The isospin 0 $J^P=0^-$ πNN resonance

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(Received 10 June 1997)

We studied the resonance behavior of the πNN system in the configuration $J^P=0^-$, isospin 0. We applied nonrelativistic Faddeev equations with realistic local potentials for the pion-nucleon and nucleon-nucleon interactions in the S_{11} and $^1S_0$ channels, respectively. A resonance was found in this specific state; it lies slightly above the πNN threshold and has a mass of 2018 MeV and a width of 1.75 MeV. [S0556-2813(97)03510-3]

PACS number(s): 14.20.Gk, 11.80.Jy, 13.75.Gx, 14.20.Pt

I. INTRODUCTION

A πNN resonance that lies 47 MeV above the πNN threshold has been observed in pionic double charge exchange reactions in nuclei [1]. Analysis of these data indicated that this resonance has a mass of 2065 MeV, angular momentum and parity $J^P=0^-$, and isospin even [1]. According to [1,2] the width of the resonance would be $\Gamma = 0.5$ MeV if it has isospin 0 or $\Gamma = 1.0$ MeV if it has isospin 2. A signal of this resonance [3] has been observed in the reaction $pp\rightarrow pp\pi^-\pi^+$ which corresponds to a $pp\pi^-$ invariant mass of 2063 MeV and a width of $\sim$ 5 MeV. A considerable amount of discussion has been going on recently whether the isospin of the d' resonance is 0 or 2 [4–9].

Three-body calculations of the πNN system in the isospin 0 sector have been restricted hitherto to the use of separable potentials for the two-body interactions [10,11] and there are some indications that a state with the required quantum numbers may exist near the πNN threshold [11].

Our motivation for performing a three-body calculation of the isospin 0 $J^P=0^-$ state is the recent availability of local πN interactions obtained from the application of an inverse scattering method [12,13]. It has been kindly pointed out to us by von Geramb and Sander [14] that the local pion-nucleon interaction in the S_{11} channel exhibits a very strong attraction at short distances, a feature not present in a separable potential description. It is thus interesting to investigate if this strong short-range attraction could be the driving mechanism giving rise to the d' resonance.

Since local potentials have been proven to be very successful in the description of the NN and NNN systems at low energies, we expect that a local potential description of both the πN and NN subsystems will provide also a reliable framework to study the πNN system in the region near threshold.

II. FORMALISM

In order to investigate the location of the πNN resonance in the complex plane we will use nonrelativistic Faddeev equations with local potentials. In general, a nonrelativistic formalism is appropriate for the description of the nucleon-nucleon system up to kinetic energies of about 300 MeV, i.e., up to about one-third of the nucleon mass. Since the d' resonance lies just 47 MeV above the πNN threshold which is about one-third of the pion mass, we consider a nonrelativistic description also appropriate for this system [12].

The πNN $J^P=0^-$ state contains two main three-body configurations. There is a $(\pi N)N$ configuration in which the pion and the nucleon interact in the S_{11} channel while the spectator nucleon is in a relative S state with respect to the interacting pair. There is also a $(NN)\pi$ configuration where the two nucleons interact in the $^1S_0$ channel while the spectator pion is in a relative S state with respect to the interacting pair. Thus, all three particles are in orbital angular momentum states $l=0$ for the two main configurations of the $J^P=0^-$ state. The next important three-body configurations involve either both the angular momentum of the pair and of the spectator, being $l=1$ or at least one of them being $l=2$. Thus, these higher configurations are not expected to play any important role.

Since for the two main configurations of the system all three particles are in S waves and one has only one channel for each configuration, the Faddeev equations for the bound-state (or resonance) problem are

$$\langle p_i q_i|T_i|\phi_0\rangle = \sum_{j\neq i} b^{ij}\frac{1}{2}\int_0^\infty q_j^2 dq_j\int_{-1}^1 d\cos\theta\, t_i(p_i,p'_i;E-q_i^2/2\nu_i) \times \frac{1}{E-p_j^2/2\eta_j-q_j^2/2\nu_j}\langle p_j q_j|T_j|\phi_0\rangle, \tag{1}$$

where p_i and q_i are the usual Jacobi coordinates and η_i and ν_i the corresponding reduced masses

$$\eta_i=\frac{m_j m_k}{m_j+m_k}, \tag{2}$$

$$\nu_i=\frac{m_i(m_j+m_k)}{m_i+m_j+m_k}, \tag{3}$$

with ijk an even permutation of 123. The momenta p'_i and p_j in Eq. (1) are given by

$$p'^2_i=q_j^2+\frac{\eta_i^2}{m_k^2}q_i^2+2\frac{\eta_i}{m_k}q_i q_j\cos\theta, \tag{4}$$

$$p_j^2 = q_i^2 + \frac{\eta_j^2}{m_k^2} q_j^2 + 2\frac{\eta_j}{m_k} q_i q_j \cos\,\theta. \qquad (5)$$

b^{ij} are the spin-isospin coefficients

$$b^{ij} = (-)^{S_j + s_j - S}\sqrt{(2S_i+1)(2S_j+1)}W(s_j s_k S s_i ; S_i S_j) \times (-)^{I_j + i_j - I}\sqrt{(2I_i+1)(2I_j+1)}W(i_j i_k I i_i ; I_i I_j), \qquad (6)$$

where W is the Racah coefficient and s_i, S_i, and S (i_i, I_i, and I) are the spins (isospins) of particle i, of the pair jk, and of the three-body system.

The two-body amplitudes t_i are obtained by the solution of the Lippmann-Schwinger equation

$$t_i(p_i, p'_i; e) = V_i(p_i, p'_i) + \int_0^\infty p''^2_i dp''_i V_i(p_i, p''_i) \times \frac{1}{e - p''^2_i/2\eta_i} t_i(p''_i, p'_i; e), \qquad (7)$$

with

$$e = E - q_i^2/2\nu_i. \qquad (8)$$

Since the variables p_i in Eqs. (1) and (7) run from 0 to ∞ it is convenient to make the transformation

$$x_i = \frac{p_i - b}{p_i + b}, \qquad (9)$$

where the new variable x_i runs from -1 to 1 and b is a scale parameter. With this transformation Eq. (1) takes the form

$$\langle x_i q_i | T_i | \phi_0 \rangle = \sum_{j \neq i} b^{ij} \frac{1}{2} \int_0^\infty q_j^2 dq_j \int_{-1}^{1} d\,\cos\,\theta\; t_i(x_i, x'_i; E - q_i^2/2\nu_i) \times \frac{1}{E - p_j^2/2\eta_j - q_j^2/2\nu_j} \langle x_j q_j | T_j | \phi_0 \rangle. \qquad (10)$$

Since in the amplitude $t_i(x_i, x'_i; e)$ the variables x_i and x'_i run from -1 to 1, one can expand this amplitude in terms of Legendre polynomials as

$$t_i(x_i, x'_i; e) = \sum_{nm} P_n(x_i) \tau_i^{nm}(e) P_m(x'_i), \qquad (11)$$

where the expansion coefficients are given by

$$\tau_i^{nm}(e) = \frac{2n+1}{2} \frac{2m+1}{2} \int_{-1}^{1} dx_i \times \int_{-1}^{1} dx'_i P_n(x_i) t_i(x_i, x'_i; e) P_m(x'_i). \qquad (12)$$

Applying expansion (11) in Eq. (10) one gets

$$\langle x_i q_i | T_i | \phi_0 \rangle = \sum_n T_i^n(q_i) P_n(x_i), \qquad (13)$$

where $T_i^n(q_i)$ satisfies the one-dimensional integral equation

$$T_i^n(q_i) = \sum_{j \neq i} \sum_m \int_0^\infty dq_j A_{ij}^{nm}(q_i, q_j; E) T_j^m(q_j), \qquad (14)$$

with

$$A_{ij}^{nm}(q_i, q_j; E) = b^{ij} \sum_l \tau_i^{nl}(E - q_i^2/2\nu_i) \frac{q_j^2}{2} \times \int_{-1}^{1} d\,\cos\,\theta\; \frac{P_l(x_i) P_m(x_j)}{E - p_j^2/2\eta_j - q_j^2/2\nu_j}. \qquad (15)$$

The three amplitudes $T_1^l(q_1)$, $T_2^m(q_2)$, and $T_3^n(q_3)$ in Eq. (14) are coupled together. The number of coupled equations can be reduced, however, since two of the particles are identical. The reduction procedure for the case where one has two identical fermions has been described before [10,15] and will not be repeated here. With the assumption that particle 1 is the pion and particles 2 and 3 are the nucleons, only the amplitudes $T_1^n(q_1)$ and $T_2^m(q_2)$ are independent from each other and they satisfy the coupled integral equations

$$T_1^l(q_1) = \sum_n \int_0^\infty dq_3 A_{13}^{ln}(q_1, q_3; E) T_2^n(q_3), \qquad (16)$$

$$T_2^m(q_2) = \sum_n (-)^{\mathrm{Iden}} \int_0^\infty dq_3 A_{23}^{mn}(q_2, q_3; E) T_2^n(q_3) + \sum_l \int_0^\infty dq_1 A_{31}^{ml}(q_2, q_1; E) T_1^l(q_1), \qquad (17)$$

with the identical-particles factor

$$\mathrm{Iden} = 1 + s_1 + s_3 - S_2 + i_1 + i_3 - I_2. \qquad (18)$$

Substitution of Eq. (16) into Eq. (17) yields an equation with only the amplitude T_2

$$T_2^m(q_2) = \sum_n \int_0^\infty dq_3 K_{23}^{mn}(q_2, q_3; E) T_2^n(q_3), \qquad (19)$$

where

$$K_{23}^{mn}(q_2, q_3; E) = (-)^{\mathrm{Iden}} A_{23}^{mn}(q_2, q_3; E) + \sum_l \int_0^\infty dq_1 A_{31}^{ml}(q_2, q_1; E) A_{13}^{ln}(q_1, q_3; E). \qquad (20)$$

Since we want to find the solutions of Eqs. (19) and (20) corresponding to resonances we follow a well-known procedure [16–18]. We let the variables q_i become complex and run along a ray in the complex plane by making the replacement

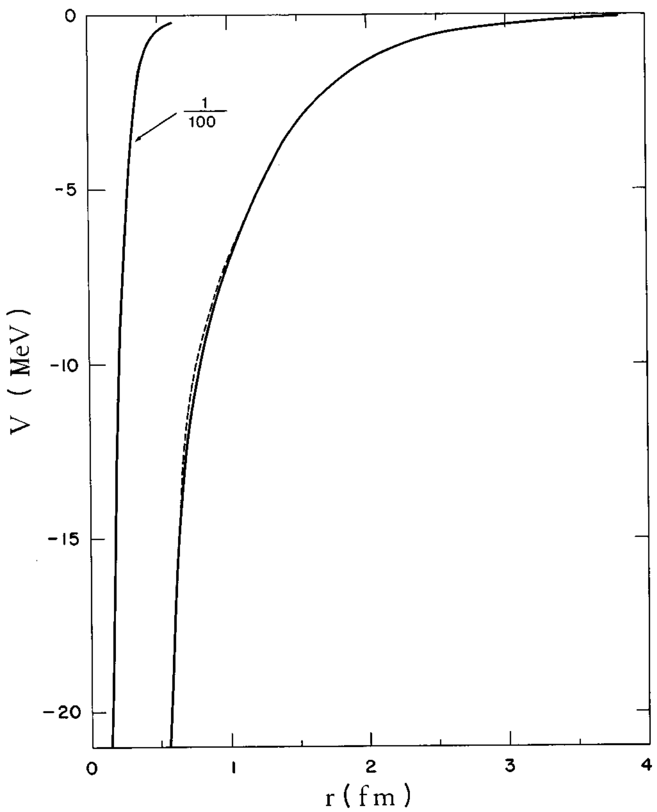

FIG. 1. The pion-nucleon local potential in the S_{11} channel (solid line) obtained from the inverse-scattering method. The dashed line is the result of the fit [Eq. (25)] as a sum of Yukawa terms.

$$q_i \to |q_i| e^{-i\phi}, \tag{21}$$

where ϕ is a constant phase. This opens up large portions of the second Riemann sheet so that one can search for the poles of Eq. (19) for energies of the form

$$E = E_r - \frac{i}{2}\Gamma, \tag{22}$$

which correspond to resonances. It is easy to see from Eqs. (22), (7), and (8) that the propagator in the Lippmann-Schwinger equation (7) will never hit any singularity if we take the variables p_i also along a ray in the complex plane as

$$p_i \to |p_i| e^{-i\phi}. \tag{23}$$

Finally, in transformation (9) we also take the scale parameter b complex by setting

$$b \to |b| e^{-i\phi}. \tag{24}$$

As a consequence of Eqs. (23) and (24), the variable x_i defined by Eq. (9) remains real and running from -1 to 1 so that expansion (11) in terms of Legendre polynomials is still valid.

In order to find the solutions of Eq. (19) we replace the integral by a sum applying a numerical integration quadrature. In this way Eq. (19) becomes a set of homogeneous linear equations. This set of linear equations has solutions only if the determinant of the matrix of the coefficients (the Fredholm determinant) vanishes for certain energies. Thus, the procedure to find the resonances of the system consists simply in searching for the zeroes of the Fredholm determinant in the complex energy plane.

TABLE I. Parameters μ_i and a_i of the S_{11} pion-nucleon local potential given by Eq. (25).

i	μ_i (fm^{-1})	a_i (MeV fm)
1	1.70826	-147.64313
2	2.70826	737.54486
3	3.70826	-1782.89172
4	4.70826	231.05701
5	5.70826	5072.75977
6	6.70826	-1033.93457
7	7.70826	-7917.12305
8	8.70826	-1232.88196
9	9.70826	3545.22607
10	10.70826	2297.15698

III. RESULTS

As we mentioned in the Introduction, the pion-nucleon potential in the S_{11} channel obtained recently from the inverse scattering method [12,13] has a very strong attraction at short distances. We show this potential in Fig. 1 together with a parametrization of it as a sum of Yukawa terms of the form

$$V(r) = \sum_{i=1}^{10} a_i \frac{e^{-\mu_i r}}{r} \tag{25}$$

that we used in our calculations. The parameters a_i and μ_i are given in Table I. For the nucleon-nucleon interaction in the $^1S_0$ channel we used the Reid soft-core potential [19] which is also a linear combination of Yukawa terms similar to Eq. (25).

In order to check our program we solve the equations also with separable interactions. In that case, the solution of Eq. (7) has the simple form

$$t_i(p_i, p'_i; e) = g_i(p_i)\tau_i(e) g_i(p'_i), \tag{26}$$

and Eqs. (14) and (15) are replaced by

$$T_i(q_i) = \sum_{j \neq i} \int_0^\infty dq_j A_{ij}(q_i, q_j; E) T_j(q_j) \tag{27}$$

and

$$A_{ij}(q_i, q_j; E) = b^{ij} \tau_i(E - q_i^2/2\nu_i) \frac{q_j^2}{2} \int_{-1}^{1} d\cos\theta \times \frac{g_i(p'_i) g_j(p_j)}{E - p_j^2/2\eta_j - q_j^2/2\nu_j}, \tag{28}$$

with p'_i and p_j given by Eqs. (4) and (5). Using Yamaguchi potentials [20] for the interactions, the function $A_{ij}(q_i, q_j; E)$ given by Eq. (28) is known in analytical form so that the extension into the complex plane is straight-

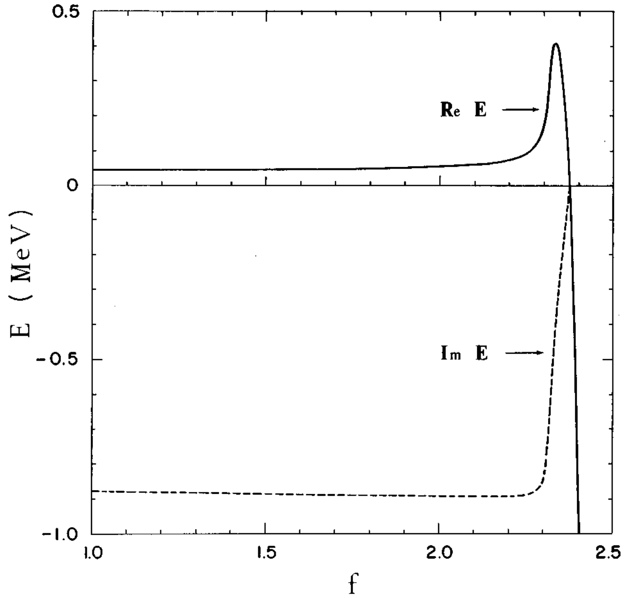

FIG. 2. Energy eigenvalue of the local-potential model as a function of the strength parameter f by which the pion-nucleon S_{11} potential has been multiplied.

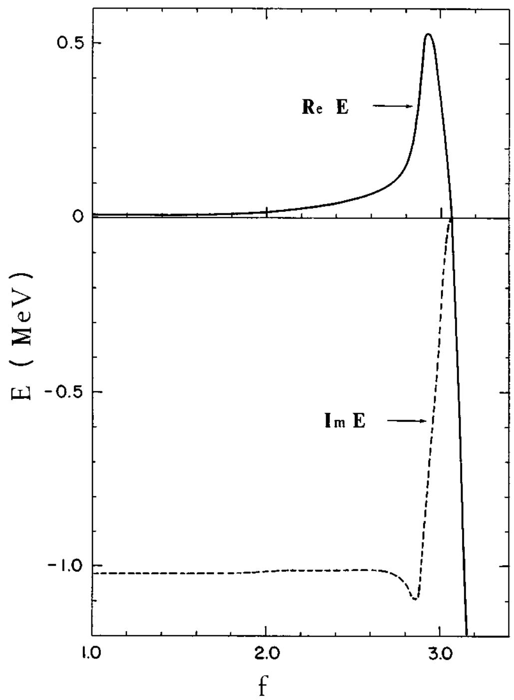

FIG. 3. Same as Fig. 2 for the model with separable potentials.

forward [16]. Thus, we checked our local-potential program by applying it to the separable-potential model.

As an additional check of our program, we also investigated the bound-state problem of the system, i.e., we solved Eqs. (19) and (27) for E on the real axis and $E<0$. In that case, one does not need to extend the equations into the complex plane so that one can take $\phi=0$ in Eqs. (21), (23), and (24). The πNN system in the $J^P=0^-$ channel does not possesses bound-state solutions. A bound-state solution appears, however, when one multiplies the pion-nucleon S_{11} interaction by a factor f larger than one. After having found the bound-state energies for several values of f, we repeated the calculation with $\phi\neq 0$ in Eqs. (21), (23), and (24) and obtained the same energy eigenvalues (notice that by increasing the amount of attraction in the NN interaction one can generate a NN bound state but this does not give rise to a πNN bound state).

Thus, after we generated a πNN bound state by introducing the factor f, we then started to decrease this factor and followed the resonance into the complex plane up to its physical position corresponding to $f=1$. We show in Fig. 2 the energy eigenvalue E as a function of this parameter f for the local-potential model. As can be seen from this figure, for $f>2.37$ the energy eigenvalue is purely real and negative so that one is in the bound-state problem region. Between $f=2.2$ and $f=1$ the eigenvalue remains practically constant. The point $f=1$ represents the physical problem and for that point the energy eigenvalue has the value $E=0.042-i0.876$ MeV. Thus, the resonance lies slightly above the πNN threshold with a mass of 2018 MeV and a width of 1.75 MeV. These values of the mass and width of the resonance are not far from those of the d' resonance reported in [1,3].

It is interesting to see how a totally different description like that of separable potentials compares with the results of Fig. 2. Therefore, we show in Fig. 3 the corresponding results of the model using pion-nucleon [21] and nucleon-nucleon [20] separable potentials. In that case the bound-state region corresponds to $f>3.06$, which reflects the fact that the separable S_{11} potential is less attractive than the local one. The physical point $f=1$ has an eigenvalue $E=0.009-i1.022$ MeV, which is comparable to that of the local-potential model.

Thus, we conclude that the d' resonance observed in [1,3] may perhaps be explained as a $J^P=0^-$ πNN resonance with isospin 0.

ACKNOWLEDGMENTS

I like to thank Dr. L. Mathelitsch who participated in the early stages of this work for his very useful suggestions and comments. This work was supported in part by COFAA-IPN.

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