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ρ-ω oscillation and charge symmetry breaking

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(Received 3 March 1997)

We propose an oscillation mechanism for meson mixing that causes charge symmetry breaking in nucleon-nucleon interactions. The mixing probabilities of the ρ meson component in the ω meson and vice versa are estimated in a nuclear medium by using the data of the branching ratio $B(\omega\to 2\pi)$ and proton-proton (p-p) and neutron-neutron (n-n) scattering lengths. It is shown that the mixing probability depends on the nuclear density and is very small at normal density. In a nuclear medium, ρ-ω mixing is larger in the p-p interaction than in the n-n interaction because of Coulomb repulsion. [S0556-2813(97)04511-1]

PACS number(s): 12.90.+b, 21.30.Cb, 11.30.Hv, 13.75.Cs

I. INTRODUCTION

Charge symmetry breaking in nuclear interactions is one of the interesting problems of meson physics [1]. Significant differences in experimental values of scattering lengths between proton-proton (p-p) [2] and neutron-neutron (n-n) [3] interactions have been reported and suggest the breaking of charge symmetry in nucleon-nucleon interactions. Many effects that contribute to this discrepancy have been investigated, e.g., the mass difference between a proton and a neutron, π-η mixing, and ρ-ω mixing. The effect of ρ-ω mixing seems to make its largest contribution to the discrepancy of scattering lengths [4–6].

Another example of charge symmetry breaking is the contradiction between experimental values and theoretical calculations of mass differences of mirror nuclei [7,8]. Some calculations, including the effects of symmetry breaking in a nuclear medium, have been carried out by considering ρ-ω mixing [9,10] and the u-d quark mass difference [11], and these calculations imply an improvement of the discrepancy by considering charge symmetry breaking. The mechanism of ρ-ω mixing and the momentum dependence in a nuclear medium also have been discussed recently [12].

Here we propose a different mechanism for ρ-ω mixing. The basic idea is that a vector meson is produced as an isospin eigenstate and travels as a mass eigenstate. The existent probability of isospin eigenstates (ρ and ω) in the mass eigenstate vector meson changes with time due to their mass difference. A similar model is applied to the solar neutrino problem [13] and CP violation of K^0 meson decay [14].

We first explain the ρ-ω mixing through the isospin eigenstate oscillation and then estimate the two-pion decay branching ratio of ω by this model with decay width. Next we consider the discrepancy between p-p and n-n scattering lengths. From these two analyses the mixing angle θ defined in the next section is determined. With this θ we calculate the ρ-ω mixing probability in the nuclear medium and discuss the effects of charge symmetry breaking.

II. ρ-ω OSCILLATION AND 2π DECAY

The original Lagrangian density we assume is given by

$$\mathcal{L}=\bar{\psi}\{\gamma^{\mu}(i\partial_{\mu}-g_{\omega}\omega_{\mu}-g_{\rho}\tau_3\rho_{\mu})-M\}\psi-\frac{1}{4}F^{(\omega)}_{\mu\nu}F^{(\omega)\mu\nu}+\frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu}-\frac{1}{4}F^{(\rho)}_{\mu\nu}F^{(\rho)\mu\nu}+\frac{1}{2}m_{\rho}^2\rho_{\mu}\rho^{\mu}+\epsilon\omega_{\mu}\rho^{\mu}, \tag{1}$$

where ψ is the nucleon field with mass M, ω is the omega meson with mass m_{ω} and isospin 0, ρ is the neutral rho meson with mass m_{ρ} and isospin 1, $F^{(\omega)}_{\mu\nu}=\partial_{\mu}\omega_{\nu}-\partial_{\nu}\omega_{\mu}$, and $F^{(\rho)}_{\mu\nu}=\partial_{\mu}\rho_{\nu}-\partial_{\nu}\rho_{\mu}$. To eliminate the isospin mixing term $\epsilon\omega_{\mu}\rho^{\mu}$, we introduce new vector meson fields defined by

$$A_{\mu}=\omega_{\mu}\cos\theta+\rho_{\mu}\sin\theta, \tag{2}$$

$$B_{\mu}=-\omega_{\mu}\sin\theta+\rho_{\mu}\cos\theta. \tag{3}$$

The Lagrangian density (1) is rewritten as

$$\mathcal{L}=\bar{\psi}\{\gamma^{\mu}(i\partial_{\mu}-g_{A}A_{\mu}-g_{B}B_{\mu})-M\}\psi-\frac{1}{4}F^{(A)}_{\mu\nu}F^{(A)\mu\nu}+\frac{1}{2}m_{A}^2A_{\mu}A^{\mu}-\frac{1}{4}F^{(B)}_{\mu\nu}F^{(B)\mu\nu}+\frac{1}{2}m_{B}^2B_{\mu}B^{\mu}, \tag{4}$$

where the mixing angle θ is determined by

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$$\tan 2\theta = \frac{2\epsilon}{m_\omega^2 - m_\rho^2}. \tag{5}$$

The new coupling constants and masses are given by

$$g_A = g_\omega \cos\theta + g_\rho \tau_3 \sin\theta, \tag{6}$$

$$g_B = -g_\omega \sin\theta + g_\rho \tau_3 \cos\theta, \tag{7}$$

$$m_A^2 = \frac{m_\omega^2 \cos^2\theta - m_\rho^2 \sin^2\theta}{\cos 2\theta}, \tag{8}$$

and

$$m_B^2 = \frac{-m_\omega^2 \sin^2\theta + m_\rho^2 \cos^2\theta}{\cos 2\theta}. \tag{9}$$

The vector fields A and B are mass eigenstates without nucleons, but not isospin eigenstates. In most previous works it is assumed that a vector meson is created as a mass eigenstate A or B and decays or is absorbed as a mass eigenstate. The mass eigenstate A or B includes isospin eigenstates ω and ρ at a fixed rate. With this assumption, the mixing angle θ is determined to reproduce the two-pion decay branching ratio of ω [6] or to explain the discrepancy between p-p and n-n scattering lengths [4].

Here we try to consider that an initially created vector meson is an isospin eigenstate, i.e., ω or ρ. The isospin eigenstate is represented by the linear combination of mass eigenstates A and B. These mass eigenstates are diagonalized without nucleons and their time dependences are given by

$$A(t) = e^{-iE_A t} A(0) \tag{10}$$

and

$$B(t) = e^{-iE_B t} B(0), \tag{11}$$

where E_A and E_B are the eigenenergies of A and B. Hereafter we neglect four-vector indices for simplicity.

Their time-dependent phases vary with time. The phase variation mixes another isospin eigenstate with the initial isospin eigenstate vector meson. The mixing rate changes with time. Thus the vector meson initially created as a pure isospin eigenstate decays or is absorbed as a vector meson with mixed isospin.

The mechanism of isospin mixing is demonstrated as an example of the ω meson decay into pions. We assume ω is created at time $t=0$ as a pure isospin eigenstate. The ω state with isospin 0 is represented as the mixing state of A and B at $t=0$:

$$\omega(0) = A(0)\cos\theta - B(0)\sin\theta. \tag{12}$$

Similarly, at time t the ω state is given by

$$\begin{aligned}\omega(t) &= A(t)\cos\theta - B(t)\sin\theta \\ &= e^{-iE_A t} A(0)\cos\theta - e^{-iE_B t} B(0)\sin\theta, \end{aligned}\tag{13}$$

where we use Eqs. (10) and (11). Using the transformations (2) and (3), this state is rewritten as

$$\begin{aligned}\omega(t) = {}& (e^{-iE_A t}\cos^2\theta + e^{-iE_B t}\sin^2\theta)\,\omega(0) \\ & + (e^{-iE_A t} - e^{-iE_B t})\sin\theta\cos\theta\,\rho(0). \end{aligned}\tag{14}$$

Here $\omega(0)$ and $\rho(0)$ are pure isospin eigenstates, but $\omega(t)$ is an isospin mixed state. The mixing probability of the ρ component (isospin 1) in ω (mixed state) at t is defined by

$$P(\omega|\rho) = |(e^{-iE_A t} - e^{-iE_B t})\sin\theta\cos\theta|^2 \tag{15}$$

and that of the ω component (isospin 0) in ω (mixed state) at t is given by

$$P(\omega|\omega) = |e^{-iE_A t}\cos^2\theta - e^{-iE_B t}\sin^2\theta|^2. \tag{16}$$

The ω meson with isospin 0 decays into three pions, but cannot decay into two pions because of G-parity conservation. The physical ω meson, though, has a branching ratio $B(\omega\to 2\pi)$ of two-pion decay because it contains the isospin 1 (ρ) component.

The branching ratio of two-pion decay of the ω meson is estimated by

$$\begin{aligned} B(\omega\to 2\pi) &= \frac{\Gamma(\omega\to 2\pi)}{\Gamma(\omega\to \mathrm{total})} \\ &= \frac{\int_0^\infty P(\omega|\rho)\Gamma_\rho dt}{\int_0^\infty P(\omega|\omega)\Gamma_\omega dt + \int_0^\infty P(\omega|\rho)\Gamma_\rho dt}, \end{aligned}\tag{17}$$

where Γ_ρ is the full width of the ρ meson and Γ_ω is the width of the ω meson except for the two-pion-decay width. Science $P(\omega|\rho)$ means the probability that the initial ω meson changes into ρ at time t, $(1/T)\int_0^T P(\omega|\rho)\Gamma_\rho dt$ represents the time average of the width that the initial ω changes into ρ and ρ decays into two pions, where we set $T\to\infty$. The first term of the denominator of Eq. (17) is the width that the initial ω decays as a pure isospin eigenstate ω. Assuming $E_A = m_A - (i/2)\Gamma_A$ and $E_B = m_B - (i/2)\Gamma_B$, we obtain

$$\begin{aligned}\int_0^\infty P(\omega|\rho)\Gamma_\rho dt = \Gamma_\rho \Bigg\{ & \frac{1}{\Gamma_A} + \frac{1}{\Gamma_B} \\ & - \frac{\Gamma_A + \Gamma_B}{(\Delta m)^2 + \frac{1}{4}(\Gamma_A+\Gamma_B)^2} \Bigg\} \sin^2\theta\cos^2\theta \end{aligned}\tag{18}$$

and

$$\begin{aligned}\int_0^\infty & P(\omega|\omega)\Gamma_\omega dt \\ &= \Gamma_\omega \Bigg\{ \frac{\cos^4\theta}{\Gamma_A} + \frac{\sin^4\theta}{\Gamma_B} \\ &\quad + \frac{\Gamma_A + \Gamma_B}{(\Delta m)^2 + \frac{1}{4}(\Gamma_A+\Gamma_B)^2} \sin^2\theta\cos^2\theta \Bigg\}, \end{aligned}\tag{19}$$

where $\Delta m = |m_A - m_B|$. The mass difference Δm can be calculated by Eqs. (8) and (9) as a function of θ. Unfortunately, Γ_A and Γ_B, which are widths of A and B, are as yet unknown. Though we do not know how to relate Γ_A and Γ_B to Γ_ω and Γ_ρ, here we assume a similar transformation as in the fields:

$$\Gamma_A = \Gamma_\omega \cos\theta + \Gamma_\rho \sin\theta \tag{20}$$

and

$$\Gamma_B = -\Gamma_\omega \sin\theta + \Gamma_\rho \cos\theta. \tag{21}$$

From the particle data [15] we set $m_\rho = 768.5$ MeV, $m_\omega = 781.94$ MeV, $\Gamma_\rho = 150.7$ MeV, and $\Gamma_\omega = 8.43$ MeV. A comparison of the calculation and the experiments of the branching ratio $B(\omega \to 2\pi) = 0.0221 \pm 0.003$ gives the mixing angle $\theta = 2.2° - 2.5°$.

III. MIXING RATIO IN THE TWO-NUCLEON INTERACTION AND SCATTERING LENGTH

Low-energy scattering experiments show the discrepancy between the proton-proton scattering length [2] and that for neutron-neutron scattering [3]. Some works suggest that the discrepancy can be explained if the ρ meson component included in the ω meson amplitude in the nuclear interaction is about 7% [5,6,10]. The off-shell mixing probability of the isospin eigenstates ρ and ω in mixed ω at time t propagating between two nucleons is given by

$$P(\omega|\rho) = |(e^{-im_A t} - e^{-im_B t}) \sin\theta \cos\theta|^2 \tag{22}$$

and

$$P(\omega|\omega) = |e^{-im_A t} \cos^2\theta - e^{-im_B t} \sin^2\theta|^2. \tag{23}$$

The total mixing probability of the ρ meson in the ω meson in two-nucleon scattering is given by

$$I(\rho) = \frac{\int_0^\infty P(\omega|\rho)\,dt}{\int_0^\infty \{P(\omega|\omega) + P(\omega|\rho)\}\,dt} = 2\,\sin^2\theta \cos^2\theta. \tag{24}$$

If the charge symmetry breaking in low-energy scattering is mostly caused by ρ-ω mixing, the mixing probability is about 0.0049. Considering other possible origins of charge symmetry breaking, this value is probably the maximum. Here we look for θ assuming the mixing probability $I(\rho) = 0.0025 - 0.0049$. We obtain $\theta = 2.0° - 2.8°$. The range of these values covers that obtained for the explanation of the two-pion-decay branching ratio of ω. Hereafter we use 2.4° as the typical value of the mixing angle θ.

IV. MIXING PROBABILITY IN THE NUCLEAR MEDIUM

The off-shell mixing probability (22)

$$P(\omega|\rho) = 2(1 - \cos\Delta m t) \sin^2\theta\, \cos^2\theta \tag{25}$$

is given as a function of time t in which a vector meson propagates between nucleons. If we discuss the meson exchange nuclear interaction, the meson propagating time t is replaced by the internucleon distance $\overline{r}$. In a nuclear medium the ρ mixing probability in the exchange omega is given by

$$P(\omega|\rho) = 2(1 - \cos\Delta m \overline{r}) \sin^2\theta \cos^2\theta. \tag{26}$$

For nuclear matter with density $\rho = 1/(\frac{4}{3}\pi r^3)$, the nearest-neighbor internucleon distance $\overline{r}$ is given by

$$\overline{r} = 2r = \left(\frac{6}{\pi\rho}\right)^{1/3}. \tag{27}$$

At the normal density $\rho = 0.17$ fm^{-3} the internucleon distance $\overline{r}$ is 2.24 fm and the ρ mixing probability is 0.000 041 with $\theta = 2.4°$. The density dependence of the mixing probability in nuclear matter is shown in Fig. 1. At extremely low density the mixing probability oscillates intensively and has a maximum value of 0.007 at 2.46×10^{-6} fm^{-3}. From that density to 0.02 fm^{-3} it decreases rapidly, and at higher densities, including the normal density, it is very small and decreases gradually with density.

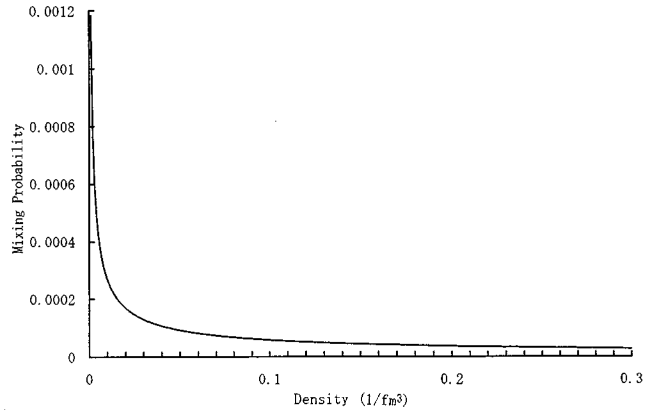

FIG. 1. The ρ-ω mixing probability is shown as a function of nuclear density ρ.

V. CONCLUSION AND DISCUSSION

We have shown that the mixing angle θ is 2.2°–2.5° in an effort to explain the branching ratio of the two-pion decay of the ω meson. The mixing angle θ is determined on the supposition that a vector meson initially created is a pure isospin eigenstate. On the contrary, if it is assumed that a vector meson produced is a mass eigenstate, the mixing probability of ρ and ω does not depend on time and the mixing angle is larger [4,6,10].

Our calculation is ambiguous in terms of how to choose the values of Γ_A and Γ_B, but small changes in Γ_A and Γ_B do not affect the value of θ very much. The equation $B(\omega \to 2\pi) = 0.0221$ has another set solution, i.e., $\theta = -2.0°$ to $-2.2°$. A negative value for θ gives a large branching ratio to the three-pion decay of ρ. In fact, if we set $\theta = -2.0°$, the branching ratio $B(\rho \to 3\pi)$ is 0.003. On the other hand, $\theta = 2.4°$ gives a small branching ratio $B(\rho \to 3\pi) = 0.0007$. Since the three-pion decay of the ρ meson is rare, we should adopt a positive value for θ.

The off-shell mixing probability is also calculated by removing the decay widths. If we want to explain the discrepancy of the nucleon-nucleon scattering lengths using only ρ-ω mixing, the mixing angle θ must have the large value of 2.8°, which gives a value of 0.0049 to the mixing probability of ρ in ω and vice versa. Other factors in addition to the ρ-ω mixing must make some contribution to this discrepancy [5,6].

To calculate the ρ-ω mixing probability in a nuclear medium we use $\theta = 2.4°$. Near the normal density $\rho = 0.17\ \mathrm{fm}^{-3}$ the mixing probability is small and decreases with density. In finite nuclei the proton density is somewhat smaller than the neutron density because of Coulomb repulsive interaction. This means that the ρ-ω mixing probability in the p-p interaction is larger than that in the n-n interaction. Therefore, if we try to calculate the masses of mirror nuclei, the effect of charge symmetry breaking due to ρ-ω mixing is stronger in proton-rich nuclei.

Another candidate for charge symmetry breaking is π-η mixing, to which we apply the same oscillation model. In the case of ω and ρ mesons, their masses are so large that they propagate only to the nearest-neighbor nucleons. Pions though are light enough to arrive at far nucleons. If nucleons are distributed uniformly with the most neighboring internucleon distance $\overline{r}$, the nucleon existent probability on the spherical shell of the radius $n\overline{r}$ is proportional to $4\pi(n\overline{r})^2$. Meson intensity with mass m is proportional to $e^{-mn\overline{r}}/mn\overline{r}$ at the distance $n\overline{r}$ from the nucleon source. Thus the total mixing probability of η in the pion interaction is given as

$$I(\pi|\eta) = \frac{\sum_{n=1}^{\infty} n\overline{r} e^{-m_\pi n\overline{r}} P(\pi|\eta)}{\sum_{n=1}^{\infty} n\overline{r} e^{-m_\pi n\overline{r}}}, \tag{28}$$

where

$$P(\pi|\eta) = 2(1 - \cos\Delta m n\overline{r}) \sin^2\theta \cos^2\theta. \tag{29}$$

The mixing probability of π-η is shown in Fig. 2. Because the mixing angle of π-η is not known, the absolute value of the probability is arbitrary and the mass difference Δm is indefinite. We use $m_A = m_\pi = 135.0$ MeV and $m_B = m_\eta = 547.45$ MeV for the calculation. The total mixing probability of π in η is also shown in Fig. 2. Since the mass of η is large enough, the summation over n is not required, i.e.,

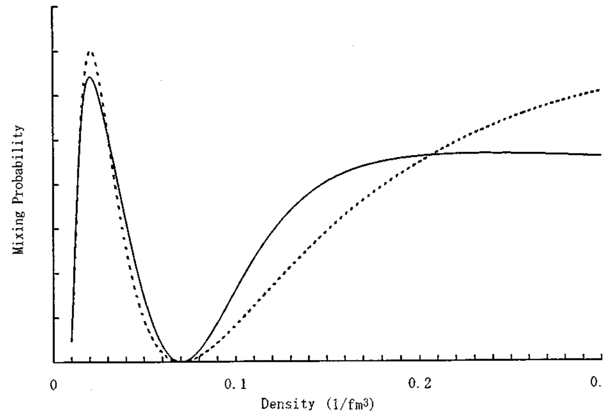

FIG. 2. The π-η mixing probability is shown as a function of nuclear density ρ. The unit of the vertical axis is arbitrary. —— represents the total mixing probability of η in π and ········· represents that of π in η.

$$I(\eta|\pi) \approx P(\eta|\pi). \tag{30}$$

Near normal density, both the mixing probabilities increase with density. Thus, contrary to the case of ρ-ω mixing, the mixing probability of π-η is larger for the neutron interaction than for the proton interaction in finite nuclei. The π-η mixing makes a larger contribution to charge symmetry breaking of neutron-rich nuclei than that of proton-rich nuclei, while ρ-ω mixing contributes more to that of proton-rich nuclei. The vector mesons ρ and ω work repulsively while the nuclear interaction and the pseudoscalar mesons π and η work attractively. Thus the effects of charge symmetry breaking are additive. We should also mention that there is some difference in the total mixing ratios between the pion interaction and the η interaction and as a consequence in the charge symmetry breaking between them.

ACKNOWLEDGMENT

H.T. would like to thank Professor S. Midorikawa (Aomori University) for his useful discussion.

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