Fluctuations in ''Brown-Rho scaled'' chiral Lagrangians

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We develop arguments for ''mapping'' the effective chiral Lagrangian whose parameters are given by ''Brown-Rho'' (BR) scaling to a Landau Fermi-liquid fixed-point theory for nuclear matter in describing fluctuations in various flavor (e.g., strangeness) directions. We use for this purpose the effective Lagrangian used by Furnstahl, Tang, and Serot that incorporates the trace anomaly of QCD in terms of a light-quark (quarkonium) degree of freedom with the heavy (gluonium) degree of freedom integrated out. The large anomalous dimension $d_{\rm an}\approx 5/3$ for the scalar field found by Furnstahl *et al.* to be needed for a correct description of nuclear matter is interpreted as an indication for a strong-coupling regime and the ground state given by the BR-scaled parameters is suggested as the background around which fluctuations can be rendered weak so that mean-field approximation is reliable. We construct a simple model with BR-scaled parameters that provides a satisfactory description of the properties of matter at normal nuclear matter density. Given this, fluctuations around the BR-scaled background are dominated by tree diagrams. Our reasoning relies heavily on recent developments in the study of nucleon and kaon properties in normal and dense nuclear matter, e.g., nucleon and kaon flows in heavy-ion processes, kaonic atoms, and kaon condensation in dense compact-star matter. [S0556-2813(97)00810-8]

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I. INTRODUCTION

In computing fluctuations in various flavor directions (such as strangeness) in nuclear processes, the standard procedure has been to assume that the ground state of hadronic matter is given by the conventional nuclear matter and append in an arbitrary fashion flavor fluctuations on top of the assumed ground state using effective chiral Lagrangians at low chiral order. In doing this, one usually takes a theory for the ground state from a standard many-body treatment and adds mesonic fluctuations using a chiral Lagrangian with, however, no constraints imposed for consistency between the ground state and the fluctuations. This is clearly an unsatisfactory procedure for going beyond the normal matter condition, although with some astute intuitive input, one can make a fairly successful phenomenology of a variety of meson-fluctuation processes at the normal matter density.

In this paper we make an initial step toward bridging the physics of the ground state to that of fluctuations on top of it in the framework of an effective chiral Lagrangian field theory. The problem can be stated as follows. Suppose one wants to describe the property of light-quark mesons in a dense nuclear medium as, for instance, probed in dilepton productions in heavy-ion collisions (e.g., CERES [1]) or in electroproduction of vector mesons inside nuclear medium (e.g., CEBAF [2]). As has been recently shown by Li, Ko, and Brown [3], such a process can be most economically and remarkably well described in terms of a chiral Lagrangian in the mean field with the parameters of the Lagrangian scaled according to ''Brown-Rho (BR) scaling'' [4]. In this approach, however, one treats the ''matter'' (i.e., nuclear) property in a way disconnected from, although not inconsistent with, the BR-scaled chiral Lagrangian that is used to describe the vector meson property. The underlying assumption here is that the ground state is given by the same *effective* chiral Lagrangian that is supposed to include high-order quantum corrections, perhaps as a ''chiral liquid'' as suggested by Lynn [5] or as mean field of the BR-scaled chiral Lagrangian as suggested in [6] (''BR conjecture''). It is not yet fully understood how the Fermi surface is obtained in this scheme. However, given the matter with a Fermi surface given by such a description, one can then *map* the BR-scaled chiral Lagrangian to Landau Fermi-liquid fixed-point theory in the way explained in [7]. This mapping has been tested and found to be phenomenologically successful in such static properties of nuclei as the nuclear gyromagnetic ratio $g_A^\star$, the nucleon effective mass $m_N^\star$, etc. [7]. We take this success as the first justification for the BR conjecture. This provides a link between the baryon property and meson property inside a dense medium. It also enables one to extrapolate from normal nuclear matter at equilibrium to hadronic matter under extreme conditions.

Further support comes from processes involving kaons in nuclear matter. Given the ground state of the matter with the scaled parameters, fluctuations on top of it into the kaonic flavor direction seem to give correct properties of the $K^\pm$ in medium as seen in kaonic atom, subthreshold productions, and flows of $K^\pm$ in heavy-ion collisions [8] (e.g., KaoS [9]

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and FOPI [10]). We take this as the second justification.

A basic problem, however, remains when we apply the theory to kaon condensation in compact-star matter, one of the most fascinating phenomena associated with strangeness in dense matter. Here one is dealing with a change of the ground state from that of nonstrange to strange matter and hence the whole system, that is, the bulk involving the ground state and excitations on top of it has to be treated on the same footing. In works up to date [11,12], this matter has not been consistently treated. It is the aim of this paper to attempt to remedy this defect.

This paper is organized as follows. In Sec. II a general strategy of an effective chiral Lagrangian as applied to a dense medium is presented and the model of Furnstahl *et al.* [13] (referred to as FTS1) that incorporates both chiral symmetry and the trace anomaly of QCD is presented in this framework. In Sec. III, the role of anomalous dimension for the scalar field that enters into the trace anomaly of the FTS1 model on the structure of many-body forces and the compression modulus of nuclear matter is examined. Section IV is devoted to the proposition that the mean-field theory with the FTS1 Lagrangian corresponds to Lynn's nontopological soliton or a chiral liquid. We discuss how this chiral liquid can be identified with Landau's Fermi-liquid structure of drop of nuclear matter in terms of renormalization-group flow arguments using developments in condensed-matter physics. In Sec. V BR scaling is incorporated into a chiral Lagrangian to obtain a weak-coupling description of the same physics as the (strong-coupling) FTS1 mean-field theory. This defines the background at a given finite density around which fluctuations can be made. In Sec. V the BR-scaled parameters introduced in the preceding section can be mapped to Landau Fermi-liquid parameters and a contact with low-energy nuclear properties as well as kaon-nuclear interactions at normal matter and higher densities be made through the mapping of the parameters. A summary and conclusions are given in Sec. VI. The Appendix shows how sensitive the equation of state (EOS) is to the correlation parameters for $\rho > \rho_0$.

II. EFFECTIVE CHIRAL LAGRANGIAN FOR NUCLEAR MATTER

We begin by recalling the main result of [6]. Let an effective Lagrangian $\mathcal{L}^{\rm eff}$ be defined as

$$S^{\rm eff} = \int d^4x\, \mathcal{L}^{\rm eff}, \tag{1}$$

where $S^{\rm eff}$ is a Wilsonian effective action arrived at after integrating out high-frequency modes of the nucleon and other heavy degrees of freedom. This action is then given in terms of sum of terms organized in chiral order in the sense of effective theories at low energy. The key point of Ref. [6] is that the mean-field solution of the chiral effective Lagrangian with the parameters given by the BR scaling [4] approximates the solution

$$\delta S^{\rm eff} = 0. \tag{2}$$

Our ultimate aim in this paper (and subsequent papers) is to ''derive'' the results of Refs. [6,8] starting with a chiral Lagrangian description of the ground state as specified above, around which fluctuations in various flavor sectors are to be made. To do this, we take a phenomenologically successful mean-field model of Walecka type to describe the ground state. In a recent publication, Furnstahl, Tang, and Serot [13] constructed an effective *quantum* nonlinear chiral model that in mean field reproduces quite well all basic nuclear properties. This model that we shall refer to as FTS1 model incorporates the trace anomaly of QCD in terms of a light (''quarkonium'') scalar field S and a heavy (''gluonium'') scalar field χ. In a general framework of chiral dynamics, it is possible to avoid the use of the conformal anomaly of QCD by appealing to other notions of effective-field theories such as ''naturalness condition'' as in [14] (that we shall refer to as FTS2) leading to an effective-mean-field theory that gives an equally satisfactory phenomenology to the FTS1. For our purpose, however, it proves to be more convenient to exploit the role of the light scalar field that figures in the trace anomaly. In particular, it makes the successful description of the nucleon flow in heavy-ion collisions obtained by Li *et al.* [15] (who use the FTS1 theory) more readily understandable.

As in FTS1, we shall assume the heavy scalar field to have the canonical scale dimension $(d=1)$, while the light scalar field is taken to transform under scale transformation as

$$S(\lambda^{-1}x) = \lambda^d S(x), \tag{3}$$

with d a *parameter* that can differ from unity, the canonical dimension. The assumption here is that radiative corrections in the scalar channel can be summarized by an anomalous dimension $d_{an} = d-1 \neq 0$. A heuristic justification for this assumption will be given below in terms of a renormalization-group flow argument. One further assumption that FTS1 adopt from Ref. [16] is that there is no mixing between the light scalar $S(x)$ and the heavy scalar χ in the trace anomaly. Their Lagrangian has the form

$$\mathcal{L}^{\rm eff} = \mathcal{L}_s - H_g \frac{\chi^4}{\chi_0^4}\left(\ln\frac{\chi}{\chi_0} - \frac{1}{4}\right) - H_q\left(\frac{S^2}{S_0^2}\right)^{2/d}\left(\frac{1}{2d}\ln\frac{S^2}{S_0^2} - \frac{1}{4}\right), \tag{4}$$

where $\mathcal{L}_s$ is the chiral- and scale-invariant Lagrangian containing χ, S, N, π, ω, etc. Here χ_0 and S_0 are the vacuum expectation values with the vacuum $|0\rangle$ defined in the matter-free space:

$$\chi_0 \equiv \langle 0|\chi|0\rangle, \quad S_0 \equiv \langle 0|S|0\rangle. \tag{5}$$

The trace of the improved energy-momentum tensor [17] from the Lagrangian is

$$\partial_\mu D^\mu = \theta_\mu^\mu = -H_g \frac{\chi^4}{\chi_0^4} - H_q\left(\frac{S^2}{S_0^2}\right)^{2/d}, \tag{6}$$

where D_μ is the dilatation current. The mass scale associated with the gluonium degree of freedom is higher than that of chiral symmetry, $\Lambda_\chi \sim 1$ GeV, so it is integrated out in favor of the light scalar, in which case the FTS1 effective Lagrangian takes the form

$$\mathcal{L}=\overline{N}[i\gamma_\mu(\partial^\mu+iv^\mu+ig_v\omega^\mu+g_A\gamma_5 a^\mu)-M+g_s\phi]N$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{4!}\zeta g_v^4(\omega_\mu\omega^\mu)^2+\frac{1}{2}\left(1+\eta\frac{\phi}{S_0}\right)$$
$$\times\left[\frac{f_\pi^2}{2}\,\mathrm{tr}(\partial_\mu U\partial^\mu U^\dagger)+m_v^2\omega_\mu\omega^\mu\right]+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$
$$-\frac{m_s^2}{4}S_0^2 d^2\left\{\left(1-\frac{\phi}{S_0}\right)^{4/d}\left[\frac{1}{d}\ln\left(1-\frac{\phi}{S_0}\right)-\frac{1}{4}\right]+\frac{1}{4}\right\}, \tag{7}$$

where $S=S_0-\phi$, η and ζ are unknown parameters to be fixed, and

$$\xi^2=U=e^{i\vec{\pi}\cdot\vec{\tau}/f_\pi},$$

$$v_\mu=-\frac{i}{2}(\xi^\dagger\partial_\mu\xi+\xi\partial_\mu\xi^\dagger),$$

$$a_\mu=-\frac{i}{2}(\xi^\dagger\partial_\mu\xi-\xi\partial_\mu\xi^\dagger).$$

It is important to note that the FTS1 Lagrangian is an effective (quantal) Lagrangian in the sense specified above. The effect of high-frequency modes of the nucleon field and other massive degrees of freedom appears in the parameters of the Lagrangian and in the counterterms that render the expansion meaningful. It presumably includes also vacuum fluctuations in the Dirac sea of the nucleons [13,18]. In general, it must be a lot more complicated. Indeed, one does not yet know how to implement this strategy in full rigor given that one does not know what the matching conditions are. In [13,14], *the major work is, however, done by choosing to fit the relevant parameters of the FTS1 Lagrangian to empirical informations*.

The energy density for uniform nuclear matter with the static mean fields obtained from Eq. (7) is

$$\varepsilon=\frac{\gamma}{(2\pi)^3}\int^{k_F}d^3k\sqrt{\vec{k}^2+(M-g_s\phi_0)^2}$$
$$-\frac{m_v^2}{2}\left(1+\eta\frac{\phi_0}{S_0}\right)\omega_0^2+g_v\rho_B\omega_0-\frac{\zeta}{4!}g_v^4\omega_0^4$$
$$+\frac{m_s^2}{4}S_0^2d^2\left\{1\left(-\frac{\phi_0}{S_0}\right)^{4/d}\left[\frac{1}{d}\ln\left(1-\frac{\phi_0}{S_0}\right)-\frac{1}{4}\right]+\frac{1}{4}\right\}. \tag{8}$$

Here γ is the degeneracy factor.

III. ANOMALOUS DIMENSION

The best fit to the properties of nuclear matter and finite nuclei is obtained with the parameter set $T1$ when the scale dimension of the scalar S is near $d=2.7$. In this section we analyze how this results and present what we understand of the role of the large anomalous dimension $d_{\mathrm{an}}=d-1\approx1.7$ in nuclear dynamics. In what follows, the parameter $T1$ with this anomalous dimension will be taken as a canonical parameter set.[1]

A. Scale anomaly

Following Coleman and Jackiw [17], the scale anomaly can be discussed in terms of an anomalous Ward identity. Define $\Gamma_{\mu\nu}(p,q)$ and $\Gamma(p,q)$ by

$$G(p)\Gamma_{\mu\nu}(p,q)G(p+q)$$
$$=\int d^4x\,d^4y\,e^{iq\cdot x}e^{ip\cdot y}\langle 0|T^*\theta_{\mu\nu}(x)\varphi(y)\varphi(0)|0\rangle, \tag{9}$$

$$G(p)\Gamma_G(p+q)$$
$$=\int d^4x\,d^4y\,e^{iq\cdot x}e^{ip\cdot y}\langle 0|T^*\partial_\mu D^\mu\varphi(y)\varphi(0)|0\rangle, \tag{10}$$

with the renormalized propagator $G(p)$ and the renormalized fields $\varphi(x)$. Here T^* is the covariant T product and $D_\mu(x)$ the dilatation current. A naive consideration on Ward identities would give

$$g_{\mu\nu}\Gamma^{\mu\nu}(p,q)=\Gamma(p,q)-idG^{-1}(p)-idG^{-1}(p+q), \tag{11}$$

with d the scale dimension of $\varphi(x)$. However, Γ is ill defined due to singularity and so has to be regularized. With the regularization, the Ward identity reads

$$g_{\mu\nu}\Gamma^{\mu\nu}(p,q)=\Gamma(p,q)-idG^{-1}(p)-idG^{-1}(p+q)$$
$$+A(p,q), \tag{12}$$

$$A(p,q)\equiv\lim_{\Lambda\to\infty}\Gamma(p,q,\Lambda)-\Gamma(p,q), \tag{13}$$

where the additional term A is the anomaly. This anomaly corresponds to a shift in the dimension of the field involved at the lowest loop order, but at higher orders there are vertex corrections. One obtains, however, a simple result when the β functions vanish at zero momentum transfer [17]. Indeed, in this case, the only effect of the anomaly will appear as an anomalous dimension. In general, this simplification does not occur. However, it can take place when there are nontrivial fixed points in the theory. Now using the reasoning developed in condensed-matter physics [19], we argue as in [7] and further elaborated later that nuclear matter is given in the absence of BCS channel by a Landau Fermi-liquid fixed-point theory with vanishing β functions of the four-Fermi interactions and that all quantum fluctuation effects would appear in the anomalous dimension of the scalar field S. That nuclear matter is a Fermi-liquid fixed point seems to be well verified at least phenomenologically as suggested in [7]. However, that fluctuations into the scalar channel can be summarized into an anomalous dimension is a conjecture that requires a proof. We conjecture here that this is one way we can understand the success of the FTS1 model.

[1] Explicitly the $T1$ parameters are $d=2.7$, $g_s^2=99.3$, $m_S=509$ MeV, $S_0=90.6$ MeV, $g_V^2=154.5$, $\xi=0.0402$, and $\eta=-0.496$.

TABLE I. Equilibrium Fermi momentum k_{eq} and binding energy $B=M-E/A$ as a function of d for Fig. 1.

d	K (MeV)	k_{eq} (MeV)	B (MeV)
2.3	1960	313	50.4
2.4	1275	308	37.0
2.5	687	297	27.1
2.6	309	279	20.4
2.7	196	257	16.4
2.8	184	241	14.0
2.9	180	231	12.4
3.0	175	223	11.2
3.1	169	217	10.3

B. Nuclear matter properties at $d_{\rm an}\approx 5/3$

The FTS1 theory has some remarkable features associated with the large anomalous dimension. Particularly striking is the dependence on the anomalous dimension of the compression modulus and many-body forces.

1. Compression modulus K

Listed in Table I are the compression modulus K and the equilibrium Fermi momentum $k_{\rm eq}$ vs the d of the scalar field ϕ. As the d increases, the K drops very rapidly and stabilizes at $K\sim 200$ MeV for $d\approx 2.6$ and stays nearly constant for $d>2.6$. This can be seen in Fig. 1. The equilibrium Fermi momentum, on the other hand, slowly decreases uniformly as the d increases.

Unfortunately, we have no simple understanding on the mechanism that makes the compression modulus K stabilize at the particular value $d_{an}\approx 5/3$. We believe there is a nontrivial correlation between this behavior of K and the observation made below that the scalar logarithmic interaction brought in by the trace anomaly is entirely given *at the saturation point* by the quadratic term at the same d_{an} with the higher polynomial terms (i.e., many-body interactions) contributing more repulsion for increasing anomalous dimension. At present our understanding is purely numerical and hence incomplete. We plan to report the results of the extensive numerical analyses we have performed and our interpretation thereof elsewhere [20].

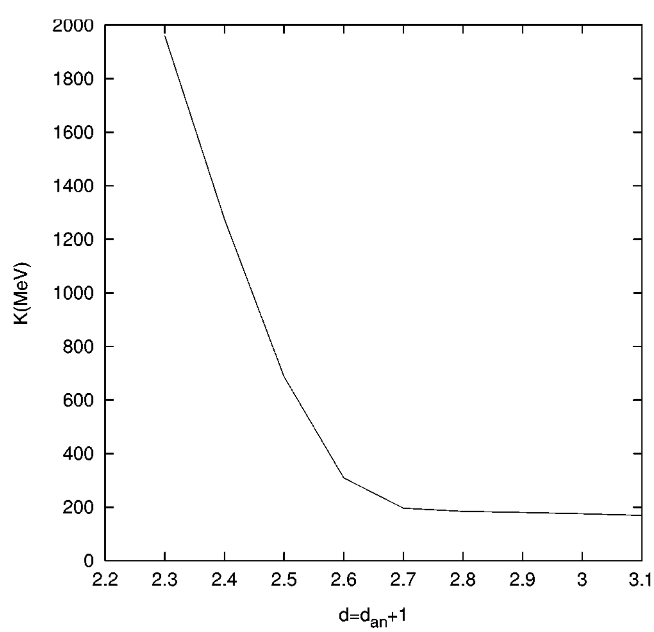

FIG. 1. Compression modulus vs anomalous dimension. The parameter set used here is the $T1$ in FTS1. This shows the sensitivity of the compression modulus to the anomalous dimension.

2. Many-body forces

In the mean field, the logarithmic potential in Eqs. (7) and (8) contains n-body-force (for $n\geqslant 2$) contributions to the energy density. For the FTS1 parameters, these n-body terms are strongly suppressed for $d\gtrsim 2.6$. This is shown in Fig. 2, where it is seen that the entire potential term is accurately reproduced by the quadratic term $\frac{1}{2}m_s^2\phi^2$ for $d_{an}\sim 5/3$. Furthermore, a close examination of the results reveals that each of the n-body terms is separately suppressed. This phenomenon is in some sense consistent with chiral symmetry [21] and is observed in the spectroscopy of light nuclei [22]. Since there are additional polynomial terms in vector fields (i.e., terms such as $\phi\omega^2$), the nearly complete suppression of the scalar polynomials does not mean the same for the total many-body forces. In fact, we should not expect it. To explain why this is so, we plot in Fig. 3 the three-body contributions of the ϕ^3 and $\phi\omega^2$ forms. We also compare the FTS1 results with the FTS2 [14] results that are based on the naturalness condition. In FTS1, the ϕ^3 term that turns to repulsion from attraction for $d>8/3$ contributes little, so the main repulsion arises from the $\phi\omega^2$-type term. This, together with an attraction from an ω^4 term, is needed for saturation of the nuclear matter at the right density.[2]

C. Anomalous dimension and the scalar-meson mass

We would like to understand how the large anomalous dimension needed here could arise in the theory and its role in the scalar sector. Since the trace anomaly arises from the necessity to regularize the theory in the ultraviolet, it cannot depend on density as long as the Fermi momentum involved is less than the chiral scale Λ_χ. Thus the anomalous dimension cannot be due to an effect of density on the trace anomaly. This means instead that the large anomalous dimension reflects a strong-coupling regime with the fluctuation around the matter-free vacuum being strong.

As suggested in [7] and elaborated more in Sec. IV, one appealing way of understanding the FTS1 mean-field theory

[2]This raises the question as to how one can understand the result obtained by Brown, Buballa, and Rho [23], where it is argued that the chiral phase transition in dense medium is of mean field with the bag constant given entirely by the quadratic term $\sim\frac{1}{2}m_s^2\phi^2$. The answer to this question is as follows. First, we expect that the anomalous dimension will stay locked at $d_{an}=d-1\sim 5/3$ near the phase transition (this is because the anomalous dimension associated with the trace anomaly, a consequence of ultraviolet regularization, is not expected to depend upon density), so the ϕ^n terms for $n>2$ will continue to be suppressed as density approaches the critical value. Second, near the chiral phase transition, the gauge coupling of the vector mesons, as argued in [24], will go to zero in accordance with the Georgi vector limit [25], so the many-body forces associated with the vector mesons will also be suppressed.

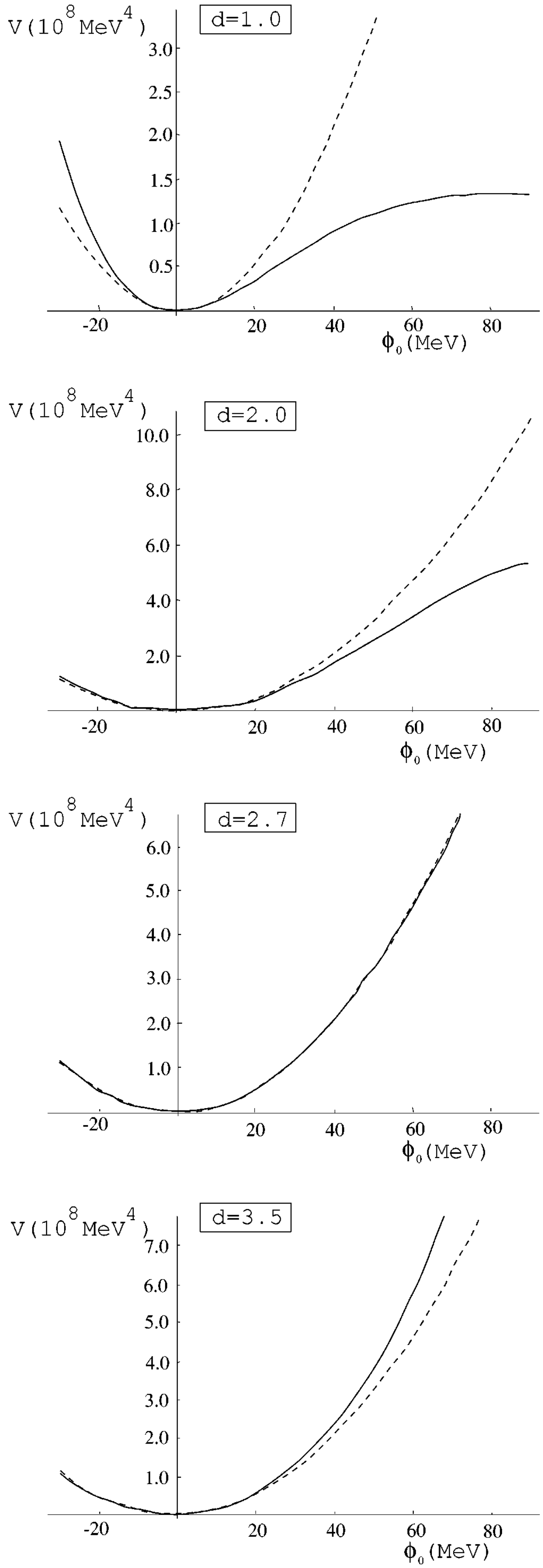

FIG. 2. Comparison between the ϕ^2 interaction and the logarithmic self-interaction of the scalar field with FTS1 parameters. The dashed lines represent $V=(m_s^2/2)\,\phi^2$ and the solid lines $V=(m_s^2/4)\,S_0^2 d^2[(1-\phi/S_0)^{4/d}[(1/d)\ln(1-\phi/S_0)-\frac{1}{4}]+\frac{1}{4}]$ for (from top to bottom) $d=1.0$, 2.0, 2.7, and 3.5, respectively.

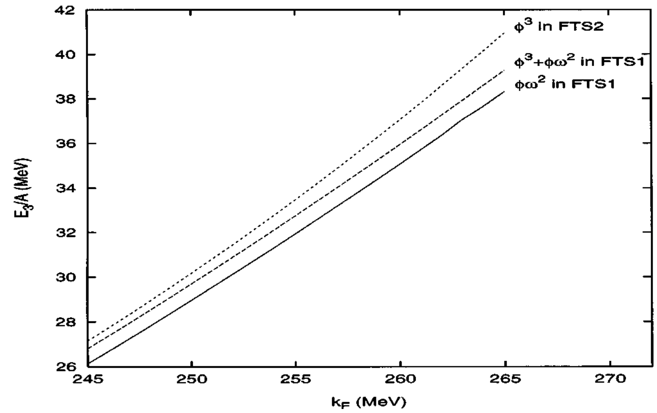

FIG. 3. Three-body contributions to the energy per nucleon vs Fermi momentum in FTS models. The short-dashed line represents the contribution of the ϕ^3 term in FTS2 with the $Q1$ parameters. The long-dashed and the solid lines represent the contributions of the cubic terms ($\phi\omega^2$ and ϕ^3, respectively) in FTS1 with the $T1$ parameters for $d=2.7$.

is to consider *all channels* to be at Fermi-liquid fixed points except that because of the trace anomaly, the scalar field develops an anomalous dimension, thereby affecting the four-Fermi interaction in the scalar channel resulting when the scalar field is integrated out. If the anomalous dimension were sufficiently negative so that marginal terms became marginally relevant, then the system would become unstable as in the case of the Nambu–Jona-Lasinio (NJL) model or superconductivity, with the resulting spontaneous symmetry breaking. However, if the anomalous dimension is positive, then the resulting effect will instead be a screening. The positive anomalous dimension we need here belongs to the latter case. We can see this as follows. Consider the potential given with the low-lying scalar S (with the gluonium component integrated out):

$$V(S,\cdots)=\frac{1}{4}\;m_S^2 d^2 S_0^2\left(\frac{S}{S_0}\right)^{2/d}\left(\frac{1}{d}\ln\frac{S}{S_0}-\frac{1}{4}\right)+\cdots\;, \tag{14}$$

where m_S is the light-quarkonium mass in free space (~ 700 MeV) and the ellipses stand for other contributions such as pions and quark masses that we are not interested in. The scalar excitation on a given background $S^\star$ is given by the double derivative of V with respect to S at $S=S^\star$,

$$m_S^{\star 2}=m_S^2\left(\frac{S^\star}{S_0}\right)^{4/d\,-2}\left[1+\left(\frac{4}{d}-1\right)\ln\frac{S^\star}{S_0}\right]. \tag{15}$$

We may identify the ratio $S^\star/S_0$ with the BR scaling factor Φ [7]:

$$\frac{S^\star}{S_0}=\Phi=\frac{f_\pi^\star}{f_\pi}=\frac{m_V^\star}{m_V}, \tag{16}$$

with the subscript V standing for light-quark vector mesons ρ and ω. Then we have

$$\frac{m_S^\star}{m_S}=\Phi(\rho)\kappa_d(\rho), \tag{17}$$

with

$$\kappa_d(\rho)=\Phi^{2/d-2}\left[1+\left(\frac{4}{d}-1\right)\ln\Phi\right]^{1/2}. \qquad (18)$$

One can see that for $d=1$, which would correspond to the canonical dimension of a scalar field, the scalar mass falls much faster, for a $\Phi(\rho)$ that decreases as a function of density, than what would be given by BR scaling. Increasing the d (and hence the anomalous dimension) makes the scalar mass fall less rapidly. With $d\approx 2$, $\kappa_d\approx 1$ and we recover the BR scaling. Since the dropping scalar mass is associated with an increasing attraction, we see that the anomalous dimension plays the role of bringing in an effective repulsion. One may therefore interpret this as a screening effect of the scalar attraction. In particular, that $d-2\approx 0.7>0$ means that in FTS1, an additional screening of the BR scaled scalar exchange (or an effective repulsion) is present.

IV. CHIRAL LIQUID AND FERMI-LIQUID FIXED POINT

In a more recent paper, Furnstahl, Serot, and Tang [14] reformulated their theory in terms of a chiral Lagrangian constructed by applying the ''naturalness'' condition for all relevant fields. In doing this, Georgi's ''naive dimensional analysis'' [26] was used instead of the trace anomaly and the large anomalous dimension. It was argued therein that a Lagrangian so constructed contains in principle higher-order terms in chiral counting including those loop corrections that can be expressed as counterterms involving matter fields (e.g., baryons). This is essentially equivalent to Lynn's effective action [5] that purports to include all orders of quantum loops in chiral expansion supplemented with counterterms consistent with the order to which loops are calculated. This means that the mean-field solution with the FTS1 (or equivalently [14]) should correspond to the ''chiral liquid'' as the ground-state matter that arises as a nontopological soliton proposed by Lynn. Fluctuations around this mean field should then give an accurate description of the observables that we are dealing with.

We shall here extend this argument further and make a connection with Landau's Fermi-liquid theory of nuclear matter by using the argument of Matsui [27], who described the link between Walecka model in mean-field and Landau-Migdal Fermi-liquid theory. This will allow us to understand BR scaling in terms of chiral Lagrangians and Fermi-liquid fixed-point theory, thereby giving a unified picture of ordinary nuclear matter and the extreme state of matter probed in heavy-ion collisions, e.g., CERES [1]. As far as we know, this is the first such attempt to connect the physics of the two vastly different regimes. The seed for such a scheme and the basic idea were mentioned in the work of Friman and Rho [7].

The basic assumption we start with is that the chiral liquid arises from a quantum effective action resulting from integrating out the degrees of freedom lying above the chiral scale $\Lambda_\chi\sim 4\pi f_\pi\sim 1$ GeV. This corresponds to the first stage of ''decimation'' [30] in our scheme. The mean-field solution of this action is then supposed to yield the ground state of nuclear matter with the Fermi surface characterized by the Fermi momentum k_F. In FTS1, the effective Lagrangian was given in terms of the baryon, pion, quarkonium scalar, and vector fields with the gluonium scalars integrated out. Instead of treating the scalar and vector fields *explicitly* as in FTS1, we will consider here integrating them out further from the effective Lagrangian. This would lead to four-Fermi, six-Fermi, etc., interactions in the Lagrangian with various powers of derivatives acting on the Fermi field. The resulting effective Lagrangian will then consist of the baryons and pions coupled bilinearly in the baryon field and four-Fermi and higher-Fermi interactions with various powers of derivatives, all consistent with chiral symmetry. A minimum version of such a Lagrangian in the mean field can be shown to lead to the original (naive) Walecka model [28]. In principle, a sophisticated version of this procedure should give a theory equivalent to the full FTS1 theory or a generalization thereof.

Leaving out the pion for the moment[3] and formulated nonrelativistically,[4] the next step is to decimate successively the degrees of freedom present in the excitations with the scale $E<\Lambda_\chi$ as follows.[5] To do this we consider excitations near the Fermi surface, which we shall take to be spherical for convenience characterized by k_F. First integrate out the excitations with momentum $p\geqslant\pm\Lambda$ (where $p=|\vec{p}|$ and $\Lambda<\Lambda_\chi$) measured relative to k_F (corresponding to the particle-hole excitations with momentum greater than 2Λ). We are thus restricting ourselves to the physics of excitations whose momenta lie below 2Λ. This defines the starting point of an *in-medium* renormalization-group procedure. The appropriate action to consider can be written in a simplified and schematic form as

$$S=\int_\Lambda \bar{\psi}[i\omega-v_F^\star k]\psi+\delta\mu^\star\int_\Lambda\bar{\psi}\psi+\int_\Lambda u\,\overline{\psi\psi}\psi\psi, \quad (19)$$

where

$$\int_\Lambda := \int\frac{d\Omega}{(2\pi)^2}\int_{-\Lambda}^{\Lambda}\frac{dk}{(2\pi)}\int_{-\infty}^{\infty}\frac{d\omega}{(2\pi)}. \qquad (20)$$

Here $v_F^\star=k_F/m^\star$, where $m^\star$ is the effective mass of the nucleon, which will be equal to the Landau mass $m_L^\star$ as will be elaborated on later. The term with $\delta\mu^\star$ is a counter term added to ensure that the Fermi momentum is fixed (that is, the density is fixed). What his term does is to cancel loop contributions involving the four-Fermi interaction to the nucleon self-energy (i.e., the ''tadpole'') so that the $v_F^\star$ is at the fixed point. This means that the counterterm essentially ensures that the effective mass $m^\star$ be at the fixed point.

[3]The pion will be introduced in Sec. V in terms of a nonlocal four-Fermi interaction that enters in the ground-state property and gives the nucleon Landau mass formula in terms of BR scaling and pionic Fock term. See later.

[4]One could do this relativistically as shown by Baym and Chin [29], which will be necessary for heavy-ion collisions, but we will present the arguments in nonrelativistic form.

[5]Here we are relying on the procedure of decimation formulated rigorously by Chen, Fröhlich, Seifert [30] in condensed-matter physics.

Without this procedure, the term quadratic in the fermion field would be ''relevant'' and hence would be unnatural [19].

In nuclear matter, the spin and isospin degrees of freedom need to be taken into account into the four-Fermi interaction. We have written all these symbolically in the action (19). The function u in the four-Fermi interaction term can therefore contain spin and isospin factors as well as a space dependence that takes into account nonlocality and derivatives. For simplicity we will consider it to be a constant depending in general on spin and isospin factors. Nonconstant terms will be ''irrelevant.'' We shall ignore in the following sections the spin dependence, which will be considered elsewhere, thus confining ourselves to the Landau parameters F and F' corresponding to the particle-hole vibrational channel. In our discussions, the BCS channel that corresponds to a particle-particle channel does not figure and hence will not be considered explicitly.

The upshot of the analyses in [19] and [30] that we apply to our system is that in addition to the Fermi surface fixed point with the $m^\star$, the four-Fermi interactions in the phonon channel F are also at the fixed points. In general, four-Fermi interactions are irrelevant except for special kinematics for which the interaction becomes marginal leading to fixed points. Six-Fermi and higher-Fermi interactions are always irrelevant and can contribute at most to screening of the fixed-point constants. Since the parameters of the fixed-point theory are taken from experiments, we need not worry about this renormalization. The resulting theory is the Fermi-liquid fixed-point theory. Shankar arrives at this theory by showing, in the absence of BCS interactions, that in the large-N limit, where $1/N=\Lambda/k_F$, only one-loop contributions survive. Fröhlich *et al.* obtain the same result in the $1/N$ expansion, where their N is taken to be $N\sim\lambda$, with $1/\lambda$ being the width of the effective wave-vector space around the Fermi sea, which can be considered as the ratio of the microscopic scale to the mesoscopic scale. More specifically, if one rescales the four-Fermi interaction such that one defines the dimensionless constant g, $u_0\sim g/k_F^2$, where u_0 is the leading term (i.e., constant term) in the Taylor series of the quantity u in Eq. (19), then the fermion wave-function renormalization Z, the Fermi velocity v_F, and the constant g are found not to flow up to $O(g^2/N)$. Thus, in the large-N limit, the system flows to Landau fixed-point theory to all orders of loop corrections. This result is correct provided there are no long-range interactions and the BCS channel is turned off. One can show this also in terms of bosonization, which turns out to be possible because of dimensional reduction of the Fermi-liquid system to an effective one-dimensional Dirac system as shown in [31].

In sum, we arrive at the picture where the chiral liquid solution of the quantum effective chiral action gives the Fermi-liquid fixed-point theory. The parameters of the four-Fermi interactions in the phonon channel are then identified with the fixed-point Landau parameters. This identification would allow the mapping of the BR-scaled parameters to the quantities governed by the Landau parameters F and F' discussed in the following section.

V. BR SCALING AND MORE EFFECTIVE CHIRAL LAGRANGIANS

A. The power of BR scaling

If the large anomalous dimension of the scalar field in FTS1 is a symptom of a strong-coupling regime, it suggests that one should redefine the vacuum in such a way that the fluctuation around the new vacuum becomes weak coupling. This is the basis of the BR scaling introduced in [4]. The basic idea[6] is to fluctuate around the ''vacuum'' defined at $\rho\approx\rho_0$ characterized by the quark condensate $\langle\bar{q}q\rangle_\rho \equiv\langle\bar{q}q\rangle^\star$. In [4,24], this theory was developed with a chiral Lagrangian implemented with the trace anomaly of QCD. The Lagrangian used was the one valid in the large-N_c limit of QCD and hence given entirely in terms of boson fields from which baryons arose as solitons (skyrmions): Baryon properties are therefore dictated by the structure of the bosonic Lagrangian, thereby leading to a sort of *universal scaling* between mesons and baryons. One can see using a dilated chiral quark model that the BR scaling is a generic feature also at high temperature in the large-N_c limit [32].

In this description, one is approximating the complicated strong interaction process at a nuclear matter density in terms of ''quasiparticle'' excitations for both baryons and bosons in medium. This means that the properties of fermions and bosons in medium at $\rho\approx\rho_0$ are given in terms of tree diagrams with the properties defined in terms of the masses and coupling constants universally determined by the quark condensates at that density.

The question then is, How can one ''marry'' the FTS1 Lagrangian with the BR-scaled Lagrangian? The next question is how to identify BR-scaled parameters with the Landau parameters. In the rest of this section we will provide some answers to these two questions.

B. A hybrid model

As a first attempt to answer this question we consider the hybrid model in which the ground state is given by the mean field of the FTS1 Lagrangian $\mathcal{L}_{\rm FTS1}$ and the fluctuation around the ground state is described by the tree diagrams of the BR-scaled Lagrangian $\Delta\mathcal{L}$,

$$\mathcal{L}^{\rm eff}=\mathcal{L}_{FTS1}+\Delta\mathcal{L}. \tag{21}$$

Note that the fluctuation in the strangeness direction (58) discussed below corresponds to one of the terms figuring in $\Delta\mathcal{L}$. This model with the canonical parameters $(T1)$ for the

[6]For completeness, we briefly summarize the key argument of [4]. Consider an extended chunk of nuclear matter. If the system is sufficiently dilute, one can start with a chiral Lagrangian constructed with parameters fixed in the matter-free space characterized by a corresponding scale, say, Λ_0. Now suppose that the matter is ''squeezed'' to a density ρ with its scale characterized by, say, Λ_ρ. Our basic assumption is that to describe this dense system, we may impose the same symmetry (such as chiral symmetry and conformal anomaly) constraints as in the matter-free space while replacing in the effective chiral Lagrangian the free-space parameters, masses and coupling constants, by those defined at that density. BR scaling is one specific way of defining these modified parameters.

ground state and a BR-scaled fluctuation Lagrangian in the nonstrange flavor sector was recently used by Li, Brown, Lee, and Ko [15] for describing *simultaneously* nucleon flow and dilepton production in heavy-ion collisions. The nucleon flow is sensitive to the parameters of the baryon sector, in particular the repulsive nucleon vector potential at high density, whereas the dilepton production probes the parameters of the meson sector. With a suitable momentum dependence implemented to the FTS1 mean-field equation of state, the nucleon flow results in good agreement with experiments. Furthermore, the scaling of the nucleon mass in the FTS1 theory in dense medium, say, at $\rho \sim 3\rho_0$, is found to be essentially the same as that given by the NJL model. Therefore, we can conclude that the nucleon in FTS1 scales in the same way as BR scaling.

The dilepton production involves both baryon and meson properties, the former in the scaling of the nucleon mass and the latter in the scaling of the vector meson (ρ) mass. The equation of state correctly describing the nucleon flow and the BR-scaled vector meson mass is found to fit the dilepton data equally well, comparable to the fit obtained in [3] using the Walecka mean field. What is important in this process is the scalar mean field that governs the BR scaling and hence the production rate is essentially the same for FTS1 and Walecka mean fields. The delicate interplay between the attraction and the repulsion that figures importantly in the compression modulus [20] does not play an important role in the dilepton process.

Let us see how the particles behave in the background of the FTS1 ground state given by $\mathcal{L}_{\rm FTS1}$. The nucleon of course scales in the manner of Brown and Rho as mentioned above. We can say nothing on the pion and the ρ meson with the FTS1 theory. However there is nothing that would preclude the ρ scaling in the manner of Brown and Rho and the pion nonscaling within the scheme. What is encoded in the FTS1 theory is the behavior of the ω and the scalar S that figure importantly in Walecka mean fields. Let us therefore focus on these two fields in the medium near normal nuclear matter density.

We have already shown in Sec. III C that the mass of the scalar field S drops less rapidly than BR scaling for $d>2$. One can think of this as a screening of the four-Fermi interaction in the scalar channel that arises when the scalar meson with the BR scaled mass is integrated out. This feature and the property of the ω field can be seen by the toy model of the FTS1 Lagrangian (that includes terms corresponding up to three-body forces)

$$\mathcal{L}_{toy\ FTS1}=\mathcal{L}_{\rm BR}+\frac{m_\omega^2}{2}(2+\eta)\frac{\phi}{S_0}\omega^2-\frac{m_s^2\phi^3}{3S_0}, \tag{22}$$

where

$$\mathcal{L}_{\rm BR}=\overline{N}[i\gamma_\mu(\partial^\mu+ig_v\omega^\mu)-M+g_s\phi]N+\frac{m_\omega^2}{2}\omega^2\left(1-\frac{2\phi}{S_0}\right)-\frac{m_s^2}{2}\phi^2\left(1-\frac{2\phi}{3S_0}\right). \tag{23}$$

We have written $\mathcal{L}_{\rm BR}$ such that the BR scaling is incorporated *at the mean-field level* as[7]

$$\Phi(\rho)=\frac{M^\star}{M}=\frac{m_s^\star}{m_s}=\frac{m_\omega^\star}{m_\omega}\approx 1-\frac{\phi}{S_0}, \tag{24}$$

with

$$S_0=\langle 0|S|0\rangle = M/g_s. \tag{25}$$

We can see from Eq. (22) that the FTS1 theory brings in an additional repulsive three-body force coming from a cubic scalar field term, while if one takes $\eta=-2$, the ω field will have a BR-scaling mass in nuclear matter. The fit to experiments favors $\eta\approx -1/2$ instead of -2, thus indicating that the FTS1 theory requires a many-body suppression of the repulsion due to the ω exchange two-body force. (In the simple model with BR scaling that we will construct below, we shall use this feature by introducing a ''running'' vector coupling $g_v^\star$ that drops as a function of density.) The effective ω mass may be written as

$$m_\omega^{\star 2}\approx\left[1+\eta\frac{\phi_0}{S_0}\right]m_\omega^2. \tag{26}$$

For $\eta<0$, we have a falling ω mass corresponding to BR scaling (modulo, of course, the numerical value of η). In FTS1, there is a quartic term $\sim\omega^4$, which is attractive and hence *increases* the ω mass. In fact, because of the attractive quartic ω term, we have

$$\frac{m_\omega^\star}{m_\omega}\approx 1.12 \tag{27}$$

at the saturation density with the $T1$ parameter set. This would seem to suggest that due to higher polynomial (many-body) effects, the ω mass does not follow BR scaling in the medium. Furthermore, the ω effective mass increases slowly around this equilibrium value:

$$\frac{\partial m_\omega^\star}{\partial k_F}\sim\frac{0.0004}{\rm MeV^2}\alpha, \tag{28}$$

with $\alpha\equiv(\gamma/2\pi^2)\,k_F^2$, if one uses

$$\phi_0\approx\frac{g_s}{m_s^2}\frac{\gamma}{6\pi^2}k_F^3, \tag{29}$$

$$\omega_0\approx\frac{g_v}{m_\omega^2}\frac{\gamma}{6\pi^2}k_F^3, \tag{30}$$

with the degeneracy factor γ and the $T1$ parameters.

C. Model with BR scaling

The above hybrid model suggests how to construct an effective Lagrangian model that implements BR scaling and

[7] Here we are ignoring the deviation of the scaling of the effective nucleon mass (denoted later as $m_L^\star$) [7] from the universal scaling $\Phi(\rho)$. This will be incorporated in Sec. V C.

contains the same physics as FTS1 theory. The crucial point is that such a Lagrangian is to give in the mean field the chiral liquid soliton solution. This can be done by making the following replacements in Eq. (23):

$$M-g_s\phi_0 \to M^\star,$$

$$m_\omega^2\left(1-\frac{2\phi_0}{S_0}\right)\to m_\omega^{\star 2},$$

$$m_s^2\left(1-\frac{2\phi_0}{S_0}\right)\to m_s^{\star 2} \tag{31}$$

and write

$$\mathcal{L}_{\rm BR}=\overline{N}[i\gamma_\mu(\partial^\mu+ig_v\omega^\mu)-M^\star+h\phi]N-\frac{1}{4}F_{\mu\nu}^2+\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m_\omega^{\star 2}}{2}\omega^2-\frac{m_s^{\star 2}}{2}\phi^2, \tag{32}$$

with

$$\frac{M^\star}{M}=\frac{m_\omega^\star}{m_\omega}=\frac{m_s^\star}{m_s}=\Phi(\rho). \tag{33}$$

The additional term $\overline{N}h\phi N$ is put in to account for the difference between the Landau mass $m_L^\star$ to be given later and the BR-scaling mass $M^\star$. In the chiral Lagrangian approach with BR scaling, the difference comes through the Fock term involving nonlocal pion exchange [7]. This will be discussed further in Sec. V D. For simplicity we will take the scaling in the form

$$\Phi(\rho)=\frac{1}{1+y\rho/\rho_0}, \tag{34}$$

with $y=0.28$, so as to give $\Phi(\rho_0)=0.78$ (corresponding to $k_F=260$ MeV) found in QCD sum-rule calculations [7] as will be discussed shortly, as well as from the in-medium Gell-Mann-Oakes-Renner relation [24]. Note that the Lagrangian (32) treated at the mean-field level would give a Walecka-type model with the meson masses replaced by the BR-scaling mass.

Now in order to describe nuclear matter in the spirit of the FTS1 theory, we introduce terms cubic and higher in ω and ϕ fields *to be treated as perturbations around the BR background* as

$$\mathcal{L}_{n\ \rm body}=a\phi\omega^2+b\phi^3+c\omega^4+d\phi^4+e\phi^2\omega^2+\cdots, \tag{35}$$

where $a-e$ are ''natural'' (possibly density-dependent) constants to be determined. By inserting for the ϕ and ω fields the solutions of the static mean field equations given by $\mathcal{L}_{\rm BR}$,

$$m_s^{\star 2}\phi=h\sum_i \overline{N_i}N_i, \tag{36}$$

$$m_\omega^{\star 2}\omega=g_v\sum_i N_i^\dagger N_i, \tag{37}$$

TABLE II. Parameters for the Lagrangian (39) with $y=0.28$, $m_s=700$ MeV, $m_\omega=783$ MeV, and $M=939$ MeV.

Set	h	g_v	z
$S1$	6.62	15.8	0.28
$S2$	5.62	15.3	0.30
$S3$	5.30	15.2	0.31

we see that at the mean-field level $\mathcal{L}_{n\ \rm body}$ generates three- and higher-body forces with the exchanged masses density dependent in the manner of Brown and Rho. Note that at this point, the scaling factor Φ and the mean-field value (36) are not necessarily locked to each other by self-consistency.

As the first trial, we will consider the drastically simplified model by dropping the n-body term (35) and minimally modifying the BR Lagrangian (32). We shall do this by letting, as mentioned above, the vector coupling run as a function of density. For this, we use the observation made in [15] that the nucleon flow probing higher density requires that $g_v^\star/m_v^\star$ be independent of density at low densities and decrease slightly at high densities. We shall therefore take, to simulate this particular many-body correlation effect, the vector coupling to scale as

$$\frac{g_v^\star}{g_v}=\frac{1}{1+z\rho/\rho_0}, \tag{38}$$

with z equal to or slightly greater than y.[8] The truncated Lagrangian that we shall consider then is

$$\mathcal{L}_{\rm BR}=\overline{N}[i\gamma_\mu(\partial^\mu+ig_v^\star\omega^\mu)-M^\star+h\phi]N-\frac{1}{4}F_{\mu\nu}^2 +\frac{1}{2}(\partial_\mu\phi)^2+\frac{m_\omega^{\star 2}}{2}\omega^2-\frac{m_s^{\star 2}}{2}\phi^2. \tag{39}$$

[8]This scaling seems at odds with the prediction made with the Skyrme model [33] where using the Skyrme model with the quartic Skyrme term inversely proportional to the coupling e, it was found that $e/e^\star\sim\sqrt{g_A^\star/g_A}$. It is tempting to identify [via SU(6) symmetry] e with the g_v that we are discussing here since the Skyrme quartic term can formally be obtained from a hidden gauge-symmetric Lagrangian by integrating out the ρ meson field. If this were correct, one would predict that the vector coupling increases (not decreases) as density increases since we know that $g_A^\star$ is quenched in dense matter. This identification could be too naive and incomplete in two respects, however. First of all, this skyrmion formula is a large-N_c relation and second, the Skyrme quartic term embodies *all* short-distance physics in one dimension-four term in a derivative expansion. Thus the constant $1/e$ must represent a lot more than just the vector-meson (ρ) degree of freedom. Furthermore, we are concerned with the ω degree of freedom that in a naive derivative expansion would give a six-derivative term. The BR-scaled model we are constructing should involve not only short-distance physics presumably represented by the $1/e$ term (consistent with the understanding that the quenching of g_A is a short-distance phenomenon) but also longer-range correlations. Therefore, the qualitative difference should surprise no one.

TABLE III. Nuclear matter properties predicted with the parameters of Table II. The effective nucleon mass (later identified with the Landau mass) is $m_L^\star = M^* - h\phi_0$.

Set	$E/A-M$ (meV)	k_{eq} (MeV)	K (MeV)	$M_L^\star/M$	Φ (k_{eq})
$S1$	−16.0	257.3	296	0.619	0.79
$S2$	−16.2	256.9	263	0.666	0.79
$S3$	−16.1	258.2	259	0.675	0.78

In Table II three sets of parameters are listed. We take the measured free-space masses for the ω and the nucleon, and for the scalar ϕ for which the free-space mass cannot be precisely given we take $m_s = 700$ MeV (consistent with what is argued in [24]), so that at nuclear matter density it comes close to what enters in the FTS1. The resulting fits to the properties of nuclear matter are given in Table III for the parameters given in Table II. These results are encouraging. Considering the simplicity of the model, the model, in particular with the $S2$ and $S3$ sets, is remarkably close in nuclear matter to the full FTS1. The compression modulus comes down toward the low value that is currently favored. In fact, the somewhat higher value obtained here can be easily brought down to about 200 MeV without modifying other quantities if one admits a small admixture of the *residual* many-body terms (35), as we shall shortly show. The effective nucleon Landau mass $m_L^\star/M \approx 0.67$ is in good agreement with what was obtained in QCD sum-rule calculations (see [7]) and also below (i.e., 0.69) by mapping BR scaling to Landau-Migdal Fermi-liquid theory. We shall see below that this has strong support from low-energy nuclear properties. What is also noteworthy is that the ratio $\mathcal{R} \equiv (g_v^\star/m_v^\star)^2$ forced upon us, though not predicted, is independent of the density (set $S1$) or slightly decreasing with density (sets $S2$ and $S3$), as required in the nucleon flow data as found by Li, Brown, Lee, and Ko [15].[9]

The assumption that the many-body correlation terms in Eq. (35) can be entirely subsumed in the dropping vector coupling may seem too drastic. Let us see what small residual three-body and four-body terms in Eq. (35) as many-body correlations (over and above what is included in the running vector coupling constant) can do to nuclear matter properties. For convenience we rewrite Eq. (35) by inserting dimensional factors as

$$\mathcal{L}_{n\ \mathrm{body}} = \frac{\eta_0}{2} m_\omega^2 \frac{\phi}{f_\pi}\omega^2 - \frac{\kappa_3}{3!} m_s^2 \frac{\phi^3}{f_\pi} + \frac{\zeta_0}{4!} g_v^2 \omega^4 - \frac{\kappa_4}{4!} m_s^2 \frac{\phi^4}{f_\pi^2} + \frac{\eta_1}{2} m_\omega^2 \frac{\phi^2}{f_\pi^2}\omega^2 \tag{40}$$

and demand that the coefficients η, ζ, and κ so defined be natural. The results of this analysis are given in Table IV and

[9]In FTS1 theory, it is the higher polynomial terms in ω and ϕ defining the mean fields that are responsible for the reduction in $\mathcal{R}$ needed in [15]. In Dirac-Brueckner-Hartree-Fock theory, it is found [34] that while $\mathcal{R}$ takes the free-space value $\mathcal{R}_0$ for $\rho \approx \rho_0$, it decreases to $\mathcal{R} \approx 0.64\mathcal{R}_0$ at $\rho \approx 3\rho_0$ due to rescattering terms, which in our language would correspond to the many-body correlations.

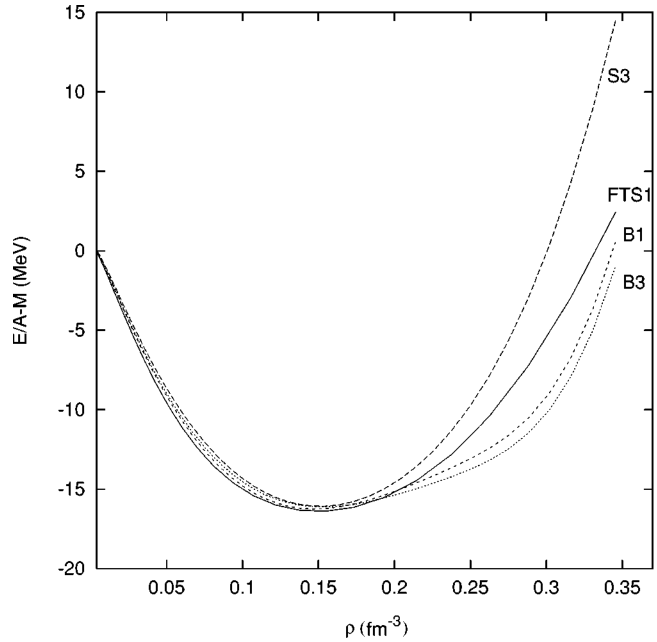

FIG. 4. $E/A-M$ vs ρ for FTS1 theory ($T1$ parameter). The $S3$, $B1$, and $B3$ models are defined in Table IV.

Fig. 4 for various values of the residual many-body terms and compared with those of the truncated model (39) with the $S3$ parameter set. The coefficients are chosen somewhat arbitrarily to bring our points home, with no attempt made for a systematic fit. (It would be easy to fine-tune the parameters to make the model as close as one wishes to FTS1 theory.) It should be noted that while the equilibrium density or Fermi momentum k_{eq}, the effective nucleon mass $m_L^\star$, and the binding energy B stay more or less unchanged, within the range of the parameters chosen, from what is given by the BR-scaled model (39) with the $S3$ parameters, the compression modulus K can be substantially decreased by the residual many-body terms. Figure 4 shows that, as expected, lowering of the compression modulus is accompanied by softening of the equation of state at $\rho > \rho_0$. While the equilibrium property other than the compression modulus is insensitive to the many-body correlation terms, the EOS at larger density can be quite sensitive to them. This is because, for the generic parameters chosen, the $m_L^\star$ can vanish at a given density above ρ_0 at which the approximation is expected to break down and hence the resulting result cannot be trusted. The $B2$ and $B4$ models do show this instability at $\rho \gtrsim 1.5\rho_0$. (See Fig. 5 in the Appendix.)

It is quite encouraging that the simple minimal model (39) with BR scaling captures so much of the physics of nuclear matter. Of course, by itself, there is no big deal in what is obtained by the truncated model: It is not a prediction. What is not trivial, however, is that once we have a Lagrangian of the form (39), which defines the mean fields, then we are able to control with some confidence the background around which we can fluctuate, which was the principal objective we set at the beginning of our paper. The power of the simple Lagrangian is that we can now treat fluctuations at *higher densities* as one encounters in heavy-ion collisions, not just at an equilibrium point. The description of such fluctuations does not suffer from the sensitivity with which the EOS de-

TABLE IV. Effect of many-body correlations on nuclear matter properties using the Lagrangian (39) plus (40). We have fixed the free-space masses $m_s = 700$ MeV, $m_\psi = 783$ MeV, and $M = 939$ MeV and set $\eta_1 = 0$ for simplicity. The equilibrium density k_{eq}, the compression modulus K, and the binding energy $B = M - E/A$ are all given in units of MeV.

Set	h	g_v	y	z	η_0	ζ_0	κ_3	κ_4	k_{eq}	$m_L^\star/M$	K	B
$S3$	5.30	15.2	0.28	0.31					258.2	0.675	259	16.1
$B1$	5.7	15.3	0.28	0.30			0.5	−4.9	256.0	0.666	209	16.2
$B2$	5.7	15.3	0.28	0.30	−0.055	0.18			257.3	0.661	201	16.1
$B3$	5.6	15.27	0.28	0.30			0.31	−4.1	259.1	0.659	185	16.1
$B4$	5.6	15.3	0.28	0.31			0.9	−8.1	256.4	0.669	191	16.1
$C1$	5.7	15.3	0.28	0.30	−0.05	0.155			256.3	0.665	218	16.2
$C2$	5.8	15.3	0.28	0.30	−0.11	0.35			256.1	0.662	161	16.2

pends at $\rho > \rho_0$ on the many-body correlation terms (35). Some of these issues are illustrated in the next subsection.

D. Some consequences

1. The ω in medium

Suppose one probes the propagation of an ω meson in nuclear medium as in HADES [35] or CEBAF [2] experiments, say, through dilepton production. The ω's will decay primarily outside of the nuclear medium, but let us suppose that experimental conditions are chosen so that the leptons from the ω decaying inside dense matter can be detected. See [36] for discussions on this issue. The question is whether the dileptons will probe the BR-scaled mass or the quantity given by Eq. (27). The behavior of the ω mass would differ drastically in the two scenarios. A straightforward application of FTS1 theory would suggest that at a density $\rho \lesssim \rho_0$, the ω mass as ''seen'' by the dileptons will increase slightly instead of decrease. Since in FTS1 theory the vector coupling g_v does not scale, this means that $g_v^\star/m_v^\star$ will effectively decrease. On the other hand, if the vector coupling constant drops together with the mass at increasing density as in the BR-scaling model,[10] the situation could be quite different, particularly if dileptons are produced at a density $\rho \sim 3\rho_0$ as in the CERES experiments [1]: The ω will then be expected to BR scale up to the phase transition.[11] Thus measuring the ω mass shift could be a key test of the BR-scaling idea as opposed to the FTS1-type interpretations. This interesting issue is planned to be studied in forthcoming experiments at GSI [35] and CEBAF [2].

2. Nuclear static properties

Given the link between BR-scaled chiral Lagrangians and Fermi-liquid fixed-point theory, one should be able to make a connection between the parameters that enter into such nuclear static properties as δg_l, referred to in the literature as the ''exchange-current'' contribution to the orbital gyromagnetic ratio, and the effective mass of the vector mesons ω and ρ. In this subsection we shall show that this is indeed possible. The results were already reported in [7], but we shall discuss them in the context developed in this paper. The key element that is intimately related to the Landau parameter F_1 is the universal scaling factor Φ, not the FTS1 effective mass discussed above that includes many-body correlations. To clarify this point, consider the Landau effective mass of the nucleon $m_L^\star$ given in terms of the Landau parameter F_1:

$$\frac{m_L^\star}{m_N} = 1 + \frac{F_1}{3} = \left(1 - \frac{\tilde{F}_1}{3}\right)^{-1}, \tag{41}$$

where $\tilde{F}_1 = (m_N/m_L^\star)F_1$. Including the pion contribution, we have a short-range term and a long-range term

$$\tilde{F}_1 = \tilde{F}_1^\omega + \tilde{F}_1^\pi, \tag{42}$$

where

$$\tilde{F}_1^\omega = \frac{m_N}{m_L^\star} F_1^\omega = -C_\omega^2 \frac{2k_F^3}{\pi^2 m_N^\sigma}, \tag{43}$$

$$\tilde{F}_1^\pi = -3\frac{m_N}{k_F}\frac{d}{dp}\Sigma_\pi(p)\Big|_{p=k_F}, \tag{44}$$

where the superscript denotes the relevant meson exchanged, Σ_π is the nucleon self-energy (Fock term) involving one-pion exchange (a nonlocal four-Fermi interaction), and

$$m_N^\sigma := m_N \Phi, \tag{45}$$

the BR-scaled nucleon mass in the absence of pions.[12] It follows from the quasiparticle velocity at the Fermi surface [7]

[10] It is interesting that the dropping ω mass is also found in a recent QCD sum-rule calculation based on current correlation functions by Klingl, Kaiser, and Weise [37] who, however, do not see the dropping of the ρ mass.

[11] It has been suggested recently [38] that at some high density, Lorentz symmetry can be spontaneously broken, giving rise to light ω mesons as ''almost Goldstone'' bosons. Such mesons could be a source of copious dileptons at some density higher than normal matter density.

[12] Note that m_N^σ corresponds to $M^\star$ in the toy model with BR scaling (39).

$$\frac{d}{dp}\epsilon(p)\Big|_{p=k_F}=\frac{k_F}{m_L^\star}=\frac{k_F}{m_N^\sigma}+\frac{d}{dp}\Sigma_\pi(p)\Big|_{p=k_F}, \qquad (46)$$

given by the BR-scaled Lagrangian together with Eqs. (41) and (42), that the ω contribution to the Landau parameter F_1 is governed only by the factor Φ:

$$\widetilde{F}_1^\omega=3(1-\Phi^{-1}). \qquad (47)$$

This is the key relation that links the nucleon scaling present in mean-field theories to the scaling of the vector mesons in a medium derived via chiral symmetry plus scale anomaly. It is also this relation that connects the behavior of hadrons in heavy-ion collisions to low-energy nuclear spectroscopic properties as we shall describe below. Understanding this relation would be crucial if one wanted to have a unified description based on an effective chiral Lagrangian.

The scaling factor $\Phi(\rho)$ is known from the QCD sum-rule calculation for the in-medium mass of the ρ meson at $\rho=\rho_0$ [39],

$$\Phi(\rho_0)=0.78\pm 0.08, \qquad (48)$$

which can also be extracted from an in-medium Gell-Mann-Oakes-Renner formula for the pion mass [20]. Since the contribution from the pion exchange is fixed by chiral symmetry for a given density, i.e., at $\rho=\rho_0$,

$$\frac{1}{3}\widetilde{F}_1^\pi=-\frac{3f_{\pi NN}^2 m_N}{8\pi^2 k_F}\left[\frac{m_\pi^2+2k_F^2}{2k_F^2}\ln\frac{m_\pi^2+4k_F^2}{m_\pi^2}-2\right]\approx -0.153, \qquad (49)$$

the Landau mass for the nucleon is entirely given once we assume the ω mass scales in the manner of Brown and Rho [6]:

$$\begin{aligned}\frac{m_L^\star}{m_N}&=\Phi\left(1+\frac{1}{3}F_1^\pi\right)\\&=\left(\Phi^{-1}-\frac{1}{3}\widetilde{F}_1^\pi\right)^{-1}\\&=(1/0.78+0.153)^{-1}=0.69(7),\end{aligned} \qquad (50)$$

which should be identified with the nucleon effective mass determined by QCD sum rule at $\rho=\rho_0$ [40],

$$\frac{m_N^\star}{m_N}=0.69^{+0.06}_{-0.14}. \qquad (51)$$

The effective mass for the nucleon found with the toy model with BR scaling (39) (with the set $S3$) denoted there as $m_L^\star$, $m_L^\star/m_N\approx 0.68$, is consistent with this QCD sum-rule value. This provides more support for our assertion.

The strongest support for this identification comes from the role that the Φ factor plays in δg_l, the exchange-current correction to the orbital gyromagnetic ratio of nuclei. The response to a slowly varying electromagnetic field of an odd nucleon with momentum $\vec{p}$ added to a closed Fermi sea can, in Landau theory, be represented by the current

$$\vec{J}=\frac{\vec{p}}{m_N}\left(\frac{1+\tau_3}{2}+\frac{1}{6}\frac{F_1'-F_1}{1+F_1/3}\tau_3\right), \qquad (52)$$

where m_N is the nucleon mass in medium-free space. The long-wavelength limit of the current is not unique. The physically relevant one corresponds to the limit $q\to 0$, $\omega\to 0$, with $q/\omega\to 0$, where (ω,q) is the four-momentum transfer. The current (52) defines the gyromagnetic ratio

$$g_l=\frac{1+\tau_3}{2}+\delta g_l, \qquad (53)$$

where

$$\delta g_l=\frac{1}{6}\frac{F_1'-F_1}{1+F_1/3}\tau_3=\frac{1}{6}(\widetilde{F}_1'-\widetilde{F}_1)\tau_3. \qquad (54)$$

This expression is recovered simply if one calculates the exchange of an ω and a ρ with BR-scaling masses. The result obtained recently in [7] is

$$\delta g_l=\frac{4}{9}\left[\Phi^{-1}-1-\frac{1}{2}\widetilde{F}_1^\pi\right]\tau_3. \qquad (55)$$

This result is highly nontrivial in that (i) the ω contribution restores the single-particle moment defined in terms of the free-space mass m_N, not of the BR-scaled mass,[13] as required by Ward identities and (ii) the correction occurs *only* in the isovector part. The numerical value for δg_l at nuclear matter density

$$\delta g_l=0.227\tau_3 \qquad (56)$$

agrees perfectly with the experimental value obtained from giant dipole resonances in heavy nuclei [41]

$$\delta g_l^p=0.23\pm 0.03. \qquad (57)$$

We should emphasize that the link between the Landau parameter that figures in the Fermi-liquid structure of nuclear matter and the BR scaling that figures in an effective chiral Lagrangian supplies a stringent consistency check of the theory. Another nontrivial consistency check is given in the strange-flavor sector, which will be described below although the results have been reported elsewhere.

3. Fluctuations in the strange-flavor direction

In considering kaonic fluctuations inside nuclear medium, the general argument developed above suggests that we are to take the $O(Q^2)$ SU(3) chiral Lagrangian with BR-scaled parameters and with bilinears in the baryon field taken in the

[13] This is reminiscent of the Kohn theorem for the cyclotron frequency of an electron in the metal in a magnetic field where the free-space mass of the electron, not the Landau mass, enters in the formula for the frequency.

mean field. In the kaon direction, this then gives (modulo the ''range term'' discussed below) for symmetric nuclear matter

$$\mathcal{L}_{KN}^{\rm eff}=\frac{-6i}{8f_\pi^{\star 2}}\overline{K}\partial_t K\langle B^\dagger B\rangle+\frac{\Sigma_{KN}}{f_\pi^{\star 2}}\overline{K}K\langle\overline{B}B\rangle\equiv\mathcal{L}_\omega+\mathcal{L}_\sigma, \tag{58}$$

where $K^T=(K^+K^0)$. The constant $f_\pi^\star$ in Eq. (58) can be identified as the pion decay constant scaling as the square-root of the quark condensate $\langle\overline{q}q\rangle$ [4,24]. The appearance of $f_\pi^\star$ indicates the BR scaling.[14] The potential felt by the kaon in the background of nuclear matter is then given by

$$V_{K^\pm}=\pm\frac{3}{8f_\pi^{\star 2}}\rho, \tag{59}$$

$$S_{K^\pm}=-\frac{\Sigma_{KN}}{2m_K f_\pi^{\star 2}}\rho_s, \tag{60}$$

where $\rho=\langle B^\dagger B\rangle$ and $\rho_s=\langle\overline{B}B\rangle$. At nuclear matter density $\rho=\rho_0$, we can identify these results as one-third of the corresponding potentials for nucleons, so we write

$$V_{K^\pm}\approx\pm\frac{1}{3}V_N \tag{61}$$

and

$$S_{K^\pm}\approx\frac{1}{3}S_N. \tag{62}$$

One way of understanding this result is that when written in terms of BR scaling, we are essentially getting a *quasiquark* description and the factor 1/3 represents that the kaon carries 1/3 of the number of chiral quarks lodged in the nucleon. We expect the quasiquark description to be good once the meson mass has decreased substantially with density as in the K^- case [6], but possibly not in the K^+ case where the mass does not move down with density. In the latter case the pseudo-Goldstone description should continue to be correct. (In particular, the range term is important for the K^+.)

Given Walecka-type mean fields for the nucleons, we can now calculate the corresponding mean-field potential for K^- nuclear interactions in symmetric nuclear matter. From the results obtained above we have

$$S_{K^-}+V_{K^-}\approx\frac{1}{3}(S_N-V_N). \tag{63}$$

Phenomenology in Walecka-type mean-field theory gives $S_N-V_N\lesssim-600$ MeV for $\rho=\rho_0$ [42]. This leads to the prediction that at nuclear matter density

$$S_{K^-}+V_{K^-}\lesssim-200\ \text{MeV}. \tag{64}$$

This seems to be consistent with the result of the analysis in K mesic atoms made by Friedman, Gal, and Batty [43], who fond attraction at $\rho\approx 0.97\rho_0$ of

$$S_{K^-}+V_{K^-}=-200\pm 20\ \text{MeV}. \tag{65}$$

As noted in [6], there is a correction called ''range term'' that appears at the same order of the chiral counting as the scalar potential (60) that is proportional to second derivative on the kaon field and hence $\sim\omega_K^2$ for an S-wave kaon where ω_K is the frequency of the kaon field. This correction can be approximately implemented by multiplying the scalar term in Eq. (60) by the factor $(1-0.37\omega_K^2/m_K^2)$. With this correction, we find for $\rho=\rho_0$ [6]

$$S_{K^-}+V_{K^-}\sim-192\ \text{MeV}. \tag{66}$$

For the K^-, the range correction is not numerically significant. However, the situation is different for K^+ nuclear interaction. In fact, including the range correction makes the K^+ effective mass *increase* with density in contrast to the K^-: We find the K^+ potential at nuclear matter density to be effectively repulsive by the amount

$$S_{K^+}+V_{K^+}\sim 25\ \text{MeV}\quad\text{at}\ \rho=\rho_0. \tag{67}$$

4. Going to higher densities in strange matter

The simple description given by the Lagrangian (58), corresponding to the tree order with the BR-scaled Lagrangian, seems to work fairly well up to $\rho\sim\rho_0$, but it must require corrections as density is further increased. This is already indicated in the construction of the BR-scaled chiral Lagrangian that reproduces FTS1 theory, i.e., the effective scaling of the coupling constant g_v needed in describing nuclear matter. More significantly, the Lagrangian (58), when naively extrapolated to $\rho\sim 3\rho_0$, would be inconsistent with what was observed in the KaoS kaon flow data [8].

The most efficient way to go higher in density is to bring in massive fields in Eq. (58). To do this, one can think of the first term of Eq. (58) as arising from an ω exchange (and a ρ exchange for nonsymmetric nuclear matter) and the second term as coming from a scalar ϕ exchange. This means that $1/f_\pi^{\star 2}$ in the first term is to be replaced, in the notation of the Lagrangian (39), by $2g_v^{\star 2}/m_v^{\star 2}$ and $\Sigma_{KN}/f_\pi^{\star 2}$ in the second term by $2m_K h^2/3m_s^{\star 2}$. (This also means that the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation for the vector meson mass cannot be naively applied in medium.) From the foregoing discussion, we expect that the first term will remain unscaled and the second term scaled[15]

[14] As noted in [6], there can be no nonderivative direct coupling between a Goldstone boson and a baryon like the second term of Eq. (58) in the chiral limit. Thus the direct coupling arises entirely through a chiral symmetry breaking, or quark masses, in QCD. Pions couple nonderivatively to baryons in the same way with the coefficient $\Sigma_{\pi N}/f_\pi^{\star 2}$. In [6], this relation is given an interpretation in terms of the S exchange, the identification exploited below.

[15] In order to compare with the analysis of [8], one should note that h is smaller (by about 1/2) than the scalar coupling in FTS1 theory. In addition, one should not forget the range term that tends to compensate the $1/\Phi^2$ scaling.

as Φ^{-2} as one increases the density up to the regime probed in the KaoS and FOPI experiments [8].

5. Kaon condensation in compact-star matter

The Lagrangian (58) or, more precisely, the vector-meson-implemented version of it was used in [6] to calculate the critical density for condensing K^-'s in dense neutron matter. For this, nuclear matter information provided by the FTS1 Lagrangian needs to be supplemented by isovector degrees of freedom to describe the neutron matter initially present in compact stars. This has not been yet worked out in terms of the FTS1 Lagrangian, although this could be effectuated by incorporating the isovector vector mesons ρ and a_1 into the FTS1 Lagrangian. Using the symmetry energy fitted at nuclear matter density of heavy nuclei and extrapolating it to densities greater than the normal matter density,[16] together with an estimate of the chemical potential for the electron, the Lagrangian (58) predicts [6,45]

$$\rho_c \lesssim 3\rho_0. \tag{68}$$

That the critical density is of the order of a few times normal matter density ensures that the Lagrangian (58) is an appropriate one since the same Lagrangian is checked in nuclear matter through the heavy-ion experiments FOPI [10] and KaoS [9] up to $\rho \sim 3\rho_0$. But are there any important corrections missed in this treatment?

To answer this question we should note that the mean-field prediction made above contains certain nonperturbative contributions that are not accessible in low-order chiral perturbation expansion. For instance, in [12], where the critical density is calculated to order Q^3 (or one-loop order) in chiral perturbation theory, one out of two constants that appear in the four-Fermi interaction terms in the Lagrangian was fixed to reproduce the Friedman-Gal-Batty attraction of 200 MeV in the kaonic atom data as does the Lagrangian (58). Thus it invokes an ingredient that is not directly extracted from the set of available on-shell data. Indeed, a recent calculation [46] to $O(Q^2)$ in chiral perturbation theory that is highly constrained by the ensemble of on-shell kaon-nucleon data and that includes both Pauli and short-range correlations for many-body effects is found to give at most about 120-MeV attraction at nuclear matter density. Thus the crucial input here is the strength of the K^- nuclear interaction in dense medium. If the analysis of the K mesic atom by Friedman *et al.* indicating the 200-MeV attraction turned out to be incorrect and the attraction came down to 100–120 MeV as found in [46], this would give a strong constraint on the constants that enter in four-Fermi interactions in the chiral Lagrangian. This would presumably account for the need for a dropping vector coupling $g_v^\star$ required for $\rho \gtrsim \rho_0$. This crucial information is also expected to come from ongoing heavy-ion experiments.

[16]A recent realistic calculation of the symmetry energy in the formalism of Dirac-Brueckner appraoch [44] confirms the extrapolation (to a density $\rho \sim 3\rho_0$) used in [6,12].

VI. SUMMARY AND CONCLUSIONS

In this paper an attempt is made to go from an effective (quantal) chiral Lagrangian to an effective-field theory for nuclear matter at variable densities, with the aim to build a bridge between (low-energy) nuclear spectroscopic properties under normal condition and (higher-energy) physics of dense matter under extreme conditions expected to be found in relativistic heavy-ion collisions and in compact stars such as neutron stars. A construction of this sort will be necessary to eventually understand the QCD phase transition(s) believed to take place at high temperature and/or high density. For this purpose, we take the FTS1 (the effective chiral Lagrangian model of Furnstahl *et al.*) [13], which is found to be highly successful in the phenomenology of finite nuclei and nuclear matter, to argue that an effective chiral Lagrangian constructed in high chiral orders corresponds, *in mean field*, to Lynn's chiral soliton [5] with chiral liquid structure. This provides an efficient background around which quantum fluctuations can be reliably calculated. We should perhaps stress that we are not implying that the FTS1 theory is the best one can construct as an effective theory of hadronic matter. We are simply taking it as one of the phenomenologically successful theories presently available that are constructed in a way consistent with chiral symmetry of QCD.

Next, using the renormalization-group-flow arguments developed in condensed-matter physics, we proceed to propose that the chiral liquid theory with the FTS1 Lagrangian (in the mean field) corresponds to Landau's Fermi-liquid fixed-point theory [19,30]. We develop the notion that the FTS1 theory in the mean field is at fixed points, except for the scalar sector, which develops a large anomalous dimension that we attribute to a strong-coupling situation. We then suggest that the strong-coupling theory with the parameters defined in matter-free space can be transformed into a weak-coupling theory if the chiral Lagrangian is rewritten in terms of BR-scaled parameters. We construct a simple model with BR-scaled masses that gives a fairly good description of ground-state properties with fits comparable to the full FTS1 theory. The simple BR-scaling Lagrangian provides the background at an arbitrary density around which fluctuations can be calculated with the tree diagrams yielding the dominant contributions. We have thus obtained a quasiparticle picture of a strongly correlated system at densities away from the equilibrium point.

The identification of the BR-scaling parameter Φ with the Landau-Migdal Fermi-liquid parameter F_1 leads to a set of relations that connect the physics that governs heavy-ion collisions, e.g., the CERES [1] dilepton data and the nucleon and kaon flow data of FOPI [10] and KaoS [9], etc., to such low-energy spectroscopic properties as effective (Landau) nucleon mass, effective g_A, and the exchange-current correction to the orbital gyromagnetic ratio, δg_l, etc. These relations are found to be satisfied to a surprising accuracy. Finally, the formalism allows a consistent calculation of kaon condensation in dense star matter, which is proposed to play an important role in supernovae explosion with the remnant forming ''nucleon'' or ''nuclear'' stars or going into small black holes [45,47].

While for an exploration our results are satisfying, there are several crucial links that remain conjectural in the work and require a lot more work. We have not yet established in a convincing way that a nontopological soliton coming from a high-order effective chiral Lagrangian accurately describes nuclear matter that we know of. The first obstacle here is that a realistic effective Lagrangian that contains sufficiently high-order loop corrections including nonanalytic terms has not yet been constructed. Lynn's argument for the existence of such a soliton solution and identification with a drop of nuclear matter is based on a highly truncated Lagrangian (ignoring nonanalytic terms). We are simply assuming that the FTS1 Lagrangian is a sufficiently realistic version (in terms of explicit vector and scalar degrees of freedom that are integrated out by Lynn) of Lynn's effective Lagrangian. To prove that this assumption is valid is an open problem. Our argument for interpreting the FTS1 with the anomalous dimension $d_{an}\approx 5/3$ for the quarkonium scalar field as a strong-coupling theory that can be reinterpreted in terms of a weak-coupling theory expressed with BR scaling is heuristic at best and needs to be sharpened, although our results strongly indicate that it is correct. Furthermore, transcribing the renormalization-group arguments developed in condensed-matter physics to dense hadronic matter, involving more degrees of freedom and more length scales, remains to be made rigorous. This is an issue that is of the same nature as transcribing Landau Fermi-liquid theory to nuclear matter as in the work of Migdal and also as going from the relativistic mean-field theory of Walecka type to Landau Fermi-liquid theory as in the work of Matsui and others.

There is also the practical question as to how far in density the predictive power of the BR-scaled effective Lagrangian can be pushed. In our simple numerical calculation, we used a parametrization for the scaling function $\Phi(\rho)$ of the simple geometric form, which can be valid, if at all, up to the normal matter density as seems to be supported by QCD sum-rule and dynamical model calculations. At higher densities, the form used has no reason to be accurate. By using the empirical information coming from nucleon and kaon flows, one could infer its structure up to, say, $\rho\sim 3\rho_0$ and if our argument for kaon condensation is correct, and hence kaon condensation takes place at $\rho\lesssim 3\rho_0$, then this will be good enough to make a prediction for the critical density for kaon condensation. In calculating compact-star properties in supernovae explosions, however, the EOS for densities considerably higher than the normal matter density, say, $\rho\gtrsim 5\rho_0$, is required. It is unlikely that this high density can be accessed within the presently employed approximations. Not only will the structure of the scaling function Φ be more complicated but also the correlation terms that are small perturbations at normal density may no longer be so at higher densities, as pointed out by Pandharipande *et al.* [48], who approach the effect of correlations from the high-density limit. In particular, the notion of the scaling function Φ will have to be modified in such a way that it will become a nonlinear function of the fields that figure in the process. This would alter the structure of the Lagrangian field theory. Furthermore, there may be a phase transition (such as spontaneously broken Lorentz symmetry, Georgi vector limit, chiral phase transition, or meson condensation) lurking nearby, in which case the present theory would have already broken down. These caveats will have to be carefully examined before one can extrapolate the notion of BR scaling to a high-density regime as required for a reliable calculation of the compact-star structure.

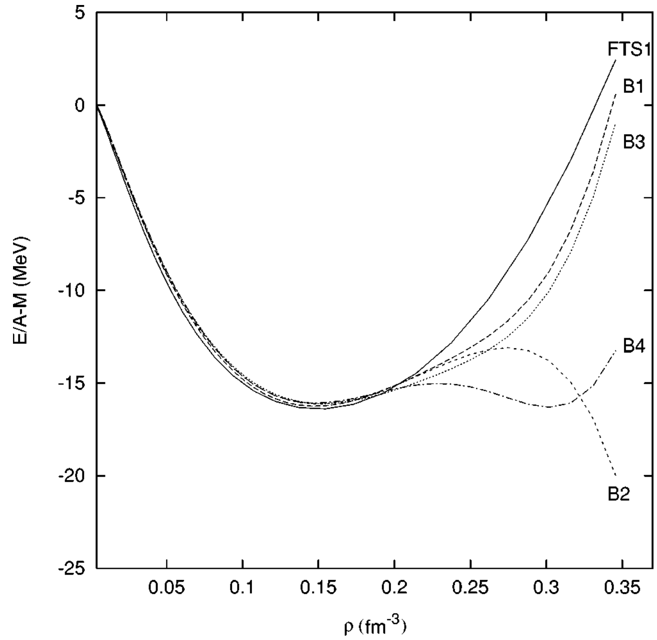

FIG. 5. $E/A-M$ vs ρ for the $B1$, $B2$, $B3$, and $B4$ models given in Table IV compared with FTS1 theory.

Finally, one could ask more theoretical questions as to in what way our effective Lagrangian approach is connected to QCD proper and if the theory is to be fully predictive, how one can proceed to calculate the corrections to the tree-diagram results we have obtained. The second issue is of course closely tied with what the appropriate expansion parameter is in the theory. These matters are addressed in the paper, but they are somewhat scattered all over the place and it might be helpful to summarize them here. The answers to these questions are not straightforward since there are two stages of ''decimation'' in the construction of our effective Lagrangian: The first is the elimination of high-energy degrees of freedom for the effective Lagrangian that gives rise to a soliton (i.e., chiral liquid) and here the relevant scale is the chiral symmetry-breaking scale ~ 1 GeV; the second is that given a chiral liquid, which we argued can be identified as the Fermi-liquid fixed point, the decimation involved here is for the excitations of scale Λ above (and below) the Fermi surface for which the expansion is made in $1/N$. As discussed in Sec. IV, $1/N\sim\Lambda/k_F$, where Λ is the cutoff in the Fermi system. In bringing in a ''BR-scaled'' chiral Lagrangian, we are relying on chiral symmetry considerations *applied* to a system with a density defined by nuclear matter. Thus the link to QCD proper of the effective theory we use for describing fluctuations around the nuclear-matter ground state must be tenuous at best. However, as recently reemphasized by Weinberg [49], low-energy effective theories need not be in a one-to-one correspondence with a ''fundamental theory'' meaning that one low-energy effective theory could arise through decimation from several different ''fundamental'' theories. This applies not only to theories with global symmetry but also to those with local gauge symmetry. In the present case, this aspect is more relevant since there is a change in degrees of freedom between the nonperturbative regime in which we are working and the perturbative regime in which QCD proper is operative.

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APPENDIX: EFFECT OF MANY-BODY CORRELATIONS ON THE EOS

In this appendix we briefly discuss the sensitivity of the EOS to the correlation parameters of Eq. (40) at a density $\rho > \rho_0$. This is shown in Fig. 5. While the parameter sets $B1$, $B2$, $B3$, and $B4$ give more or less the same equilibrium density and binding energy (see Table IV), the parameter set $B2$ has an instability and $B4$ a local minimum at ~ 2 times the normal matter density, whereas the sets $B1$ and $B3$ give a stable state at all densities, possibly up to meson condensations and/or chiral phase transitions. It is not clear what this means for describing fluctuations at a density above ρ_0, but it indicates that given data at ordinary nuclear matter density, it will not be feasible to extrapolate in a unique way to higher densities unless one has constraints from experimental data at the corresponding density. In our discussion, we relied on the data from KaoS and FOPI collaborations to avoid the fine-tuning of the parameters.

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