

Cross section and analyzing power of $\vec{p}p \rightarrow pn\pi^+$ near threshold

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The cross section and analyzing power of the $\vec{p}p \rightarrow pn\pi^+$ reaction near threshold are estimated in terms of data obtained from the $\vec{p}p \rightarrow d\pi^+$ and $pp \rightarrow pp\pi^0$ reactions. A simple final state interaction theory is developed which depends weakly upon the form of the pion-production operator and includes some Coulomb corrections. Within the uncertainties of the model and the input data, the approach reproduces well the measured energy dependence of the total cross section and the proton analyzing power at a fixed pion center-of-mass angle of 90° , from threshold to $T_p = 330$ MeV. The variation of the differential cross section with pion angle is also very encouraging. [S0556-2813(97)03410-9]

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I. INTRODUCTION

The $pp \rightarrow d\pi^+$ reaction has been measured for many years [1], either for its own intrinsic physics interest or as a calibration for other reactions. Due to the importance of establishing the precise beam energy, accurate experiments near threshold could be carried out much easier with the advent of cooled proton beams [2,3]. The region within a few MeV of threshold is very interesting because it shows the interplay of the dominant p -wave production, induced by the Δ resonance, and the small s -wave rescattering terms, which becomes even more dramatic when going to heavier nuclei [4].

Since the phase space for producing a three-body final state varies like Q^2 , where Q is the center-of-mass (c.m.) kinetic energy in the exit channel, the determination of the beam energy is even more crucial in such experiments near threshold. The accurate measurements of the $pp \rightarrow pp\pi^0$ total cross section at the Indiana [5] and CELSIUS storage rings [6] down to $Q \approx 1$ MeV provoked great theoretical interest because the comparatively large cross section revealed the influence of unexpected short-range physics [7].

The total and differential cross sections for charged pion production in the allied $pp \rightarrow pn\pi^+$ reaction have also been measured very close to threshold [8,9] and these results have recently been complemented by the determination of the proton analyzing power [10]. At the same value of Q , charged pion production is typically a factor of 5 larger than neutral and the interesting practical question to ask is to what extent the extra information will constrain theory further.

The general features of the first results on the $pp \rightarrow pn\pi^+$ total cross section near threshold [8] can in fact be understood within a nucleon-nucleon final state interaction model using measurements of the $pp \rightarrow d\pi^+$ and $pp \rightarrow pp\pi^0$ reac-

tions as input [11]. This approach uses a representation of the np spin-triplet scattering wave function in terms of the deuteron bound-state wave function which does not depend upon the details of the np potential [12,13]. This result, which is exact when the np scattering energy is extrapolated to $-\epsilon_d$, where ϵ_d is the deuteron binding energy, is generally a very good approximation for small energies and distances [14]. Assuming the pion production operator to be of short range, the formalism allows one to estimate the cross section for the $pp \rightarrow pn\pi^+$ reaction where the final np pair are in the spin-triplet S wave. The much smaller contribution from spin-singlet final states may be deduced directly from the $pp \rightarrow pp\pi^0$ data using isospin invariance.

The agreement with the measured total cross sections is generally very encouraging except for underestimating the point nearest threshold [11], which is the hardest to measure experimentally. In the definitive data set of Ref. [9], the value at this point is somewhat reduced and a more careful evaluation of Coulomb effects given in this present work increases the near-threshold prediction. These two changes both reduce the discrepancy.

Away from threshold our final state interaction approach is capable of describing the angular distribution of the TRIUMF $pp \rightarrow pn\pi^+$ data at 400 and 450 MeV [15] provided the excitation energy in the np final state is below about 10 MeV, where S waves can be expected to dominate [16]. Differential cross sections in the pion c.m. angle are now also available near threshold [9], and here we extend our formalism to describe such data. One difficulty in implementing this program is the restricted differential cross section information for the $pp \rightarrow pp\pi^0$ reaction corresponding to the total cross sections reported in Ref. [5]. If this is taken to be isotropic, we predict the $pp \rightarrow pn\pi^+$ angular distribution to be a little steeper than that observed experimentally [9], though it should be noted that there are also major uncertainties in the $pp \rightarrow d\pi^+$ input [1-3].

The proton analyzing power in low energy $\vec{p}p \rightarrow d\pi^+$ is largest at 90° and the energy dependence of this quantity for

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the three-body $pp \rightarrow pn\pi^+$ final state has just been reported by an IUCF group [10]. The extension of our FSI method to treat analyzing powers is straightforward, but its application is hindered by the dearth of A_y data in the $\vec{p}p \rightarrow pp\pi^0$ spin-singlet final state. If this is small then the broad features of the 90° energy dependence are reproduced.

Calculations within our final state interaction model are not definitive in that they do not use all of the energy and r dependence of the S -state neutron-proton scattering wave functions. This means however that they do not require detailed considerations of the range dependence of the pion production operator and are largely independent of the choice of the neutron-proton potential. Higher partial waves in the neutron-proton final state are simply ignored and this will become a bigger defect with increasing energy. Nevertheless we claim that any microscopic calculation which reproduces the measured $pp \rightarrow d\pi^+$ and $pp \rightarrow pp\pi^0$ observables should, near threshold, give results qualitatively similar to the ones presented here. The tolerable agreement with the $pp \rightarrow pn\pi^+$ data therefore suggests that the extraction of quantitatively new information on the basic pion production reaction mechanism near threshold requires accurate measurements combined with much more refined theoretical calculations.

In Sec. II we discuss the relation of the NN scattering and bound-state wave functions in the spin-triplet S state and illustrate it with results derived from the Paris potential [14], which are typical and readily obtainable. The energy dependence of the spin-singlet wave functions is investigated with and without Coulomb effects and it is shown that the major difference between the two can be accounted for by merely shifting the position of the 1S_0 pole on the second energy sheet.

Though at low energies it is reasonable to consider only S -wave nucleon-nucleon systems, pionic p waves enter quickly because of the smallness of the s -wave terms. In Sec. III we develop the formalism to include the effects of pionic s and p waves, together with the approximate wave functions, in the three-body phase space to estimate the measured $pp \rightarrow pn\pi^+$ observables. Coulomb corrections are included in the s wave in the same approximate manner as that used to treat the low energy $pp \rightarrow d\pi^+$ reaction.

As previously stressed, the agreement with the experimental data presented in Sec. IV is generally good, though not perfect. However the differences are well within the uncertainties of the model and the input and output experiments. Conclusions and suggestions for further work are to be found in Sec. V.

II. SCATTERING AND BOUND-STATE WAVE FUNCTIONS

An S -wave bound-state reduced wave function behaves asymptotically as

$$u_\alpha(r) \sim e^{-\alpha r} \quad (2.1)$$

at large radii r and is normalized such that

$$\int_0^\infty dr [u_\alpha(r)]^2 = 1. \quad (2.2)$$

The analogous scattering wave function with *real* boundary conditions is normalized by its asymptotic behavior

$$v(k, r) \sim \frac{1}{k} \sin[kr + \delta(k)], \quad (2.3)$$

where $\delta(k)$ is the phase shift at wave number k .

Despite these very different normalizations, we have shown that for finite range potentials the two types of wave functions are related quantitatively by the theorem [13]

$$\lim_{k \rightarrow i\alpha} \{ \sqrt{2\alpha(\alpha^2 + k^2)} v(k, r) \} = -u_\alpha(r). \quad (2.4)$$

This theorem is valid at the bound-state pole, independent of the details of the potential, but it is also a robust extrapolation in the sense that, in the case of a weakly bound state, at short distances r

$$v(k, r) \approx - \frac{1}{\sqrt{2\alpha(\alpha^2 + k^2)}} u_\alpha(r), \quad (2.5)$$

provided that k is small on the scale of the potential range and strength. Of course the approximation breaks down at large distances because the bound and scattering wave functions must be orthogonal when integrated over all space.

In the case of an almost bound virtual state, where the pole is on the second sheet, there is no bound-state wave function to fix the radial dependence. Nevertheless the same techniques show that the energy dependence of the scattering wave function is given by a similar factor to that of Eq. (2.5) [11,13]:

$$v(k, r) \approx - \frac{1}{\sqrt{\alpha^2 + k^2}} C(r). \quad (2.6)$$

These approximations can be tested explicitly in the cases of the Yamaguchi, square well, Bargmann, or Hulthén potentials, which can be resolved exactly. However several complications arise when looking at the more realistic potentials which are used to describe the nucleon-nucleon system. The extrapolation theorem of Eq. (2.5) has only been proved for single channel scattering in a local potential [13], whereas in the spin-triplet case there is coupling between the S and D waves.

In Ref. [11] we showed the variation of the deuteron and scattering wave functions with r for the illustrative case of the Paris nucleon-nucleon potential [14] and, despite the neglect of coupled channel effects, Eq. (2.6) seems to be broadly valid to within 5% for $r \leq 1.7$ fm for neutron-proton c.m. energies $E_{np} \leq 10$ MeV. To see this more quantitatively, we plot in Fig. 1 values of the spin-triplet function

$$Z_t = - \sqrt{2\alpha_t(k^2 + \alpha_t^2)} v(k, r)/r, \quad (2.7)$$

with $\alpha_t = 0.232 \text{ fm}^{-1}$, in terms of k^2 . The radius is fixed at $r = 1.05$ fm, which is close to the peak in the Paris wave function.

When continued to $k^2 = -\alpha_t^2$, the smooth fit

$$Z_t(k^2) = \sqrt{2\alpha_t}(0.627 + 0.122k^2 - 0.057k^4) \quad (2.8)$$

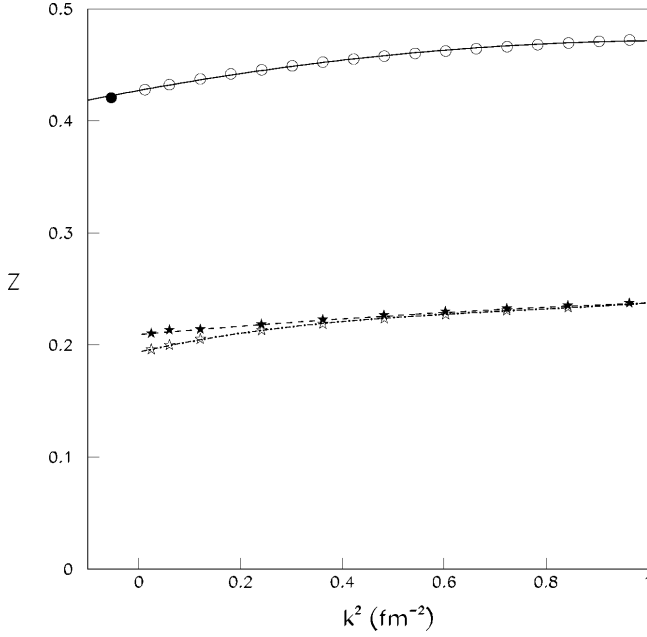


FIG. 1. The NN triplet scattering function Z_t defined by Eq. (2.7) evaluated at $r=1.05$ fm for discrete values of k^2 using the Paris potential (open circles). The smooth extrapolation of Eq. (2.8) lies about 0.5% above the deuteron point (closed circle). The points for the corresponding singlet functions Z_s , defined by Eq. (2.9), are shown without Coulomb force (open star) and with (closed star) and interpolated using Eq. (2.10). For presentational purposes the singlet values are multiplied by a scale factor of 0.3. The principal effect of the Coulomb force is to change the position of the anti-bound state pole at $k^2 = -\alpha_s^2$. Residual effects due to the Coulomb repulsion may be seen below $E_{pp}=1$ MeV.

lies about 0.5% above the value of the deuteron wave function, but such a discrepancy is close to the limit in the precision of the extrapolation. The deviation could of course be due to effects of the D state, though the Paris potential also includes some velocity dependence which may lie outside the domain of validity of the theorem.

The slow variation of $Z_t(k^2)$ with k^2 is not a peculiarity of the Paris potential but seems to be found for all simple potentials of a similar range which reproduce the deuteron binding energy. While almost all the energy dependence is given by the square-root factor in Eq. (2.6), $Z_t(k^2)$ is a steadily increasing function and this is typical for potentials with just one lightly bound state. Nevertheless, even at $E_{np}=15$ MeV it lies less than 8% above the deuteron point. It should be noted that the slope of this function depends upon the value of r and that at $r \approx 1.7$ fm the function is almost flat [14].

Though there are no channel-coupling problems in the spin-singlet case, the approximation given by Eq. (2.6) must break down at very small k^2 in the proton-proton case due to the essential singularity at $k=0$ caused by the Coulomb repulsion. This is not a problem provided $E_{NN} > 1$ MeV, as can be seen from the values of the singlet function

$$Z_s = -\sqrt{k^2 + \alpha_s^2} v(k, r) / r \quad (2.9)$$

which are also shown in Fig. 1. The results may be parametrized by

$$Z_s = 0.646 + 0.343k^2 - 0.353k^4 + 0.157k^6$$

with $\alpha_s = -0.053$ fm $^{-1}$ (without Coulomb),

$$Z_s = 0.698 + 0.131k^2 - 0.037k^4$$

with $\alpha_s = -0.100$ fm $^{-1}$ (with Coulomb). (2.10)

III. AMPLITUDES AND OBSERVABLES

Many of the elements of the FSI formalism are given in Ref. [11], but for clarity some are repeated here. We use a normalization such that the two-body spin-averaged amplitude and c.m. differential cross section are related by

$$\frac{d\sigma}{d\Omega}(pp \rightarrow d\pi^+) = \frac{p_\pi}{64\pi^2(m_d + m_\pi)^2 p_p} \times |M(pp \rightarrow d\pi^+)|^2, \quad (3.1)$$

where m_p , m_n , m_d , and m_π are the masses of the proton, neutron, deuteron, and pion, and p_π and p_p are the momenta of the final pion and the initial proton, respectively.

The corresponding three-body cross section is given by

$$\frac{d^2\sigma}{d\Omega dk}(pp \rightarrow pn\pi^+) = \frac{p_\pi k^2}{32\pi^3(m_p + m_n + m_\pi)^2 p_p} \times |M(pp \rightarrow (pn)_k \pi^+)|^2, \quad (3.2)$$

where the matrix element squared is assumed to be averaged over the angles of the np relative momentum \vec{k} . In practice we shall only estimate the contributions from S -wave np pairs, where the results are in any case isotropic in \vec{k} .

Reduced mass factors for the three-body reaction are defined by

$$\frac{1}{\mu} = \frac{1}{m_\pi} + \frac{1}{m_p + m_n}, \quad \frac{1}{m_{pn}} = \frac{1}{m_p} + \frac{1}{m_n}, \quad (3.3)$$

whereas for bound-state production the $m_p + m_n$ combination in Eq. (3.3) is replaced by the deuteron mass.

The momenta in the exit channel are linked to the value of the excitation energy Q through

$$Q = \frac{p_\pi^2}{2\mu} + \frac{k^2}{2m_{pn}}. \quad (3.4)$$

The variable η , conventionally used to describe low energy pion production, is the maximum pion momentum in pion mass units and is given by

$$\eta \equiv \frac{p_\pi^{\max}}{m_\pi} = \frac{\sqrt{2\mu Q}}{m_\pi}. \quad (3.5)$$

The cross section for pion production summed over all pn excitation energies in the three-body channel is

$$\begin{aligned} \frac{d\sigma}{d\Omega}(pp \rightarrow pn\pi^+) &= \frac{1}{32\pi^3(m_p+m_n+m_\pi)^2 p_p} \\ &\times \int_0^{\sqrt{2m_{pn}}} Q |M(pp \rightarrow (pn)_k\pi^+)|^2 p_\pi k^2 dk. \end{aligned} \quad (3.6)$$

Since the pion production operator is expected to be dominated by short-range physics, the relation of Eq. (2.4) between the bound and scattering wave functions suggests a similar relation for the spin-triplet matrix elements, viz

$$\begin{aligned} |M(pp \rightarrow (pn)_i\pi^+)|^2 &\approx \frac{|M(pp \rightarrow d\pi^+)|^2}{2\alpha_i(k^2 + \alpha_i^2)} \\ &\approx \frac{32\pi^2(m_d+m_\pi)^2 p_p}{p_\pi \alpha_i(k^2 + \alpha_i^2)} \\ &\times \frac{d\sigma}{d\Omega}(pp \rightarrow d\pi^+). \end{aligned} \quad (3.7)$$

The prediction for the spin-triplet part of the unpolarized cross section then follows immediately through the use of Eq. (3.6).

The formalism for the proton analyzing power A_y in $pp \rightarrow pn\pi^+$ can be developed identically, the only changes being A_y factors on the right-hand side of Eq. (3.7) and the left-hand side of Eq. (3.6).

It is of great practical interest that near threshold the integration in Eq. (3.6) may be performed analytically. Thus for a $pp \rightarrow d\pi^+$ observable which varies as

$$\mathcal{O}_d^{(n)}(\eta) \propto \eta^n, \quad (3.8)$$

the corresponding observable for the $pp \rightarrow (pn)_i\pi^+$ reaction is given by

$$\mathcal{O}_{pn}^{(n)}(\eta) = P^{(n)}(\eta\zeta) \times \mathcal{O}_d^{(n)}(\eta), \quad (3.9)$$

where

$$\zeta = \frac{m_\pi}{\sqrt{2\mu\epsilon_d}}. \quad (3.10)$$

A description of the low energy $pp \rightarrow pn\pi^+$ observables only requires these functions for $n=1, 2, 3$, and explicitly in these cases

$$P^{(1)}(x) = \frac{x^3}{4(1+\sqrt{1+x^2})^2}, \quad (3.11a)$$

$$P^{(2)}(x) = \frac{1}{\pi} \left\{ \frac{2}{3}x + \frac{1}{x} - \left(1 + \frac{1}{x^2} \right) \arctan(x) \right\}, \quad (3.11b)$$

$$P^{(3)}(x) = \frac{x^3}{8(1+\sqrt{1+x^2})^2} \left\{ 1 + \frac{x^2}{2(1+\sqrt{1+x^2})^2} \right\}. \quad (3.11c)$$

It is customary in the analysis of $pp \rightarrow d\pi^+$ data to assume that, due to the Coulomb repulsion in the final state, the cross section very near threshold is suppressed by a Gamow factor of

$$F_C(\eta) = \frac{2q/\eta}{\exp(2q/\eta) - 1} \approx \frac{1}{1+q/\eta}, \quad (3.12)$$

where

$$q = \frac{\pi\mu\alpha}{m_\pi}. \quad (3.13)$$

This approach, which considers all the charge in the deuteron to be concentrated at the center of the nucleus, is sufficient to explain the major differences between the $np \rightarrow d\pi^0$ [17] and $pp \rightarrow d\pi^+$ [1] total cross sections, and the simplification made in Eq. (3.12) is indistinguishable from the full Gamow factor under the conditions of such data.

An analogous suppression will be suffered by the π^+ in the $pp \rightarrow pn\pi^+$ case, but this is only likely to be of practical importance for s -wave pions because the centrifugal barrier keeps the p waves small in the region of low η . Taking the charge, as in the deuteron case, to be situated at the center of mass of the proton-neutron system, the integral for the Coulomb-modified $P^{(1)}(x)$ of Eq. (3.11) can still be performed in closed form, giving

$$\begin{aligned} P_C^{(1)}(x,c) &= \frac{x^3}{\pi(1+x^2(1-c^2))} \left\{ \frac{1}{2}c^3\sqrt{1-c^2} \ln \left(\frac{1-\sqrt{1-c^2}}{1+\sqrt{1-c^2}} \right) + \frac{\pi}{4x^4} [1+x^2(1-c^2)][2+x^2(1+2c^2)] - \frac{\pi}{2x^4} (1+x^2)^{3/2} \right. \\ &\quad \left. - \frac{c}{x^2} [1+x^2(1-c^2)] + \frac{c}{x^3} (1+x^2) \arctan(x) \right\}, \end{aligned} \quad (3.14)$$

where $c = q/\eta$.

In $pp \rightarrow pp\pi^0$ the s -wave pion production is likely to be dominant to much higher values of η than for charged pion production induced by the suppression of the Δ contributions through the selection rules for the final S -wave pp states. Though there is no bound-state production to set the normalization, the energy dependence of the total cross section should therefore be of the form $\eta P^{(1)}(x)$.

In the absence of Coulomb effects, the cross sections for π^0 and π^+ production are related by

$$\sigma(pp \rightarrow \{pn\}_{I=1}\pi^+) = \sigma(pp \rightarrow pp\pi^0). \quad (3.15)$$

Coulomb corrections must however be applied before this is used to estimate the spin-singlet contribution to $pp \rightarrow pn\pi^+$, and this is done in two stages. As can be seen from Fig. 1, most of the effects of Coulomb barrier in the final pp wave function on the right-hand side of Eq. (3.15) are removed by replacing the value of Z_s with Coulomb in Eq. (2.10) by that without. On the other hand, the repulsion in the π^+p system on the left of the equation can be handled by replacing $P^{(1)}(x)$ by $P_C^{(1)}(x, c)$.

IV. COMPARISON WITH EXPERIMENT

In the low energy region it is a good approximation to take the $pp \rightarrow d\pi^+$ differential cross section and analyzing power to be of the form

$$\frac{d\sigma}{d\Omega} = A + B\cos^2\theta, \quad (4.1a)$$

$$A_y \frac{d\sigma}{d\Omega} = C\sin\theta, \quad (4.1b)$$

where θ is the c.m. angle of the pion, and these dependences will propagate through to the spin-triplet part of the $pp \rightarrow pn\pi^+$ reaction.

The SP96 phase shift solution from the Virginia database [1] yields the following predictions for Coulomb-corrected observables. At $\theta = 0^\circ$,

$$\frac{d\sigma}{d\Omega} = (13.6\eta + 164.0\eta^3) \mu\text{b/sr}, \quad (4.2)$$

whereas at $\theta = 90^\circ$

$$\frac{d\sigma}{d\Omega} = (13.6\eta + 16.2\eta^3) \mu\text{b/sr}, \quad (4.3a)$$

$$A_y \frac{d\sigma}{d\Omega} = -18.1\eta^2 \mu\text{b/sr}. \quad (4.3b)$$

The normalization given in Eq. (4.2) is quite close to that deduced from neutron-induced data using charge independence [17]. However it lies typically 10% *above* the recent COSY $pp \rightarrow d\pi^+$ measurement [2] and 25% *below* that from IUCF [3]. The difference between these two gives a measure of the uncertainty in the input of our calculation.

Spin-singlet final states may be estimated using

$$\sigma(pp \rightarrow pp\pi^0) \approx 4\pi D \eta P^{(1)}(\zeta' \eta), \quad (4.4)$$

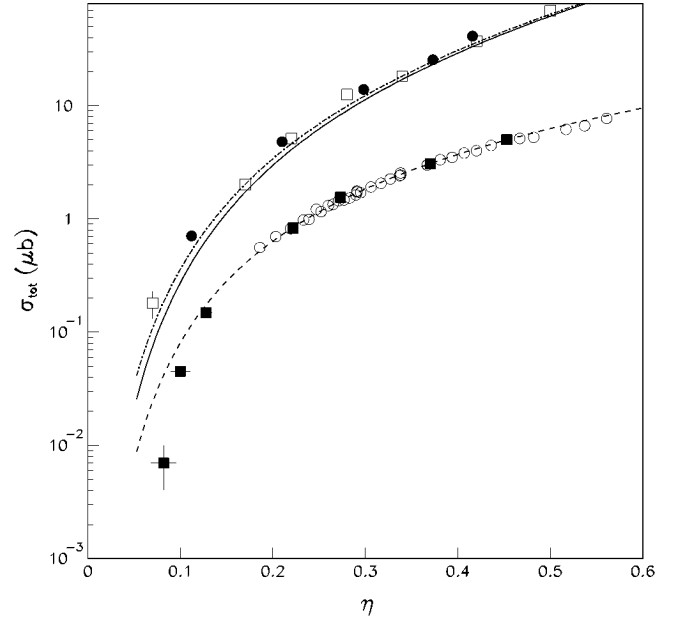


FIG. 2. The experimental $pp \rightarrow pp\pi^0$ total cross sections of Ref. [5] (open circles) and Ref. [6] (squares) are to be compared to the broken curve, which represents the fit on the basis of Eq. (4.4). This curve is very similar in shape to the microscopic calculations of Ref. [18] which include the Coulomb repulsion explicitly. The $pp \rightarrow pn\pi^+$ data of Refs. [8,9], which are very similar to the extended data set of Ref. [10], are subject to a 13% overall error. The uncertainty in the input for the solid and chain curves, which show the corresponding predictions with and without the pionic Coulomb corrections, is around 20% [1–3]. The Coulomb corrections given by Eq. (3.14) differ little in practice from those obtained by merely multiplying by the Gamow factor of Eq. (3.12).

where, as discussed in Sec. II, after including the pp Coulomb repulsion, $\zeta' = 12.6$. The best fit to the total cross section data of Ref. [5] is achieved with $D \approx 0.86 \mu\text{b}$, and the resulting curve shown in Fig. 2 is very similar in shape to microscopic calculations which include the pp final state interaction more fully [18]. The deviations for the three lowest CELSIUS points may be due to an uncertainty in the proton beam energy [6] but there may also be some influence of the breakdown in the smoothness of Z_s at very small k^2 in Fig. 1 due to residual Coulomb effects. This is of no consequence for the subsequent estimation of the $pp \rightarrow pn\pi^+$ cross sections.

Though the authors of Ref. [5] did not report values for the $pp \rightarrow pp\pi^0$ differential cross sections, their data do suggest some influence from higher partial waves at 325 MeV. There is however evidence that at 320 MeV ($\eta = 0.46$) the cross section is fairly isotropic and that the proton analyzing power is consistent with zero [19]. Introducing Coulomb corrections in both the final $pp\pi^0$ and $pn\pi^+$ systems, these data then predict that

$$\frac{d\sigma}{d\Omega}(pp \rightarrow \{pn\}_s\pi^+) \approx D' \eta P_C^{(1)}(\zeta'' \eta, c), \quad (4.5)$$

where $D' \approx 0.42 \mu\text{b/sr}$ and $\zeta'' = 25.4$. This is to be taken in conjunction with the spin-triplet predictions for the differential cross section

$$\begin{aligned}
\frac{d\sigma}{d\Omega}(pp \rightarrow \{pn\}, \pi^+) \\
&= 13.6 \eta P_C^{(1)}(\zeta'' \eta, c) \\
&\quad + (16 + 148 \cos^2 \theta) \eta^3 P^{(3)}(\zeta'' \eta) \mu\text{b/sr},
\end{aligned}
\tag{4.6}$$

and analyzing power

$$A_y \frac{d\sigma}{d\Omega}(pp \rightarrow \{pn\}, \pi^+) = -18 \eta^2 \sin \theta P^{(2)}(\zeta'' \eta) \mu\text{b/sr},
\tag{4.7}$$

where pion Coulomb corrections have only been made in the s wave.

The momentum dependence of the $pp \rightarrow pn \pi^+$ total cross section is shown in Fig. 2 both with and without pionic Coulomb corrections. There is in fact very little difference between either using the $P_C^{(1)}(x, c)$ of Eq. (3.14) or multiplying the uncorrected $P^{(1)}(x)$ of Eq. (3.11) by the Gamow factor of Eq. (3.12). The full curve lies somewhat below the IUCF measurements [9] but, apart from the lowest point, the deviations are of the same order as the Coulomb corrections. It should be noted that there are typically 20% overall normalization uncertainties in the input and output data, as well as problems in determining the value of η close to threshold [9]. The total cross sections from the extended data set of Ref. [10] are consistent with the earlier measurements, though they are subject to very similar systematic uncertainties.

Due to the differences in the pole positions and hence energy dependence, the ratio of the s -wave production of pions with spin-triplet and spin-singlet final states is estimated to be about 1.7 at low η but 7.8 at high values.

The angular dependence of the predicted $pp \rightarrow pn \pi^+$ differential cross section at $\eta=0.21$ and $\eta=0.42$ is compared in Fig. 3 with the IUCF data [9]. The theory is about 20% too low in normalization and the slopes, evaluated assuming the singlet cross section to be isotropic, a little too strong. It is clear from the larger data set of Ref. [10] that the binning effect due to the experimental averaging over angular domains is only of marginal importance.

In Fig. 4 is shown the variation of A_y at 90° as a function of η for $pp \rightarrow d \pi^+$ and $pp \rightarrow pn \pi^+$, assuming the singlet cross section to be isotropic with zero analyzing power. The curve lies a little below the experimental points of Ref. [10], where the only major systematic uncertainty is the acceptance in polar and azimuthal angles.

V. CONCLUSIONS

We have shown within the framework of a simple final state interaction theory that most of the low energy data on the total and differential cross sections and proton analyzing power for the $pp \rightarrow pn \pi^+$ reaction can be understood semi-quantitatively in terms of equivalent information deduced from $pp \rightarrow pp \pi^0$ and $pp \rightarrow d \pi^+$ measurements. We believe that any more microscopic approach with finite range pion-production operators, which fit the latter data, would give $pp \rightarrow pn \pi^+$ predictions very similar to the ones presented

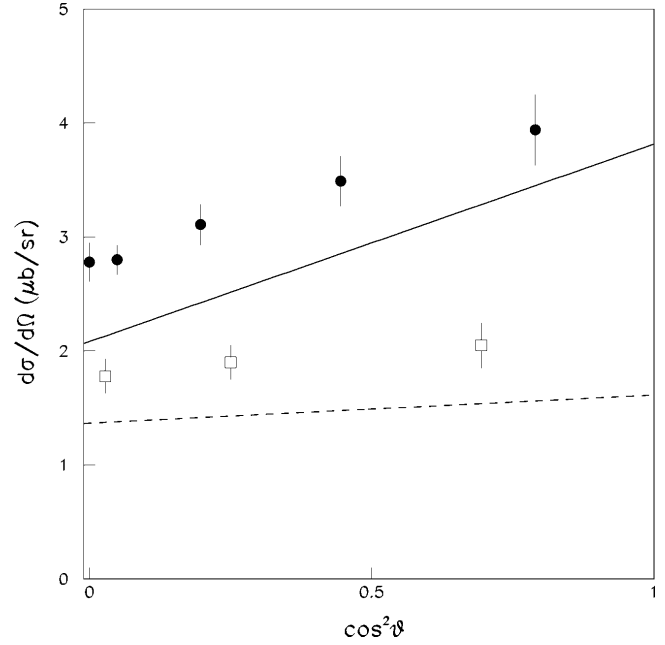


FIG. 3. Differential cross sections for $pp \rightarrow pn \pi^+$ at $\eta=0.21$ (squares) and $\eta=0.42$ (circles) taken from Ref. [9] and compared with the predictions of the model assuming the singlet cross section to be isotropic. The data and predictions at the lower energy are both multiplied by a factor of five to present them on a similar scale.

here, though with some changes at higher values of η due to the neglect of final np P -wave contributions. However it is also apparent from Fig. 1 that our formalism will tend to underestimate slightly the triplet contribution to the

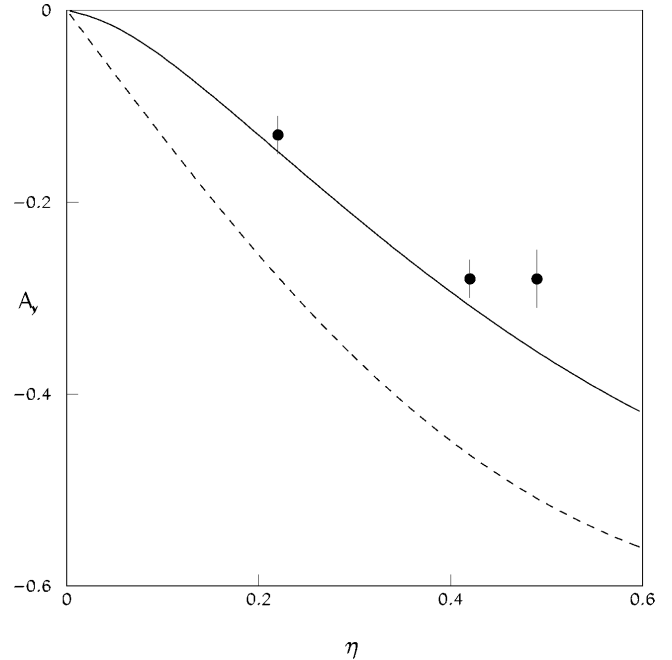


FIG. 4. Variation of the proton analyzing power in $pp \rightarrow pn \pi^+$ at $\theta=90^\circ$ as a function of η , the experimental points being taken from Ref. [10]. The broken curve is the prediction of the phase shift solution SP96 of Ref. [1] for $pp \rightarrow d \pi^+$, whereas the solid curve follows from Eq. (4.7) assuming the $pp \rightarrow pp \pi^0$ cross section to be isotropic with negligible analyzing power.

$pp \rightarrow pn\pi^+$ cross section at high η because Z_i is an increasing function of k^2 . The rate of rise depends upon the nucleon-nucleon separation and so it is impossible to quantify the effect without having a detailed model for the pion-production operator as well as the nucleon-nucleon wave function. Though we illustrated our formalism with the Paris potential, our results are independent of this choice apart from small Coulomb effects.

P -wave contributions might be responsible for some of the discrepancies observed in the angular distributions and analyzing power. However it is clear from the linear scale of Fig. 3 that the agreement would be better if the normalization of the input $pp \rightarrow d\pi^+$ data were increased by about 25%, as was found in the IUCF measurement [3] of this rate. The deviations in the total cross section predictions in Fig. 2 are larger near threshold but, having taken into account the overall normalization error on the $pp \rightarrow pn\pi^+$ data and the difficulty in determining the exact value of η [9], these could be mainly of an experimental nature. On the other hand, it is hard to reconcile the COSY normalization of $pp \rightarrow d\pi^+$ [2] with the IUCF measurement of $pp \rightarrow pn\pi^+$ [8–10] within our model.

If the disagreement over the $pp \rightarrow d\pi^+$ normalization can be resolved, to take full advantage of the accurate IUCF data

would require a sophisticated microscopic model to be accurate to the 10% level. It could then be helpful if such a model were consistently applied in the evaluation of the experimental acceptances rather than relying on the Watson FSI approach [9].

Qualitatively new information, against which to test our model, could be provided by experiments which introduce interferences between triplet and singlet final np states, as, for example, in the (\vec{p}, \vec{p}') spintransfer. These are however much harder to perform.

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