Omega^2}{2A}(\vec{r}_i-\vec{r}_j)^2\right]. \tag{6}$$

The one-body term of the Hamiltonian (6) is then rewritten as a sum of the center-of-mass term $H^\Omega_{\rm cm}=\vec{P}^{\,2}_{\rm cm}/2Am+\frac{1}{2}Am\Omega^2R^2$, $\vec{P}_{\rm cm}=\Sigma_{i=1}^{A}\vec{p}_i$, and a term depending on relative coordinates only. Shell-model calculations are carried

out in a model space defined by a projector P. In the present work, we will always use a complete $N\hbar\Omega$ model space. The complementary space to the model space is defined by the projector $Q=1-P$. In addition, from among the eigenstates of the Hamiltonian (6), it is necessary to choose only those corresponding to the same center-of-mass energy. This can be achieved by projecting the center-of-mass eigenstates with energies greater than $\frac{3}{2}\hbar\Omega$ upward in the energy spectrum. The shell-model Hamiltonian, used in the actual calculations, takes the form

$$H^{\Omega}_{P\beta}=\sum_{i<j=1}^{A} P\left[\frac{(\vec{p}_i-\vec{p}_j)^2}{2Am}+\frac{m\Omega^2}{2A}(\vec{r}_i-\vec{r}_j)^2\right]P + \sum_{i<j}^{A} P\left[V_{ij}-\frac{m\Omega^2}{2A}(\vec{r}_i-\vec{r}_j)^2\right]_{\rm eff}P + \beta P(H^{\Omega}_{\rm cm}-\tfrac{3}{2}\hbar\Omega)P, \tag{7}$$

where β is a sufficiently large positive parameter.

The effective interaction introduced in Eq. (7) should, in principle, exactly reproduce the full-space results in the model space for some subset of states. In practice, the effective interactions can never be calculated exactly as, in general, for an A-nucleon system an A-body effective interaction is required. Consequently, large model spaces are desirable when only an approximate effective interaction is used. In that case, the calculation should be less affected by any imprecision of the effective interaction. The same is true for the evaluation of any observable characterized by an operator. In the model space, renormalized effective operators are required. The larger the model space, the less renormalization is needed.

Usually, the effective Hamiltonian is approximated by a two-body effective interaction determined from a two-nucleon system. In this study, we use the procedure as described in Ref. [17]. To construct the effective interaction we employ the Lee-Suzuki [18] similarity transformation method, which gives an interaction in the form $P_2V_{\rm eff}P_2=P_2VP_2+P_2VQ_2\omega P_2$, with ω the transformation operator satisfying $\omega=Q_2\omega P_2$. The projection operators P_2, $Q_2=1-P_2$ project on the two-nucleon model and complementary space, respectively. Our calculations start with exact solutions of the Hamiltonian

$$H^{\Omega}_2\equiv H^{\Omega}_{02}+V^{\Omega}_2=\frac{\vec{p}_1^{\,2}+\vec{p}_2^{\,2}}{2m}+\frac{1}{2}m\Omega^2(\vec{r}_1^{\,2}+\vec{r}_2^{\,2})+V(\vec{r}_1-\vec{r}_2) - \frac{m\Omega^2}{2A}(\vec{r}_1-\vec{r}_2)^2, \tag{8}$$

which is the shell-model Hamiltonian (6) applied to a two-nucleon system. We construct the effective interaction directly from these solutions. Let us denote the two-nucleon harmonic-oscillator states that form the model space as $|\alpha_P\rangle$ and those that belong to the Q space as $|\alpha_Q\rangle$. Then the Q-space components of the eigenvector $|k\rangle$ of the Hamiltonian (8) can be expressed as a combination of the P-space components with the help of the operator ω,

$$\langle\alpha_Q|k\rangle=\sum_{\alpha_P}\langle\alpha_Q|\omega|\alpha_P\rangle\langle\alpha_P|k\rangle. \tag{9}$$

If the dimension of the model space is d_P, we may choose a set $\mathcal{K}$ of d_P eigenevectors, for which the relation (9) will be satisfied. Under the condition that the $d_P\times d_P$ matrix $\langle\alpha_P|k\rangle$ for $|k\rangle\in\mathcal{K}$ is invertible, the operator ω can be determined from Eq. (9). In the present application we select the lowest states obtained in each channel. Once the operator ω is determined the effective Hamiltonian can be constructed as

$$\langle\gamma_P|H_{2\,\rm eff}|\alpha_P\rangle=\sum_{k\in\mathcal{K}}\left[\langle\gamma_P|k\rangle E_k\langle k|\alpha_P\rangle + \sum_{\alpha_Q}\langle\gamma_P|k\rangle E_k\langle k|\alpha_Q\rangle\langle\alpha_Q|\omega|\alpha_P\rangle\right]. \tag{10}$$

This Hamiltonian, when diagonalized in a model-space basis, reproduces exactly the set $\mathcal{K}$ of d_P eigenvalues E_k. Note that the effective Hamiltonian is, in general, quasi-Hermitian. It can be Hermitized by a similarity transformation determined from the metric operator $P_2(1+\omega^{\dagger}\omega)P_2$. The Hermitian Hamiltonian is then given by [19]

$$\overline{H_{2\,\rm eff}}=[P_2(1+\omega^{\dagger}\omega)P_2]^{1/2}H_{2\,\rm eff}[P_2(1+\omega^{\dagger}\omega)P_2]^{-1/2}. \tag{11}$$

Finally, the two-body effective interaction used in the present calculations is determined from the two-nucleon effective Hamiltonian (11) as $V_{\rm eff}=\overline{H_{2\,\rm eff}}-H_{02}$. Note that we distinguish the two-nucleon system projection operators P_2,Q_2 from the A-nucleon system operators P,Q.

To at least partially take into account the many-body effects neglected when using only a two-body effective interaction, we employ the recently introduced multivalued effective interaction approach [20]. As a consequence, different effective interactions are used for different $\hbar\Omega$ excitations. The effective interactions then carry an additional index indicating the sum of the oscillator quanta for the spectators $N_{\rm sps}$ defined by

$$N_{\rm sps}=N_{\rm sum}-N_{\alpha}-N_{\rm spsmin}=N'_{\rm sum}-N_{\gamma}-N_{\rm spsmin}, \tag{12}$$

where $N_{\rm sum}$ and $N'_{\rm sum}$ are the total oscillator quanta in the initial and final many-body states, respectively, and N_{α} and N_{γ} are the total oscillator quanta in the initial and final two-nucleon states $|\alpha\rangle$ and $|\gamma\rangle$, respectively. $N_{\rm spsmin}$ is the minimal value of the spectator harmonic-oscillator quanta for a given system. Here, for $A=10$, $N_{\rm spsmin}=4$. Different sets of the effective interaction are determined for different model spaces characterized by $N_{\rm sps}$ and defined by projection operators

$$Q_2(N_{\rm sps})=\begin{cases}0 & \text{if } N_1+N_2\leq N_{\rm max}-N_{\rm sps}\\ 1 & \text{otherwise,}\end{cases} \tag{13a}$$

$$P_2(N_{\rm sps})=1-Q_2(N_{\rm sps}). \tag{13b}$$

In Eqs. (13), $N_{\rm max}$ characterizes the two-nucleon model space. It is an input parameter chosen in relation to the size

of the many-nucleon model space. This multivalued effective-interaction approach is superior to the traditional single-valued effective interaction, as confirmed also in a model calculation [21].

Our goal in this study is to evaluate the Fermi matrix element

$$M_F = \langle {}^{10}\mathrm{B}, 0^+ 1 | T_- | {}^{10}\mathrm{C}, 0^+ 1 \rangle, \tag{14}$$

which is equal to $\sqrt{2}$ for an isospin-invariant system. Note that for a system with isospin breaking, the isospin-lowering operator T_- should be renormalized in a similar way, as the interaction used for calculation of the eigenstates appearing in Eq. (14). In fact, we can apply the formalism described in Ref. [22] to construct a two-body effective operator $(T_-)_{\mathrm{eff}}$ consistent with the two-body effective interaction derived above and exact for the two-nucleon system. Then we could use such an operator in the A-body calculation. We studied such a possibility in a solvable-model calculation as described in Ref. [21]. Here we did two-nucleon calculations with the effective T_- operator. The observed renormalization of the bare operator for the model spaces of the size used in our calculations was, however, insignificant compared to the other effects as described further. Therefore, in the A-body calculations we used the bare T_- operator.

III. APPLICATION TO THE $A=10$ SYSTEM WITH ISOSPIN BREAKING

In order to evaluate the Fermi matrix element [Eq. (14)], we apply the formalism outlined in Sec. II for $A=10$ nuclei. In the calculations we use the Reid93 nucleon-nucleon potential [23] and consider the following isospin-breaking contributions. First, the Reid93 potential differs in the $T=1$ channels for proton-neutron (pn) and proton-proton (pp), neutron-neutron (nn) systems, respectively. Second, we add the Coulomb potential to the pp Reid93 potential. Consequently, using the Eqs. (9)–(11), we derive different two-body effective interactions for the pn, pp, and nn systems. No other mechanisms for isospin breaking are considered.

As we derive the effective interaction microscopically from the nucleon-nucleon interaction, the number of freely adjustable parameters in the calculation is limited.

First, we have the choice of the model-space size in the shell-model diagonalization. That is, however, constrained by computer capabilities. The largest model space we were able to use was the space allowing all $4\hbar\Omega$ excitations relative to the unperturbed ground state. Most of the calculations were done in the m scheme using the many-fermion-dynamics code [24] extended to allow the use of different pn, pp, and nn interactions. We also performed some calculations with the OXBASH shell-model code [25]. In the m scheme, the dimensions associated with ^{10}B and ^{10}C are 581 740 and 430 137, respectively. To study the dependence on the model-space size, we performed calculations in the $2\hbar\Omega$ space as well. In that space, the dimensions drop to 14 502 and 10 111, respectively.

Second, we have the choice of the two-nucleon model space used for the evaluation of the effective interaction. This is related to the many-nucleon model-space size and, in principle, is determined by that size. Traditionally, however, the $Q=0$ space used to determine the G matrix does not necessarily coincide with the many-particle model space [26,27]. In our calculation, the two-nucleon model space is characterized by a restriction on the number harmonic-oscillator quanta $N_1 \leq N_{\max}$, $N_2 \leq N_{\max}$, and $(N_1+N_2) \leq N_{\max}$. Here $N_i = 2n_i + l_i$ is the harmonic-oscillator quantum number for the nucleon i, $i=1,2$. This type of restriction guarantees an orthogonal transformation between the two-particle states and the relative and center-of-mass coordinate states. For the present $4\hbar\Omega$ calculation, the choice of $N_{\max}=6$ appears to be appropriate. However, it has been observed in the past [17,28,29] that when the Lee-Suzuki procedure combined with the G-matrix calculation according to Ref. [26] (which is equivalent to the procedure we are using) is applied to calculate the two-body effective interaction, the resulting interaction may be too strong. This is in particular true when the multivalued approach is used. Several possible adjustments were discussed to deal with this problem [17,28] and amounted to introducing an extra parameter. In the present calculations, we do not introduce any new parameter, but rather we treat $N_{\max}$ as a free parameter and use $N_{\max}=8$ for the $4\hbar\Omega$ calculations and $N_{\max}=6$ for the $2\hbar\Omega$ calculations, respectively. With this choice we obtain quite reasonable binding energies for the studied nuclei. We have also performed several $2\hbar\Omega$ test calculations with single-valued interactions that were derived following Ref. [30], as opposed to the multivalued interaction discussed in Sec. II. To obtain reasonable binding energies with the single-valued interaction we do not have to change the $N_{\max}$ value from that corresponding to the many-nucleon space, e.g., $N_{\max}=4$ for the $2\hbar\Omega$ calculation. This difference of treatment of the two types of interactions follows from the fact that the overall strength of the single-valued interaction is weaker.

Third, our results depend on the harmonic-oscillator frequency Ω. We have studied this dependence by performing calculations for the values $\hbar\Omega = 14$, 15.5, and 17 MeV.

Let us also mention one important feature of the present approach. For both the multi-valued and the single-valued interactions our calculations do not break the separation of the center of mass and the internal relative motion. In particular, a variation of the parameter β introduced in Eq. (7) does not change the eigenenergies and other characteristic of the physical states. This is so due to the choice of a complete $N\hbar\Omega$ many-nucleon space and the triangular two-nucleon model space for deriving the effective interaction as well as due to the procedure used to derive the effective interaction.

In Figs. 1, 2, and 3 we present the experimental and calculated spectra of ^{10}B for $\hbar\Omega = 14$, 15.5, and 17 MeV, respectively, for the $2\hbar\Omega$ and $4\hbar\Omega$ model spaces. In general, we observe an overall improvement in the spectra with the enlargement of the model space in all three cases. Also, the $4\hbar\Omega$ calculations exhibit more stability with regard to changes in the harmonic-oscillator frequency than do the $2\hbar\Omega$ results. The agreement with experiment improves when going from $\hbar\Omega = 14$ MeV to $\hbar\Omega = 17$ MeV, in particular for the ground state and the lowest states. In fact, from Fig. 3 we

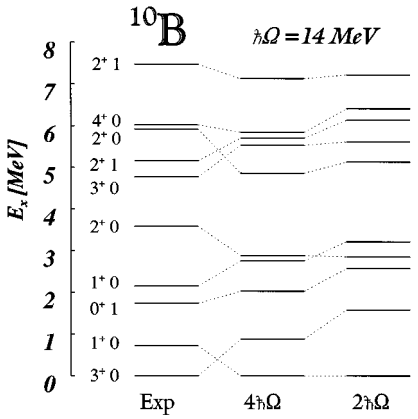

FIG. 1. Experimental and calculated excitation spectra of ^{10}B. The results corresponding to the model-space sizes of $4\hbar\Omega$ and $2\hbar\Omega$ relative to the ground-state configurations are presented, respectively. The harmonic-oscillator energy of $\hbar\Omega = 14$ MeV was used.

find that a very reasonable description of the spectra is obtained for $\hbar\Omega = 17$ MeV.

In Table I the overall behavior with respect to $\hbar\Omega$ is illustrated. In general, we observe a reasonable reproduction of the binding energy, with a moderate decrease occurring for increasing $\hbar\Omega$. Using free-nucleon effective charges, we find that although the quadruple moment for the 3^+0 state is underestimated considerably, the magnetic dipole moment is well reproduced. In addition, the point-proton rms radius exhibits a fairly strong dependence and increases with decreasing $\hbar\Omega$. For the rms radius, we find that the best agreement with experiment [32] is achieved for $\hbar\Omega = 14$ MeV.

From the point of view of the β decay of ^{10}C, a good description of the $T=1$ states is important. From Figs. 1–3 we can see that the calculated ^{10}B $T=1$ states have the right relative positions and are reasonably stable with variations of both the model-space size and $\hbar\Omega$. We have also performed $4\hbar\Omega$ calculations for ^{10}Be to study the splitting of the isospin analog states in the whole isospin-multiplet ^{10}C-^{10}B-^{10}Be. The experimental ground-state splitting between ^{10}C and ^{10}Be is 4.66 MeV, while our calculated values are 4.68, 4.83, and 4.94 MeV for $\hbar\Omega = 14$, 15.5, and 17 MeV, respectively.

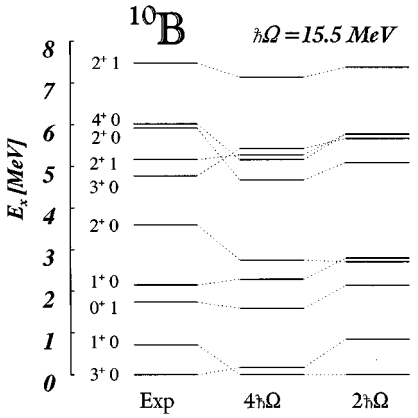

FIG. 2. Same as in Fig. 1 for the harmonic-oscillator energy of $\hbar\Omega = 15.5$ MeV.

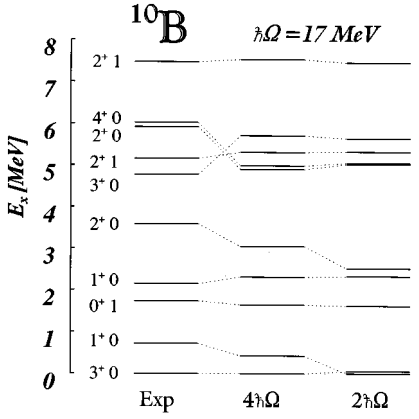

FIG. 3. Same as in Fig. 1 for the harmonic-oscillator energy of $\hbar\Omega = 17$ MeV.

The best agreement with experiment is achieved for $\hbar\Omega = 14$ MeV, where the calculated rms point-proton radius is also in agreement with the experimental value. On the other hand, the splitting between the 0^+1 states of ^{10}C and ^{10}B, which is experimentally 2.69 MeV, is overestimated in our calculations by 8%, 11%, and 14% for the $\hbar\Omega = 14$, 15.5, and 17 MeV calculations, respectively. Since the correct ^{10}C-^{10}Be splitting is obtained for $\hbar\Omega = 14$ MeV, the excess in the ^{10}C-^{10}B splitting suggests that the isospin breaking due the strong $T=1$ force may be too large. One possible explanation is that our approach for deriving the effective interaction tends to exaggerate the differences between the pn and nn, pp potentials. Such an artificial effect should decrease with increasing model-space size. On the other hand, it is also possible that the Reid93 potential itself overestimates differences between the pn and pp,nn systems in the $T=1$ channel. For the most part, we find the best overall agreement for the rms point proton radius, binding energy, and Coulomb energy splitting for $\hbar\Omega = 14$ MeV. Given that the isospin mixing is largely driven by the Coulomb interaction, which is then dependent on the size of the nucleus, we feel that the best value for the isospin-mixing correction to the Fermi matrix element will be achieved for $\hbar\Omega = 14$ MeV.

The most important results of our study are also summarized in Table I in the last two rows. The calculated isospin-mixing corrections $\delta_C = 1 - |M_F|^2/2$ (in %) are presented for all three choices of Ω and for both $4\hbar\Omega$ and $2\hbar\Omega$ model spaces. Again, a correlation between the radius and the isospin-mixing correction is clearly observed, as δ_C decreases with increasing radius. This is simply understood in terms of a larger radius implying weaker Coulomb effects. On the other hand, with an increase in the model-space size, a significant increase in the isospin-mixing correction is apparent. This is due to the fact that in the larger model space, the excitation energies of the $1p$-$1h$ 0^+1 states decrease, hence leading to greater mixing. For this reason, the more realistic multivalued effective interaction is important. We have also performed test calculations with the single-valued interaction in the $2\hbar\Omega$ space and found δ_C to be smaller by approximately 30%.

Our $4\hbar\Omega$ results suggest an isospin-mixing correction

TABLE I. Experimental and calculated binding energies, (in MeV), magnetic moments (in μ_N), and quadrupole moments, (in e fm^2) of ^{10}B. Also the experimental and calculated binding energies in MeV and the point proton radius (in fm) of ^{10}C are presented. The results correspond to the $4\hbar\Omega$ calculations. In addition, the isospin-mixing correction δ_C (in %) is shown as obtained in both the $4\hbar\Omega$ calculations and the $2\hbar\Omega$ calculations. Results of three different calculations with the harmonic-oscillator parameter taken to be $\hbar\Omega = 14$, 15.5, and 17 MeV, respectively, are presented. The effective interaction used was derived from the Reid93 nucleon-nucleon potential. The experimental values are taken from Refs. [31,32].

Property	Expt.	$\hbar\Omega = 14$ MeV	$\hbar\Omega = 15.5$ MeV	$\hbar\Omega = 17$ MeV
$E_B(^{10}\mathrm{B})$	64.75	63.61	62.78	61.53
$Q(3^+0)$	8.47(6)	5.85	5.64	5.52
$\mu(3^+0)$	1.80	1.86	1.85	1.85
$\mu(1^+0)$	0.63(12)	0.84	0.84	0.84
$E_B(^{10}\mathrm{C})$	60.32	58.68	58.19	56.83
$\sqrt{\langle r_p^2\rangle}$	2.31 ± 0.03	2.28	2.21	2.17
$\delta_C(4\hbar\Omega)$ (%)		0.084	0.091	0.097
$\delta_C(2\hbar\Omega)$ (%)		0.055	0.061	0.067

$\delta_C \approx 0.08 - 0.1$ %. This is compatible with the previously published value of $\delta_C \approx 0.15(9)\%$ by Ormand and Brown [13]. That value was a sum of two contributions. First, about 0.04% came from the shell-model wave-function renormalization due to the isospin mixing and was obtained in a $0\hbar\Omega$ shell-model calculation using phenomenological effective interactions. Second, the amount 0.09% was due to the deviation from unity of the radial overlap between the converted proton and the corresponding neutron. This effect was attributed to the influence of states lying outside the $0\hbar\Omega$ space. The radial wave functions were obtained in a Hartree-Fock calculation using Skyrme-type interactions. Because we use a multiconfiguration model space in the present calculation, we should have both effects included consistently at the same time.

Another important factor in the calculation is the position of the $2\hbar\Omega$ states. As discussed before, the position of the $1p$-$1h$ states influences the ground-state isospin mixing. Unfortunately, the excitation energy of these states is not known experimentally. However, in our calculations the multivalued effective interaction is used and a more realistic description of these states should be obtained, especially in the $4\hbar\Omega$ model space. On the other hand, in an analogous calculation for ^{4}He, it was observed that an $8\hbar\Omega$ model space is needed to get the $2\hbar\Omega$ dominated 0^+ state close to the experimental excitation energy [17,20]. There are states like 1^+0 at 5.18 MeV in ^{10}B or 0^+1 at 6.18 MeV in ^{10}Be, which are believed to be $2p$-$2h$, $2\hbar\Omega$ excitations. We do not observe any such states below 7.5 and 12 MeV, respectively, in our calculations. The first excited 0^+1 state in ^{10}Be obtained in the $4\hbar\Omega$ calculation with $\hbar\Omega = 14$ MeV lies at 9.8 MeV. It is, however, predominately a $0\hbar\Omega$ state. There can be two reasons why we do not get such states. First, these states have not yet converged in the Lanczos procedure. Second, and more likely, the $4\hbar\Omega$ model space is too small for the right description of the $2\hbar\Omega$ excitation states. Therefore, it would be desirable to extend the present calculations to a larger, e.g., $6\hbar\Omega$, model space. Unfortunately, due to the computational limitations, it is not possible at this time to perform a calculation of this magnitude. However, from Table I we observe an increase of δ_C by $\approx 0.03\%$ between the $2\hbar\Omega$ and the $4\hbar\Omega$ calculation. Therefore, we might expect an increase of similar magnitude for an increase of the model-space size beyond $4\hbar\Omega$. Therefore, the more realistic value of the isospin-mixing correction from our calculation would be $\delta_C \approx 0.12(3)\%$, where the uncertainty is estimated from the change in δ_C obtained when using an increased model space.

IV. CONCLUSION

The effects of isospin mixing on the transition matrix element for the superallowed Fermi β decay of ^{10}C were estimated within the context of a large-basis, shell-model calculation. The calculations were performed assuming no closed core and an effective interaction based on a realistic two-body nucleon-nucleon interaction, while including the Coulomb interaction between protons. Contrary to previous estimates for the isospin corrections, this calculation was carried out within a model space that included many $\hbar\Omega$ excitations. As a consequence, the conventional configuration mixing and radial mismatch contributions were evaluated within a unified framework simultaneously and the usual separation was not necessary. With regard to parameters used within the calculation, we find a correlation between the isospin-mixing correction and the Coulomb splitting between the isotopic multiplets, which in turn is governed by the nuclear size through the oscillator parameter. Given that the isospin-mixing correction is primarily a Coulomb effect, the best value for δ_C is taken to coincide with the oscillator parameter that correctly reproduces the Coulomb splittings. With regard to the model-space size, a clear improvement (or an indication towards convergence) in most observables is evident when the size of the model space is increased from $2\hbar\Omega$ to $4\hbar\Omega$, but δ_C is found to increase by only 0.03% (in magnitude) in this case. Hence our final estimate for δ_C is taken to be 0.12(3)% (where the $4\hbar\Omega$ result has been increased by

0.03% to account for the possible effects of an increased model space). This result also happens to be in excellent agreement with the previous estimates that relied on the conventional separation of the configuration and radial mismatch contributions. Finally, we note that the magnitude of the isospin-mixing correction obtained in our calculation does not lead to a resolution to the deviation from unitarity for the Cabibbo-Kobayashi-Maskawa matrix.

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