Structure of one neutron halo light nuclei ^{11}Be and ^{19}O

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(Received 28 February 1997; revised manuscript received 7 April 1997)

The low-lying excitation spectra of ^{11}Be and ^{19}O have been calculated on the basis of particle-vibration coupling model. Experimentally observed parity inversion of the ground state and first excited state of ^{11}Be has been reproduced. A fairly good agreement with experiment is obtained regarding excitation energies and spectroscopic factors for both nuclei. Calculated neutron rms radii confirm the observed neutron halo for these nuclei. [S0556-2813(97)05207-2]

PACS number(s): 21.60.Ev, 27.20.+n

I. INTRODUCTION

The study of the properties of halo nuclei drew a lot of attention from both theoreticians and experimentalists because of their novel characteristics. Of them, ^{11}Be has a very intriguing feature that the ground state parity is exactly opposite to what one expects from a shell model type of calculation. The separation energy of the last neutron of this one-neutron halo system is only 0.50 MeV. It is now well known that very low binding energies of the valence neutrons are responsible for the large matter radii of the halo nuclei. There have been many theoretical attempts [1–4] to address the problem of parity inversion of the ground state and the first excited state of ^{11}Be and evaluating the extension of its neutron halo. But in spite of having a good amount of experimental information [5–7], there is no consistent theory so far. The consistency here means the applicability of the underlying theory over a wide mass range. In [1,2] we find that the correlation between the valence nucleon and the ^{10}Be core has been taken into account phenomenologically by renormalizing the average one-body potential. Though it produces the halo nature of the nucleus, the physics of renormalization is not very much clear. By treating the single-particle energies as parameters in the shell model type of calculation [3,4,8] one gets the correct sequence of levels at the cost of transparency in the composition of the wave function. Mean field (HF) calculations using the Skyrme interaction [9,10] could not reproduce the ground state properties of ^{11}Be, showing the inadequacy of the underlying mean field. Through a very involved variational shell model calculation incorporating two-body correlation, Otsuka *et al.* [11] could bring down the $1/2^+$ state below the $1/2^-$ state. However, its numerical complicacy is a deterrant factor for its use as a general theory. In a sort of perturbative approach Vinh Mau [12] showed how the parity inversion can occur if one takes into account the coupling of the odd neutron motion with the first 2^+ vibrational state of the ^{10}Be core. Later, assuming a rotational model for the description of the 2^+, Nunes *et al.* [13] did coupled channel calculations to reproduce successfully the low-lying structure of ^{11}Be. But, there is no *a priori* reason for choosing the rotational model for the system. However, there is a striking similarity in all these calculations which is also a point of concern. One finds that the focus of all these calculations lies in the structure of ^{11}Be and related nuclei like ^{13}C. The scope of these approaches is too restrictive. It has been already observed [14] that the ground state density distribution of ^{11}Be is of more or less spherical shape which has been confirmed theoretically by Sagawa [15]. We show here that there exists a general method to study the structure of odd valence neutrons (holes) outside a vibrational closed core nuclei. The correlation between the valence neutron and the core is built up through an interaction similar in spirit to the work of Vinh Mau but quite different in structure. In the present work, we show that through this core-particle coupling model which is well tested in medium and heavy mass regions [16–18], the spectroscopic properties of the one-neutron halo nuclei ^{11}Be can be reproduced satisfactorily. In order to prove our conjecture we have calculated low energy structure of another one-neutron halo nucleus ^{19}O for which there are ample experimental results [19,20] but very few theoretical calculations. The only calculation existing to our knowledge is that of McGrory and Wildenthal [21] where they could reproduce only the strength distribution of $1d_{5/2}$ and $2s_{1/2}$ orbitals but were considerably away from producing the $1d_{3/2}$ strength distribution. In the present calculation inclusion of experimental quadrupole phonon excitations was sufficient to reproduce the observed fragmentation of the spectroscopic strengths of all the shell model states quite satisfactorily. The strength of our calculation lies in the simplicity of the model.

II. MODEL

The total Hamiltonian for a coupled system of particles and (holes) and collective excitation of the core may be written as

$$H=H_{\rm core}+H_{\rm s.p.}+H_{\rm int}, \tag{1}$$

where $H_{\rm core}$ describes the harmonic collective vibrations of the even-even core. The experimental excitation spectrum of ^{10}Be shows the behavior of a vibrator. The first 2^+ excited state occurring at 3.368 MeV can be identified as a one-quadrupole-phonon excited state and the second 0^+ and 2^+ states occurring approximately at double the energy of first 2^+ can be taken as the members of two-quadrupole-phonon excitations. The case for ^{19}O is similar. $H_{\rm s.p.}$ describes the motion of the valence nucleon (hole) in an effective potential. Considering the nucleus to be formed of incompressible and irrotational fluid, $H_{\rm core}$ is given by

TABLE I. Parameters of the Skyrme-Hartree-Fock calculation.

T_0	T_1	T_2	T_3	T_4	x_0	x_1	x_2	x_3	α
-1950.0	362.252	-104.27	11861.4	123.69	1.1717	0.0	0.0	1.7620	0.25

$$H_{\mathrm{core}}=\frac{1}{2}\sum_{\lambda\mu} B_\lambda|\dot{\alpha}_{\lambda\mu}|^2+C_\lambda|\alpha_{\lambda\mu}|^2, \qquad (2)$$

where $\alpha_{\lambda\mu}$ are the collective coordinates. The basis states which are eigenfunctions of $H_{\mathrm{core}}+H_{\mathrm{s.p.}}$ can be written as $\{[N_2V_2L_2,N_3V_3L_3]^L(nl1/2)^j\}^{JM}$, where $(N_\lambda V_\lambda L_\lambda)$ is the totally symmetric state of N_λ phonons of multipolarity λ coupled to angular momentum L_λ and V_λ represents all the additional quantum numbers necessary for complete specifications of the state; L_2 and L_3 are coupled to the angular momentum L of the core which is then coupled to j of the particle or hole to form the resultant J. $(nl1/2)$ represents a particle (hole) state having radial quantum number n, orbital qauntum number l, and spin 1/2. The core-particle interaction is given by

$$H_{\mathrm{int}}=\sum_{\lambda}^{2,3} k_\lambda(r)\sum_{\mu=-\lambda}^{\lambda} \alpha^*_{\lambda\mu}Y_{\lambda\mu}(\theta_p,\phi_p). \qquad (3)$$

$Y_{\lambda\mu}$ are the spherical harmonics corresponding to the particle coordinates. The collective coordinates $\alpha_{\lambda\mu}$ can be expressed as a linear combination of phonon creation and destruction operators $b^\dagger$, and b, respectively, for the core. After the inclusion of the pairing effects, the matrix element of the interaction Hamiltonian is given by

$$\begin{aligned}&\langle\{[N_2V_2L_2,N_3V_3L_3]^L(nl1/2)^j\}^{JM}|H_{\mathrm{int}}|\{[N_2'V_2'L_2',N_3'V_3'L_3']^{L'}(n'l'1/2)^{j'}\}^{JM}\rangle\\
&=\sum_{\lambda}^{2,3} X_\lambda\langle nl\left|\left|\left[\left(\frac{m\omega_0}{\hbar}\right)^{1/2} r_p\right]^\lambda\right|\right|n'l'\rangle(-1)^{J-1/2+\lambda}\frac{1}{2}\{[L][L'][j][j']\}^{1/2}\left(\frac{1+(-1)^{l+l'+\lambda}}{2}\right)(\delta_{\lambda,2}+\delta_{\lambda,3}(-1)^{L+L'+L_3+L_3'})\\
&\times\begin{pmatrix} j' & \lambda & j\\ 1/2 & 0 & -1/2\end{pmatrix}\begin{Bmatrix} L & L' & \lambda\\ L_\lambda' & L_\lambda & L_\eta\end{Bmatrix}(U_j'U_j-V_j'V_j)\,\delta_{N_\eta N_{\eta'}}\delta_{L_\eta L_{\eta'}}\delta_{V_\eta V_{\eta'}}[\delta_{N_\lambda N_{\lambda-1}'}(-1)^{L_2+L_3}\langle N_\lambda L_\lambda||b_\lambda^\dagger||N_{\lambda'}L_{\lambda'}\rangle\\
&+\delta_{N_\lambda' N_{\lambda+1}}(-1)^{L_2'+L_3'}\langle N_\lambda' L_\lambda'||b_\lambda^\dagger||N_\lambda L_\lambda\rangle].\end{aligned} \qquad (4)$$

X_2 and X_3 are the quadrupole and octupole interaction strengths, respectively, and $\eta=2$ when $\lambda=3$ and vice versa. X_λ's are given by

$$X_\lambda=K_\lambda\sqrt{2(\lambda+1)}\sqrt{\frac{\hbar\omega_\lambda}{2\pi C_\lambda}}. \qquad (5)$$

V_j's and U_j's are the occupation and nonoccupation probabilities of the respective single-particle states.

III. METHOD OF CALCULATION

The inputs to the model are (i) the low-lying vibrational energies of the core, ^{10}Be and ^{18}O, (ii) the $U_j(V_j)$ factors, (iii) quasiparticle energies ϵ_j, and (iv) the interaction coupling strengths X_2 and X_3. The vibrational energies for the cores ^{10}Be and ^{18}O are taken from the experimental data [22,24]. The calculations have been restricted to only two-quadrupole-phonon space as the calculated spectra have been found to be insensitive to the inclusion of octupole phonons. The $U_j(V_j)$ factors in the case of ^{11}Be are taken from the (d,p) reaction study [23]. As the available experimental neutron stripping data do not show up the full single-particle strength for ^{19}O [19], the $U_j(V_j)$ factors used in the present calculation are obtained from Skyrme-Hartree-Fock theory with Z_σ parametrization. The parameters are given in Table I. The relative quasiparticle energies ϵ_j with respect to the lowest single-particle state in the conventional shell model basis are treated as free parameters. The interaction coupling strengths X_2 and X_3 are the further parameters in the calculations. As mentioned earlier, the octupole phonon state of the core is not considered and thereby $X_3=0$. Therefore, effectively, (X_2,ϵ_j)'s are the only free parameters in the present calculations. These are varied to fit the experimental level scheme and the spectroscopic factors. The sets of best fit parameters used in the present calculations for both the systems are given in Table II.

The wave function of a state with angular momentum J with Z component M and energy E^α (α distinguishes between states of same spin and parity) is given by

$$|E^\alpha,JM\rangle=\sum_{N_2,L_2,j} C_\alpha(N_2L_2,j,J)|N_2L_2,j;JM\rangle, \qquad (6)$$

and the spectroscopic factor is defined as

$$S^\alpha(l,j=J)=U_j^2|C_\alpha(00,j,J)|^2. \qquad (7)$$

TABLE II. Parameters of the calculation.

Nucleus	U_j					ϵ_j (MeV)					X_2
	$1p_{1/2}$	$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$	$1f_{7/2}$	$1p_{1/2}$	$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$	$1f_{7/2}$	
^{11}Be	0.877	0.707	0.979			0.0	2.10	0.20			2.5
^{19}O		0.816	1.000	0.774	0.565		0.0	1.50	4.00	5.60	0.8

IV. RESULTS AND DISCUSSION

A. ^{11}Be

The experimental level scheme [22] and spectroscopic factors based on (d,p) reaction data [23] are shown in Fig. 1. The single-particle space for the odd neutron was taken to be comprised of $1p_{1/2}$, $2s_{1/2}$, and $1d_{5/2}$ orbitals. The collective states of ^{10}Be, used in the present calculation, to which the odd neutron couples were taken to be the first 2^+ state at 3.368 MeV, second 2^+ state at 5.958 MeV, and 0^+ state at 6.179 MeV. The energy of the two qudrupole phonon 4^+, not available in the experimental data, was taken to be twice the energy of the first 2^+ state of ^{10}Be [22]. The only free parameters in this calculation are $\epsilon_{2s_{1/2}}$, $\epsilon_{1d_{5/2}}$, and X_2. The calculated energies and spectroscopic factors of various states are given in Fig. 1 for comparison. It is seen from the figure that the present calculation is able to reproduce the level spectrum except for the $\frac{1}{2}{}^+_2$ state. The calculated $\frac{1}{2}{}^+_2$ is pushed up at around 5 MeV (not shown in the figure). There is a very good agreement between the calculated and the experimental level energies and spectroscopic factors. It was found that a strong correlation between the vibrational 2^+ state and $2s_{1/2}$ single-neutron single-particle state is responsible for pushing down the $\frac{1}{2}^+$ state below the $\frac{1}{2}^-$ state. The calculated total $1p_{1/2}$ single-particle strength is found to be concentrated on $\frac{1}{2}^-$ state as observed in the experimental results. The calculated level energies of $\frac{5}{2}^+$ states are very well reproduced. The observed $1d_{5/2}$ single-particle strength is found to be concentrated on the $\frac{5}{2}{}_1$ state. However, in the present calculation, the available $1d_{5/2}$ single-particle strength is found to be fragmented between $\frac{5}{2}{}_1$ and $\frac{5}{2}{}_2$ states, the major part of the strength going to the $\frac{5}{2}{}_1$ state.

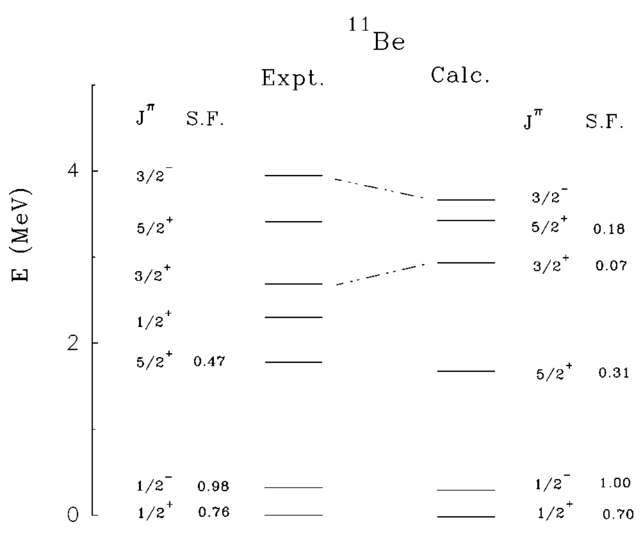

FIG. 1. Calculated and experimental level schemes of ^{11}Be. The excitation energies, spin parities, and spectroscopic factors of the levels are shown.

B. ^{19}O

The experimental level scheme [24] and spectroscopic factors based on neutron stripping data [19] are shown in Fig. 2. For generation of even- and odd-parity states of this nucleus we have taken the first 2^+ state at 1.982 MeV, second 2^+ state at 3.92 MeV, second 0^+ state at 3.633 MeV, and the 4^+ state at 3.554 MeV of ^{18}O. The single-particle space considered in the present calculation is spanned by $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, and $1f_{7/2}$ states. The quasiparticle energies $\epsilon_{2s_{1/2}}$, $\epsilon_{1d_{3/2}}$, and $\epsilon_{1f_{7/2}}$ and the quadrupole interaction coupling strength X_2 were treated as free parameters. The calculated spectrum of low-lying levels of ^{19}O is shown in Fig. 2 along with the experimental one. It is seen from Fig. 2 that there is an overall good agreement between the calculated and experimental spectra. The spectroscopic strengths of the $5/2^+$ states are very well reproduced. However, the energy of the third $5/2^+$ state is overpredicted by 500 keV. The case for the $7/2^-$ states is similar, where the spectroscopic factors agree fairly well with the experiment but the energies of the higher-lying states are underpredicted by 300–500 keV. In the present calculation it is observed that the $2s_{1/2}$ single-particle strength is distributed over many levels, with the first $1/2^+$ level having the largest value of 70%. However, experimentally the total $2s_{1/2}$ single-particle strength is observed to go to the first $\frac{1}{2}^+$ state. For the $3/2^+$ states the calculation is well able to reproduce the $1d_{3/2}$ single-particle strength distribution. However, the calculation was unable to reproduce the 100 keV first $3/2^+$ state. This state may have the structure of three-particle states with

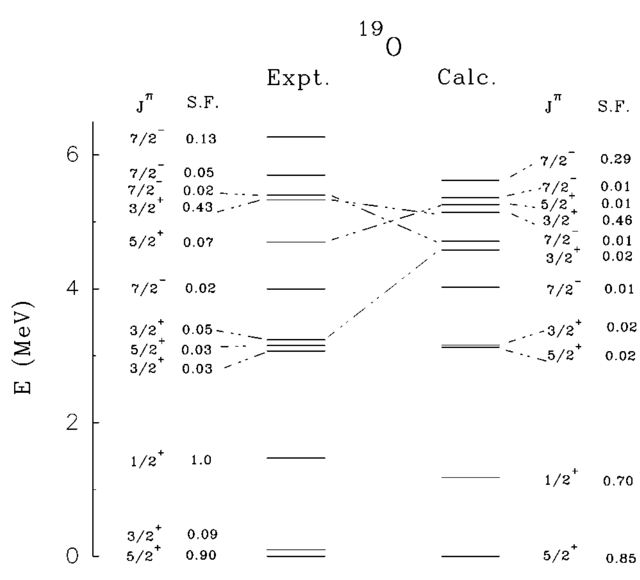

FIG. 2. Calculated and experimental level schemes of ^{19}O. The excitation energies, spin parities, and spectroscopic factors of the levels are shown.

^{16}O as the core. Last, the experimentally observed $3/2^+$ state at 3.24 MeV may correspond to the calculated $3/2^+$ state at 4.58 MeV.

V. ROOT-MEAN-SQUARE NEUTRON RADIUS

The neutron rms radii of the ground states of ^{11}Be and ^{19}O were calculated by taking the expectation value of the r_n^2 operator in the states $|E^\alpha=0,J,M=J\rangle$. The expectation value has two components, one coming from the core part (^{10}Be and ^{18}O) and the other coming from the extra particle part. The core contribution to the mean square radius is estimated as $R^2_{\rm core}=(r_0 A^{1/3})^2$. The value of r_0 is taken to be 1.2 fm. The contribution from the extra particle part is evaluated from the ground state wave function of the systems calculated earlier. For ^{11}Be the calculated value of neutron rms radius is 3.67 fm, which slightly overestimates the experimental value 3.38 ± 0.06 fm [25]. In the case of ^{19}O the present calculation predicts the neutron radius to be 3.87 fm. However, there is no available experimental result to the best of our knowledge for the neutron radius of ^{19}O to compare with.

VI. SUMMARY AND CONCLUSION

The level structures of the halo nuclei ^{11}Be and ^{19}O have been studied in the simple approach of an odd particle coupled to the neighboring even-even vibrating core. The core excitation energies, which take into account all the correlations arising out of the many particle core system, were taken from the experiment. The coupling of this correlated core with the odd valence particle is responsible for the low-lying structure of the halo nuclei. Relative single-particle energies and the quadrupole interaction strength were treated as parameters. The calculated energies and the spectroscopic factors agree quite well with the experimental results. The ground state wave functions of ^{11}Be and ^{19}O, thus calculated, have been used to derive the ground state rms neutron radii. The calculated values of rms radii slightly overestimate the experimental values. In conclusion, it may be inferred that a simple picture of a particle-vibration coupling model, without going into sophisticated calculations, can very well reproduce the experimental observables of the light odd-particle halo nuclei.

ACKNOWLEDGMENTS

The authors acknowledge useful discussions with S. Bhattacharya. One of the authors (R.B.) thank Bikash Sinha, Director, VECC, for extending hospitality and J. N. De for his encouragement during the work.

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