Δ^+ mass reexamined

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We question the reliability of current estimates for the Δ^+ mass. We explain why the standard value, determined from pion photoproduction, should not be used in conjunction with present estimates for the Δ^{++} and Δ^0 masses, determined from pion-nucleon scattering. The resultant mass splittings are internally inconsistent. We also briefly comment on discrepancies found when ''theoretical'' and ''experimental'' values for the Δ masses are compared. [S0556-2813(97)02909-9]

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What is the $\Delta(1232)$ mass? As the name implies, 1232 MeV is a reasonable estimate for the ''average'' mass. The Δ^0 and Δ^{++} masses, 1233.6 ± 0.5 MeV, and 1230.9 ± 0.3 MeV, respectively [1,2], have been estimated from pion-nucleon scattering, and the Δ^+ mass (1234.9 ± 1.4 MeV) has been extracted [3] from pion photoproduction data. Taken together, the Δ^+ mass seems inconsistent with estimates for the Δ^0 and Δ^{++} masses. To be more specific, the resultant splittings do not fit neatly into the pattern exhibited by other members of the baryon octet and decuplet [4,5].

While the above result of Ref. [3] is usually quoted [2] for the Δ^+ mass, most estimates have resulted in values which are considerably lower and closer to theoretical estimates. In the following, we first explain why the Δ^+ mass of Ref. [3] should not be used in conjunction with more recent determinations of the Δ^0 and Δ^{++} masses. We then emphasize that similar problems arise in any analysis starting from multipoles determined in a separate analysis. Finally, we note that pole parameters may be in better agreement with theoretical estimates. The splitting found in Δ pole masses [6] is given as an example.

Values for the Δ^+ mass and pole parameters, found in Ref. [3], were based upon a multipole analysis performed in Ref. [7]. In that analysis, the multipoles with $L \leq 1$ were constrained, via Watson's theorem [8], to have the phases of the corresponding pion nucleon partial waves. Phases were taken from the CBC [9] and CERN [10] analyses, and the results were compared. The authors concluded that the CERN phases were more consistent with the database being fitted. The result of this choice can be seen in Table I of Ref. [3]. The real part of the $M_{1+}^{3/2}$ multipole passes through zero for a photon energy near 350 MeV. However, in more recent analyses [11], the crossover point is closer to 340 MeV. In fact, at the time of the CERN analyses, the P_{33} resonance was named the $\Delta(1236)$ [12], the CERN value [10] being 1235.8 MeV. [Note that the Δ^+ mass (1234.9 ± 1.4 MeV) found in Ref. [3] is actually consistent with the CERN value.] Clearly the results of this analysis [3] cannot be combined with those found in more recent pion nucleon analyses [13–15]. A 4 MeV shift in the expected ''average mass'' overwhelms the splitting of charge states.

Since the use of Watson's theorem determines the phase behavior exhibited by photoproduction multipoles, we expect to find a Δ^+ mass between the Δ^0 and Δ^{++} masses. This has

generally been the result of analyses using more modern pion nucleon phases [2]. This result appears reasonable, based on the splitting seen in the Σ and Σ^* states. The most model-independent determination would require sufficient observables to drop the Watson's theorem constraint. Another approach would be to allow small shifts in the $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ phases. This was attempted in Ref. [16], utilizing the CBC phases [9]. The estimated Δ^+ mass was 1231.8 MeV, lying about halfway between the Δ^0 and Δ^{++} masses estimated in the CBC analysis [9]. If an approximate equal-spacing rule holds for the Δ states, it will be very difficult to distinguish between results with/without the Watson's theorem constraint.

While the above arguments result in a more satisfying set of Δ masses, we should remind the reader of some problems which persist. One recent paper [17], relevant to these issues, has redetermined the Δ^0 and Δ^{++} resonance and pole parameters from a fit to the Pedroni total cross sections [18].

The resulting $\Delta^0 - \Delta^{++}$ mass difference was 2.25 ± 0.68 MeV, a value consistent (within errors) with the original Pedroni determination (2.7 ± 0.3 MeV) [18]. A rough estimate of the expected Δ splitting, in terms of the $n - p$ mass difference, was given as

$$\Delta^0 - \Delta^{++} = 2(n - p), \qquad (1)$$

which again seems reasonable in light of the mass relation [5]

$$\Delta^0 - \Delta^+ = n - p. \qquad (2)$$

The predicted splitting is then about 2.6 MeV, which agrees with the values found in Refs. [17,18]. However, the more formally correct sum rule [5] for this splitting

$$\Delta^0 - \Delta^{++} = 2(n - p) - (\Sigma^+ - 2\Sigma^0 + \Sigma^-), \qquad (3)$$

agrees with the previous estimate only in the case of equally spaced charge states. Inserting experimental values [2], one obtains a 0.9 MeV splitting between the Δ^0 and Δ^{++}.

In a fit to the octet and decuplet masses, Cutkosky [4] found a $\Delta^0 - \Delta^{++}$ splitting of 1.53 MeV. This dropped to 0.81 MeV when the experimental Δ masses were excluded from the fit. (No attempt was made to fit the Δ^+ mass of Ref. [3].) These values are actually closer to the splitting between pole masses (0.40 ± 0.57 MeV) found in Ref. [17]. A simi-

larly small splitting (0.8 MeV) is found in the Virginia Tech analysis of pion-nucleon scattering data [19].

Finally, we note that the fit by Cutkosky [4] is constructed to satisfy the mass relations in Eqs. (2) and (3). Since the predicted $\Delta^0 - \Delta^{++}$ splitting is *less* than the $\Delta^0 - \Delta^+$ splitting, the Δ^+ mass falls below the Δ^{++}. This possibility provides motivation for studies where the Watson's theorem constraint is relaxed.

In this paper, we have seen that the standard value for the Δ^+ mass [3] is inconsistent with Δ^0 and Δ^{++} masses found in more recent pion-nucleon analyses. The problem can be traced to the use of outdated phase information. In fact, the use of Watson's theorem always biases the masses found in pion photoproduction analyses. While a Δ^+ mass midway between the Δ^0 and Δ^{++} masses appears more reasonable,

and is supported by the study made in Ref. [16], the fit made by Cutkosky [4] does not confirm this simpler picture.

The Δ^+ pole position [11,20] is consistent with values found for mixed charges from pion-nucleon scattering [2]. However, since the splitting in pole positions is considerably smaller than that found for the resonance masses, an accurate determination of the splitting ($\Delta^0 - \Delta^+$) will be difficult. The apparent agreement between pole mass splittings and mass formulas is suggestive, but could be accidental given the uncertainties associated with both the pole splitting and the mass formula in Eq. (3).

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