PHYSICAL REVIEW C VOLUME 56, NUMBER 4 OCTOBER 1997

Electromagnetic gauge invariance of the cloudy bag model

Gerald A. Miller,* and Anthony W. Thomas†
National Institute of Nuclear Theory, Box 351550, University of Washington, Seattle, Washington 98195-1560
(Received 13 June 1996)

We examine the question of the gauge invariance of electromagnetic form factors calculated within the cloudy bag model. One of the assumptions of the model is that electromagnetic form factors are most accurately evaluated in the Breit frame. This feature is used to show that gauge invariance is respected in this frame. [S0556-2813(97)01810-4]

PACS number(s): 12.39.Ba, 12.90.+b, 12.39.Fe, 24.85.+p

The present experimental efforts [1] to determine the strangeness content of the nucleon have led to considerable interest in nucleon models which include pionic and kaonic clouds. For example, one way to model the strangeness content of the nucleon is to regard it as having a $\overline{K}\Lambda$ component [2].

In the cloudy bag model (CBM) [3,4], the pion field required by chiral symmetry is quantized and coupled to the quarks in an MIT bag [5]. This leads to a model that includes relativistic quark wave functions and quark confinement as well as respecting chiral symmetry. The cloudy bag model was applied to computing π- [3,6] and K-nucleon scattering [7], baryon electromagnetic [8,9] and axial form factors [10], $M1$ radiative decays of mesons of both light and heavy flavor [11], and most recently deep-inelastic scattering [12]. Generally good agreement with experiment was obtained. Moreover, the model provided a framework for nuclear physics which included the old meson cloud physics along with the new quark degrees of freedom in a consistent theoretical framework [4].

Recently, Musolf and Burkardt [13] and Koepf and Henley [14] have argued that the standard computation of the nucleon electromagnetic form factors within such composite models is not gauge invariant. An attempt was made to remedy this perceived problem by adding a contact interaction, as suggested by Gross and Riska [15], or by applying the minimal substitution prescription of Ohta [16]. These ''extra'' terms can be relatively large, especially when applied to the kaon cloud.

Of course, questions of local gauge invariance arise quite generally in nuclear physics, where one often works with effective meson-nucleon interactions involving momentum dependent vertex functions. We recall that the cloudy bag model electromagnetic form factors are most accurately calculated in the Breit frame in which the initial momentum of the nucleon is $-\vec{q}/2$, the final momentum is $\vec{q}/2$, and the energy transferred to the target nucleon vanishes. The use of this special frame was necessary because the evaluations of the model were not covariant and because recoil effects were not included. The problems associated with this deficiency can be reduced by limiting the energy of each nucleon to $\sqrt{\vec{q}^2/4+m^2}$. Once obtained, the form factors can be used in any frame, under the restriction that $-q^2/4m^2$ be small.

Although the underlying, quark-level Lagrangian respects electromagnetic gauge invariance, the CBM does not respect gauge invariance in all frames. In this paper we show that the CBM respects gauge invariance in the Breit frame and, therefore, that in this frame no extra terms need to be added for calculating the nucleon electromagnetic form factors. In order to prove this result one must include the vertex correction illustrated in Fig. 1(b), which was omitted in Ref. [14].

The use of the Breit frame was essential to the CBM (as to all the older, static source, meson theories [17,18] where the source has known internal structure—the MIT bag [5]. In a static model in contrast, there is no difference between a nucleon of momentum $\vec{0}$ and $-\vec{q}/2$, so that the choice of the Breit frame was implicit in the identification of $G_E(Q^2)$ with the matrix element of j^0, while the matrix element of $\vec{j}$ was identified with $G_M(Q^2)$.

Calculations of electromagnetic form factors Γ^μ and nucleon self-energies Σ obey electromagnetic gauge invariance if the Ward identity for nucleons of mass m

$$q_\mu \Gamma^\mu(p',p) = S^{-1}(p') - S^{-1}(p) \tag{1}$$

is satisfied. Here the inverse propagator of the nucleon is $S^{-1}(p)=\gamma p - m - \Sigma(p)$ and $p'=p+q$. As we have already explained, the CBM is not covariant. All calculations of electromagnetic form factors were made in time-ordered perturbation theory with on-mass shell nucleons. Indeed there was no discussion on how to continue the model off-mass shell.

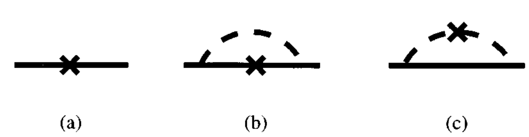

FIG. 1. Contributions to the nucleon electromagnetic form factors.

*Permanent address, Department of Physics, Box 351560, University of Washington, Seattle, Washington 98195-1560. Electronic address: miller@phys.washington.edu

†Permanent address, Department of Physics and Mathematical Physics, and Special Research Center for the Subatomic Structure of Matter, University of Adelaide, Adelaide, South Australia 5005, Australia. Electronic address: athomas@physics.adelaide.edu.au

Thus, the only legitimate concern over gauge invariance within the model, is that Eq. (1) is satisfied when evaluated between on-mass-shell spinors $\overline{u(p')}$ and $u(p)$. That is, we need to check whether

$$q_\mu \overline{u(p')}\Gamma^\mu(p',p)u(p)=\overline{u(p')}[S^{-1}(p')-S^{-1}(p)]u(p). \tag{2}$$

It is useful to start by evaluating the cloudy bag model (CBM) value of $\Sigma_{\rm CBM}(p')-\Sigma_{\rm CBM}(p)$. We use the loop integral formulation of the cloudy bag model, introduced by Koepf and Henley, to evaluate these quantities. Then

$$\Sigma(p)=3ig^2\int\frac{d^4k}{(2\pi)^4}\gamma_5 f(k)S(p-k)f(k)\gamma_5\Delta(k), \tag{3}$$

where Δ is the pion propagator, $f(k)$ is the cloudy bag model form factor $[f(k)=3j_1(|\vec{k}|R)/|\vec{k}|R$, with R the radius of the bag], and g is the pion nucleon coupling constant. We use the pseudoscalar form of the cloudy bag model, but the pseudovector version yields the same result, as we shall see.

We have already noted that the CBM involves only on-mass-shell nucleons. The effects of $N\overline{N}$ pairs were not included in the model—they were supposed to be suppressed by form factors for the composite particle. Thus, for the bare nucleon propagator it is consistent with the CBM to use the form

$$S_0(p)\rightarrow\frac{1}{2E(p)}\frac{\gamma^0E(p)-\vec{\gamma}\cdot\vec{p}+m}{p^0-E(p)+i\epsilon}=\frac{m}{2E(p)}\frac{u(p)\overline{u(p)}}{p^0-E(p)+i\epsilon}, \tag{4}$$

where $E(p)=\sqrt{\vec{p}^{\,2}+m^2}$. The pseudovector and pseudoscalar interactions have the same matrix elements between on-shell spinors, so this is the equality mentioned above. From Eq. (4) we see that all that enters the dressed nucleon propagator in the CBM is the matrix element of Σ between on-shell spinors. Therefore the self-energy in the CBM was effectively the Dirac scalar quantity $\Sigma_{\rm CBM}(p)$:

$$\Sigma_{\rm CBM}(p)=3ig^2\int\frac{d^4k}{(2\pi)^4}\frac{f^2(k)\Delta(k)}{2E(\vec{p}-\vec{k})}\times\frac{\overline{u(p)}\gamma\cdot k u(p)}{p^0-k^0-E(\vec{p}-\vec{k})+i\epsilon}. \tag{5}$$

We do the dk^0 integral over the lower half plane, so that $k^0\rightarrow\omega_k=\sqrt{\vec{k}^2+m_\pi^2}$. A simple evaluation shows that

$$\Sigma_{\rm CBM}(p)=\Sigma_{\rm CBM}[E(|\vec{p}|),\vec{p}]=\frac{E(|\vec{p}|)}{m}I_1(|\vec{p}|)+\frac{I_2(|\vec{p}|)}{m}, \tag{6}$$

where

$$I_1(|\vec{p}|)=\frac{-3g^2}{4}\int\frac{d^3k}{(2\pi)^3}\frac{f^2(|\vec{k}|)}{E(|\vec{p}-\vec{k}|)}\times\frac{1}{\omega_k+E(|\vec{p}-\vec{k}|)-E(|\vec{p}|)} \tag{7}$$

and

$$I_2(|\vec{p}|)=\frac{-3g^2}{4}\int\frac{d^3k}{(2\pi)^3}\frac{f^2(|\vec{k}|)}{E(|\vec{p}-\vec{k}|)}\times\frac{\vec{k}\cdot\vec{p}}{\omega_k+E(|\vec{p}-\vec{k}|)-E(|\vec{p}|)}. \tag{8}$$

In the Breit frame $|\vec{p}\,'|=|\vec{p}|=|\vec{q}|/2$ and $E(\vec{p}\,')=E(|\vec{p}|)=\sqrt{\vec{q}^{\,2}/4+m^2}$. Since Eq. (6) shows that the self-energy depends only on $|\vec{p}|$, it is then clear that

$$\Sigma_{\rm CBM}(p')-\Sigma_{\rm CBM}(p)=0 \tag{9}$$

in this frame.

Considering first the bare vertex γ^μ shown in Fig. 1(a), we see that its contraction with q_μ cancels the term $\gamma\cdot p'-\gamma\cdot p$ on the right hand side of Eq. (1). This means that all that is needed to demonstrate the electromagnetic gauge invariance of the CBM in the Breit frame is to show that $q_\mu\overline{u(p')}[\Gamma^\mu_{\rm CBM}(p',p)-\gamma^\mu]u(p)=0$.

There are two contributions to $\Gamma^\mu_{\rm CBM}(p',p)-\gamma^\mu$. There is a vertex contribution $V^\mu(p',p)$ in which the photon hits the nucleon while the pion is in the air [Fig. 1(b)], and the direct term $\Gamma^\mu_\pi(p',p)$ in which the photon couples to a charged pion [Fig. 1(c)]. Then $\overline{u(p')}[\Gamma^\mu-\gamma^\mu]u(p)=V^\mu+\Gamma^\mu_\pi$.

It is straightforward to obtain

$$q_\mu V^\mu(p',p)=ig^2\overline{u(p')}\int\frac{d^4k}{(2\pi)^4}\gamma_5 f(k)S(p-k)\times\gamma\cdot qS(p'-k)f(k)\gamma_5u(p)\Delta(k^2). \tag{10}$$

But

$$S(p-k)\gamma\cdot qS(p'-k)=S(p-k)-S(p'-k), \tag{11}$$

so that

$$q_\mu V^\mu=\frac{1}{3}[\Sigma(p)-\Sigma(p')], \tag{12}$$

and from Eq. (9) we have

$$q_\mu V^\mu=0. \tag{13}$$

Now consider $q_\mu\Gamma^\mu_\pi$. The Feynman graph is given by

$$q_\mu\Gamma^\mu_\pi(p',p)=i2g^2\int\frac{d^4k}{(2\pi)^4}\overline{u(p')}\gamma_5 f(k-q/2)\times S(p-k+q/2)f(k+q/2)\gamma_5u(p) \tag{14}$$

$$\Delta(k-q/2)\Delta(k+q/2)[(k+q/2)^2-(k-q/2)^2], \tag{15}$$

in which we have replaced the original integration variable k by $k-q/2$. Then use Eq. (4) in the Breit frame ($q^0=0$) to obtain

$$q_\mu \Gamma_\pi^\mu(p',p) = \frac{2ig^2}{4}\int \frac{d^4k}{(2\pi)^4}\frac{\overline{u(p')}\gamma\cdot k u(p)}{E(k)[E(q/2)-k^0-E(k)]} \times \Delta(k-q/2)f(k-q/2)\Delta(k+q/2)f(k+q/2) \times[(k+q/2)^2-(k-q/2)^2]. \tag{16}$$

Consider the d^3k integral for fixed k^0, with $\gamma\cdot k=(\gamma^0k^0-\vec{\gamma}\cdot\vec{k})$. The coefficient of the γ^0k^0 term vanishes since it is multiplied by an odd function of $\vec{k}$. Furthermore, the integral d^3k of the $\vec{\gamma}\cdot\vec{k}$ term must be proportional to $\vec{\gamma}\cdot\vec{q}$ because $\vec{p}=\vec{q}/2=-\vec{p}\,'$. However, $\overline{u(\vec{q}/2)}\vec{\gamma}\cdot\vec{q}u(-\vec{q}/2)=0$. Thus we have $q_\mu\Gamma_\pi^\mu(p',p)=0$. Other terms, such as the anomalous magnetic coupling to the intermediate nucleon, the influence of intermediate Δ states and the effects of the pion-quark-photon contact interaction [19] are individually gauge invariant. This means that the proof that the CBM respects electromagnetic gauge invariance in the Breit frame is complete.

We recognize that the utility of the result proved above is limited. It would be better to use a fully covariant model so that one could verify the Ward identity in any frame. No such model involving quarks, mesons, and photons exists. The present work is limited to the goal of showing that the cloudy bag model was not grossly incorrect.

We thank T. E. O. Ericson, E. Henley, G. Krein, D. Lu, and A. G. Williams for useful discussions and the national Institute for Nuclear Theory for its support. This work was supported in part by U.S. Department of Energy Grant No. DE-FG06-88ER40427 and by the Australian Research Council.

[1] See, e.g., D. Beck *et al.*, Nucl. Phys. **A532**, 507 (1991).
[2] A. I. Signal and A. W. Thomas, Phys. Lett. B **191**, 205 (1987); X. Ji and J. Tang, *ibid.* **362**, 182 (1995); W. Melnitchouk and M. Mahleiro, Phys. Rev. C **55**, 431 (1997).
[3] S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D **22**, 2838 (1980); **23**, 2106(E) (1981); A. W. Thomas, S. Théberge, and G. A. Miller, *ibid.* **24**, 216 (1981).
[4] A. W. Thomas, Adv. Nucl. Phys. **13**, 1 (1984); G. A. Miller, Int. Rev. Nucl. Phys. **2**, 190 (1984).
[5] A. Chodos *et al.*, Phys. Rev. D **9**, 3471 (1974).
[6] B. C. Pearce and I. R. Afnan, Phys. Rev. C **40**, 220 (1989); E. A. Veit *et al.*, Phys. Rev. D **33**, 1859 (1986).
[7] G. He and R. H. Landau, Nucl. Phys. **A585**, 379 (1995); E. A. Veit *et al.*, Phys. Rev. D **31**, 1033 (1985).
[8] A. N. Tabachenko, Mod. Phys. Lett. A **9**, 399 (1994); S. Lee, K. J. Kong, and I. T. Cheon, Phys. Lett. B **202**, 21 (1988); K. Bermuth *et al.*, Phys. Rev. D **37**, 89 (1988); G. Kalbermann and J. M. Eisenberg, *ibid.* **28**, 71 (1983); S. Théberge *et al.*, Can. J. Phys. **60**, 59 (1982).
[9] M.-S. Bae and J. A. McGovern, J. Phys. G **22**, 199 (1996); S. Théberge and A. W. Thomas, Nucl. Phys. **A393**, 252 (1983).
[10] Y. Yamaguchi *et al.*, Nucl. Phys. **A500**, 429 (1989); K. Yohama *et al.*, Phys. Lett. B **186**, 255 (1987); M. A. Morgan and G. A. Miller, *ibid.* **179**, 379 (1986).
[11] P. Singer and G. A. Miller, Phys. Rev. D **33**, 141 (1986); **39**, 825 (1989); G. A. Miller and P. Singer, *ibid.* **37**, 2564 (1988); Y. S. Zhong *et al.*, Nucl. Phys. **A559**, 579 (1993).
[12] F. M. Steffens *et al.*, Phys. Lett. B **358**, 139 (1995).
[13] M. J. Musolf and M. Burkardt, Z. Phys. C **61**, 433 (1994).
[14] W. Koepf and E. M. Henley, Phys. Rev. C **49**, 2219 (1994).
[15] F. Gross and O. Riska, Phys. Rev. C **36**, 1928 (1987).
[16] K. Ohta, Phys. Rev. C **40**, 1335 (1989).
[17] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1955).
[18] E. M. Henley and W. Thirring, *Elementary Quantum Field Theory* (McGraw-Hill, New York, 1962).
[19] M. A. Morgan, G. A. Miller, and A. W. Thomas, Phys. Rev. D **33**, 817 (1986).