

Scaling-model analysis of nuclear breathing modes in view of a realistic relativistic mean-field parametrization

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We calculate breathing-mode energies in the scaling model for several parameter sets that are under discussion in nuclear relativistic mean-field theory. The relativistic Hartree approximation is used together with a schematic approach for the surface incompressibility. Empirical data can only be reproduced reasonably with parameter sets that lead to a nuclear matter compressibility modulus not higher than 230 MeV. [S0556-2813(97)00511-6]

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The relativistic mean-field approach (RMF) in the form of the nonlinear $\sigma - \omega - \rho$ model is in full practical use for the description of nuclear properties. Despite a well founded field-theoretical background it still has some phenomenological aspects, in particular from the nonlinear terms in its Lagrangian density:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma^\mu\partial_\mu - M + g_\sigma\varphi - g_\omega\gamma^\mu\omega_\mu - g_\rho\gamma^\mu\vec{\tau}\cdot\vec{b}_\mu)\psi \\ & + \frac{1}{2}(\partial_\mu\varphi\partial^\mu\varphi - m_\sigma^2\varphi^2) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + \frac{1}{2}m_\rho^2\vec{b}_\mu\cdot\vec{b}^\mu - \frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} - \frac{1}{3}Mb(g_\sigma\varphi)^3 \\ & - \frac{1}{4}c(g_\sigma\varphi)^4, \end{aligned} \quad (1)$$

where $F_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ and $\vec{G}_{\mu\nu} \equiv \partial_\mu\vec{b}_\nu - \partial_\nu\vec{b}_\mu$, in which \vec{b}_μ denotes an isovector.

The related parameters are fitted mainly to nuclear ground-state properties. Some older approved parameter sets are still under discussion [1–3] together with more recent ones [4,5], so that there is a request for convergence of the parameter sets with an overall efficiency as already reached by nonrelativistic Skyrme-Hartree-Fock (SHF) and Gogny-HF (GHF) approaches. The latter are still in vogue, e.g., as a reliable basis to extrapolate to exotic nuclei [6,7], despite their much more phenomenological basis.

After some promising attempts no characteristic nuclear property seems to remain that is only describable in the framework of RMF. In principle, spin-orbit effects in nuclei could be such candidates since the spin-orbit energy density in RMF depends on gradients of the meson fields, and in SHF, differently, on the density gradient. This distinction might take effect on the spin-orbit splitting of high-lying single-particle states with the consequence of a change in the shell structure. However, recently, RMF spin-orbit potentials were shown to be reasonably well reproduced also by the Skyrme ansatz [8].

The RMF approach describes the nuclear saturation mechanism realistically as a consequence of a characteristic interplay between scalar and vector meson fields. Thus,

small amplitude density oscillations around the saturated ground state are a good testing field for the RMF approach. From its ansatz RMF should be superior to SHF and GHF in the description of breathing modes of nuclei, and, therefore, their RMF investigation should help to determine reliable unique RMF parameters.

The present short RMF study of nuclear breathing modes starts from a previous investigation [9], which was based on the scaling model introduced by Blaizot *et al.* [10,11]. The main deficiency of this earlier investigation came from the crude treatment of the surface incompressibility with an estimated uncertainty of around 30%. We use now a method that—although based again on a model description of the nuclear compression—does much more take into consideration the dynamics [12]. In particular, it can account for the coupling of bulk and surface vibrations. It has been worked out and used in nonrelativistic dynamics of breathing modes [13]. It is based on an approximate ground-state energy density functional that can also be regarded as an approximate relativistic representation.

Mainly in order to introduce notations we repeat first some basic quantities and relations necessary for the use of the scaling model. Following the scaling model and Blaizot [11], we first define a finite-nucleus incompressibility for a nucleus of mass number A and asymmetry $I \equiv (N-Z)/A$ through:

$$K(A, I) = \frac{M}{\hbar^2} \langle r^2 \rangle E_{\text{br}}^2, \quad (2)$$

where $\langle r^2 \rangle$ denotes the rms *matter* radius. Next we make the leptodermous expansion:

$$K(A, I) = K_v + K_{\text{sf}}A^{-1/3} + K_{\text{vs}}I^2 + K_{\text{coul}}Z^2A^{-4/3} + \dots, \quad (3)$$

where, in the scaling-model approximation,

$$K_{\text{sf}} = \left(22 - 2 \frac{K'}{K_v} \right) a_{\text{sf}} + 36\pi r_0^2 \rho_{00}^2 \ddot{\sigma}, \quad (4)$$

$$K_{\text{vs}} = K_{\text{sym}} + L \left(\frac{K'}{K_v} - 6 \right), \quad (5)$$

and

$$K_{\text{coul}} = \frac{3q_{\text{el}}^2}{5r_0} \left(\frac{K'}{K_v} - 8 \right). \quad (6)$$

One sees the possibility of fitting Eq. (3) directly to the data, and extracting empirical values for K_v , along with all the other coefficients. However, it is impossible to extract a unique value of K_v from the measured E_{br} in this way [14,15], all values over the range 100–400 MeV (and maybe over an even wider range) being compatible with the Groningen data [16–18]. Thus there is no alternative to the general strategy pioneered by Blaizot, Gogny, and Grammaticos [10], i.e., to simply trying out different proposed interaction schemes. In our present RMF investigation we start therefore from a microscopic Lagrangian (1), with parameter sets in current use that give a good reproduction of nuclear ground-state properties. Then we calculate the scaling-model values of the coefficients of the leptodermous expansion (3), and obtain $K(A, I)$ values that are compared to the experimental data extracted via Eq. (2) from the breathing-mode energies E_{br} for several nuclei using the experimental values for $\langle r^2 \rangle$.

The quantities K' , K_{sym} , L , r_0 , and ρ_{00} , like J and K_v , are defined with respect to infinite nuclear matter (INM), as follows. If we express the energy per nucleon, e , of INM as a function of the total baryon density ρ and the asymmetry $\delta = (\rho_n - \rho_p)/\rho$, where ρ_n and ρ_p refer to neutron and proton densities, respectively, and $\rho = \rho_n + \rho_p$, then the above coefficients appear in the expansion [19]:

$$e(\rho, \delta) = \left(a_v + \frac{1}{18} K_v \epsilon^2 - \frac{1}{162} K' \epsilon^3 + \dots \right) + \delta^2 \left(J + \frac{1}{3} L \epsilon + \frac{1}{18} K_{\text{sym}} \epsilon^2 + \dots \right) + \dots, \quad (7)$$

where $\epsilon = (\rho - \rho_{00})/\rho_{00}$, in which we denote by ρ_{00} the equilibrium (saturation) density in the symmetric case, $\delta = 0$. We also define the charge-radius constant by $r_0 = (3/4\pi\rho_{00})^{1/3}$.

The remaining quantities, a_{sf} and $\ddot{\sigma}$, refer to symmetric semi-infinite nuclear matter (SINM) with the limiting behavior:

$$\lim_{z \rightarrow \infty} \rho(z) = 0, \quad (8)$$

$$\lim_{z \rightarrow -\infty} \rho(z) \equiv \rho_c = \rho_{00}. \quad (9)$$

That is, deep beneath the surface the local properties of symmetric SINM tend towards those of saturated symmetric INM. A specific surface energy for symmetric SINM is now defined according to

$$\sigma_{00} = \int_{-\infty}^{\infty} \{ \mathcal{E}(z) - a_v \rho(z) \} dz, \quad (10)$$

where $\mathcal{E}(z)$ is the local energy density in SINM and a_v is the energy per nucleon in symmetric INM at saturation, as defined in Eq. (7). Then $a_{\text{sf}} = 4\pi r_0^2 \sigma_{00}$. We note that the rela-

tions given above are valid regardless of the choice of theoretical methods, in particular they hold also for RMF approaches.

The crucial quantity $\ddot{\sigma}$ in Eq. (4) is defined as the double derivative of the surface tension σ of compressed symmetric SINM with respect to the asymptotic density ρ_c , taken at the saturation density ρ_{00} . σ depends also on how the surface itself is compressed even if the basic structure of the nucleus with a bulk and a surface part remains unchanged.

The assumption of the scaling model is that at all stages during the dynamical compressional oscillations the nuclear density undergoes scaling. By microscopic random-phase-approximation (RPA) calculations [11] it has been found that at least for medium heavy and heavy nuclei breathing modes are well represented by scaled densities. In a nonrelativistic hydrodynamical model [13] the scaling prescription turned out to be practically exact for values from $A = 125$ to $A = 130$; even ^{208}Pb is well reproduced. This is corroborated by careful RPA calculations [10,11].

A recent nonrelativistic microscopic analysis [20] based on realistic finite range Gogny effective interactions comes to the conclusion that the microscopic methods are a more reliable tool to describe nuclear monopole vibrations than phenomenological liquid-drop-type expansions for $K(A, I)$ as given in the ansatz (3) which, however, remain useful for a first initial analysis. There are also expansions of $K(A, I)$ similar to the leptodermous expansion (3) based, however, on a picture where the structure of a compressed nucleus is obtained by constraining its radius (see e.g., Ref. [21]). A liquid-drop-type expansion of $K(A, I)$ is also obtained. However, the leading term differs by value from the corresponding term in Eq. (3). All terms together nevertheless seem to end up in a value for $K(A, I)$ comparable to the total value of Eq. (3). Also the generator coordinate method was used (see Ref. [22]) to describe relativistically the breathing modes. In this method—similar to that of Ref. [21]—the nucleus does not exhibit a separation into a bulk and a surface part during oscillation, and therefore it cannot be compared directly with the scaling model.

For the determination of $\ddot{\sigma}$, Eq. (4) in a full relativistic Thomas-Fermi (RTF) or Hartree (RH) calculation one would have to perform calculations of σ over a range of nonsaturating densities where the external pressure must be simulated by a constraint depending on the baryon densities, and even on meson densities. In an attempt this complicated problem was circumvented in Ref. [9] by using a pocket expression for $\ddot{\sigma}$ where RH ground-state quantities could be inserted. This preliminary formula was based on a general ground-state energy density functional (that therefore could be even relativistic). However, it was derived for the compressed static state that is lowest in energy compared to other compression modes of the surface. Since the breathing mode corresponds rather to a scaling of the density the formula has the tendency to give *magnitudes* of $\ddot{\sigma}$ ($\ddot{\sigma}$ always being negative) that are too large by up to 30%, as compared to the scaling value (see Refs. [23] and [24] for the nonrelativistic and relativistic cases, respectively).

This error in $\ddot{\sigma}$ was the largest source of error in the calculation of $K(A, I)$ in Ref. [9] compared to pure scaling calculations. Another source of error—also in the present calculation—concerned the higher-order terms that are ne-

TABLE I. RMF parameter sets. $C_i^2 = g_i^2(M/m_i)^2$, $i = \sigma, \omega, \rho$.

	NL-Z [1]	NL1 [1]	NLC [2]	NL3 [4]	NL-RA [5]	NL-SH [3]
M (MeV)	938.90	938.00	939.00	939.00	939.00	939.00
m_σ (MeV)	488.67	492.25	500.80	508.194	515.00	526.059
m_ω (MeV)	780.00	795.36	783.00	782.501	782.60	783.00
m_ρ (MeV)	763.00	763.00	770.00	763.00	763.00	763.00
C_σ^2	373.2479	373.1760	334.3711	356.3846	308.08	347.533
C_ω^2	241.4392	245.4580	214.1854	238.4424	204.00	240.997
C_ρ^2	35.6700	37.4175	27.8800	30.3161	31.00	29.0954
b	0.0027922	0.0024578	0.0028703	0.0020553	0.0019	0.0012747
c	-0.0039347	-0.0034334	-0.0036849	-0.0026508	-0.0019	-0.0013308

glected in Eq. (3). An nonrelativistic analysis based on Refs. [25] and [26] shows that the magnitude of the net contribution of these terms to $K(A, I)$ is of the order of 10 MeV or less. Finally, there is the question of the validity of the scaling model, which is implicit in Eq. (3) itself, and in the calculation of its coefficients according to Eqs. (4)–(6). By comparing with RPA calculations it has been found [27] that the scaling model overestimates $K(A, I)$ slightly, but by not more than 10 MeV.

In the present analysis we are now following the improved formalism of Refs. [12, 13] that takes into account the scaling behavior of the density so that the characteristic deficiencies of Ref. [9] can be avoided. Starting from a simple model energy density functional, that should also be able to simulate roughly the relativistic one, an analytical treatment of a one-parameter class of compression modes could be carried through. The formula thus obtained for $\ddot{\sigma}$ in the scaling mode is

$$\ddot{\sigma} \equiv \left(\frac{d^2 \sigma}{d\rho_c^2} \right)_{\rho_c = \rho_{00}} = - \frac{19 K_v \alpha}{81 \rho_{00}}, \quad (11)$$

where α is the surface diffuseness parameter of a symmetric Fermi density.

A comparison of the value of $\ddot{\sigma}$ in the recent approach for the energetically lowest static compression mode with the old formula of Ref. [9] suggests an uncertainty of less than 8%. In view of the fact that the basic quantity in our breathing-mode analysis is $K(A, I)$ with K_v as leading term, such an error would not much affect our conclusions, since the values of $K(A, I)$ would only be modified by about 10 MeV. Realistic Skyrme-extended Thomas-Fermi calculations confirmed the basic analytical model almost quantitatively (see Ref. [12]).

The pocket formula (11) may be a basis to estimate K_{sf} Eq. (4) for more complicated interactions. A symmetric Fermi function reproducing in the best way the exact density gives the equivalent α parameter in Eq. (11). To check the proposed method we refer to the exact scaling $K(A, I)$ values ($90 \leq A \leq 208$) following from Ref. [26] for the zero-range Skyrme interactions SIII, Ska, and SkM*. With the α parameters of Ref. [28] we got $K(A, I)$ values that approximate the exact ones within 2% (SIII), 6% (Ska), and 10% (SkM*). As expected from the derivation of Eq. (11) the agreement was the better the smaller the asymmetry of the density was.

Therefore, the model approach based on Eq. (11) will be a good first approximation to the RMF values of $K(A, I)$, the more since the RMF nuclear densities for the standard RMF

TABLE II. INM und SINM coefficients for the parameter sets of Table I (see text for explanation of quantities).

	NL-Z	NL1	NLC	NL3	NL-RA	NL-SH
a_v (MeV)	-16.18	-16.42	-15.77	-16.24	-16.25	-16.35
ρ_{00} (fm ⁻³)	0.1508	0.1518	0.1485	0.1482	0.1570	0.1460
r_0 (fm)	1.17	1.16	1.17	1.17	1.15	1.18
K_v (MeV)	172.8	211.1	224.5	271.5	320.5	355.3
K' (MeV)	422.5	32.7	278.1	-203.0	-216.2	-601.6
J (MeV)	41.72	43.46	35.02	37.40	38.90	36.12
L (MeV)	133.91	140.07	107.97	118.53	119.09	113.64
K_{sym} (MeV)	140.20	142.68	76.91	100.88	62.11	79.77
σ_{00} (MeV fm ⁻²)	1.038	1.098	1.021	1.069	1.169	1.092
a_{sf} (MeV)	17.71	18.66	17.61	18.47	19.43	19.04
α (fm)	0.5279	0.5160	0.4763	0.4546	0.4289	0.4125
$\ddot{\sigma}$ (MeV fm ⁴)	-141.9	-168.4	-168.9	-195.4	-205.3	-235.5
K_{sf} (MeV)	-192.9	-188.3	-234.1	-233.0	-303.4	-304.6
K_{vs} (MeV)	-335.9	-676.1	-437.1	-698.9	-732.8	-794.5
K_{coul} (MeV)	-4.12	-5.83	-4.99	-6.45	-6.52	-7.11

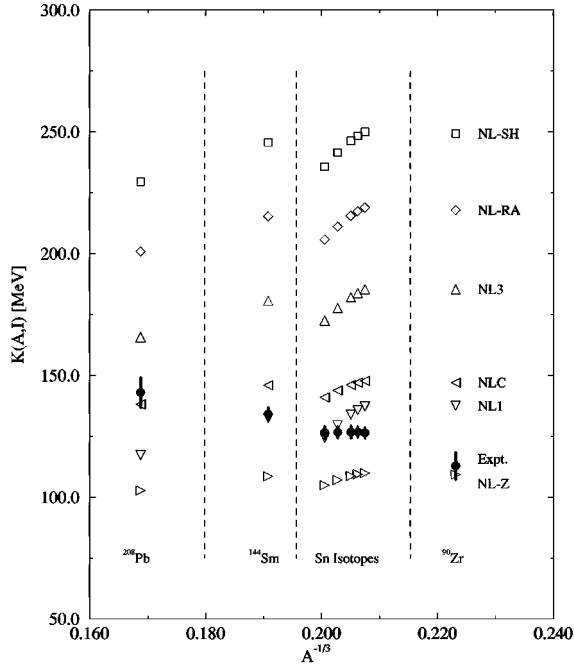


FIG. 1. Finite nucleus incompressibility $K(A, I)$ for several nuclei calculated with the RMF parameter sets given in Table I. [See Ref. [9] for experimental values of $K(A, I)$].

parameter sets come nearer to symmetric Fermi functions than most SHF densities.

In Table I the RMF parameter sets used in the present investigation are arranged in order of increasing values of K_v . Table II gives the values of all INM coefficients entering into the expansion Eq. (7) of the energy per nucleon e . This table also shows the RH values of the SINM quantity a_{sf} , of the surface tension σ_{00} , Eq. (10), and of the α parameter of the Fermi function obtained by a fit to the numerical SINM density. The α parameter is known from experiment quite precisely to be (0.53 ± 0.03) fm [29]. Thus this quantity has to be regarded also as a significant criterion for RMF parameter sets. The value of $\bar{\sigma}$ derived from Eq. (11) as well as the values of K_v , K_{vs} , and K_{Coul} obtained from Eqs. (4)–(6) are given in Table II in addition.

Figure 1 displays the calculated values of $K(A, I)$, and compares with the experimental values [9], extracted from Refs. [16–18] using Eq. (2). A sufficient correspondence between the scaling predictions and experiment is achieved only for the parameter sets NL-Z, NL1, and NLC, having a value of K_v lower than 230 MeV.

Neither the slope of $K(A, I)$ for increasing mass numbers A nor the dependence on the neutron excess I are reproduced very well by any parameter set of Table I. These deficiencies are expected to be reduced if higher-order terms are taken into account in Eq. (3) (see Ref. [26]). In particular, the higher-order terms for the isospin and curvature dependence of $K(A, I)$ are expected to change the slope of the curve that can be drawn through corresponding points in Fig. 1 in order to guide the eyes. However, the absolute changes introduced by relatively small higher-order terms in Eq. (3) do not change our conclusion since $K(A, I)$ is dominated by K_v .

In Fig. 2 the $K(A, I)$ -scaling values calculated for ^{208}Pb ,

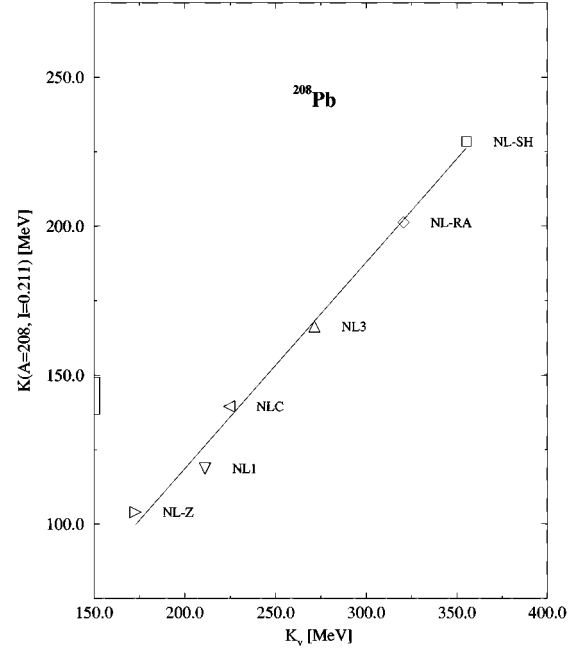


FIG. 2. Finite nucleus scaling incompressibility $K(A=208, I=0.211)$ of ^{208}Pb as a function of K_v for the RMF parameter sets of Table I. The experimental value with its error bar is indicated by the box near the vertical axis. The straight line is a linear least-squares fit: $K(208, 0.211) = -20.07 \text{ MeV} + 0.69 K_v$.

where RPA analyses show small amplitude motion and scaling to be well fulfilled (see Refs. [10, 20]), are plotted as a function of K_v for the six parameter sets under investigation. The experimental region for the value of $K_A(^{208}\text{Pb})$ is indicated on the vertical axis, and again the conclusion about K_v is supported. The six parameter sets are all obtained by fits to special ground-state properties, in particular nuclear binding energies and radii. The ground-state density fall off in the surface region was not taken systematically into account. One might call these Lagrangians ground-state equivalent. With a view to this property Fig. 2 enforces a linear relation between calculated $K_A(^{208}\text{Pb})$ values and the corresponding K_v . Obviously, K_v cannot be determined in a unique way from the ground-state properties that are involved. The inclusion of compressibility properties, however, should fix this K_v value better.

To summarize, despite all remaining uncertainties in our calculations, we conclude that RMF parameter sets being compatible with the measured breathing-mode energies should not have a value of K_v higher than about 230 MeV. This is in agreement with a recent nonrelativistic analysis based on Skyrme scaling model calculations [30] as well as with a microscopic Skyrme RPA approach [20]. One also should note that the parameter sets with K_v greater than 230 MeV give surface thicknesses that are out of the experimentally acceptable region (cf. Table II).

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