Anapole moment and nucleon weak interactions

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From the recent measurement of parity nonconservation (PNC) in the Cs atom we have extracted the constant of the nuclear spin dependent electron-nucleon PNC interaction, $\kappa=0.442(63)$; the anapole moment constant, $\kappa_a=0.364(62)$; the strength of the PNC proton-nucleus potential, $g_p=7.3\pm1.2(\text{expt})\pm1.5(\text{theor})$; the π-meson-nucleon interaction constant, $f_\pi\equiv h_\pi^1=[9.5\pm2.1(\text{expt})\pm3.5(\text{theor})]\times10^{-7}$; and the strength of the neutron-nucleus potential, $g_n=-1.7\pm0.8(\text{expt})\pm1.3(\text{theor})$. [S0556-2813(97)02609-5]

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In the work [1] the parity nonconserving (PNC) transition amplitude between the $6s$ and $7s$ states of the ^{133}Cs atom has been precisely measured:

$$E\equiv-\text{Im}(E1_{\text{PNC}})/\beta=1.5935(56)\ \text{mV/cm}. \tag{1}$$

They also observed the nuclear spin dependent contribution

$$\text{Im}(E1_a)/\beta=0.077(11)\ \text{mV/cm}. \tag{2}$$

This is a manifestation of parity violation in atomic nuclei and provides the first measurement of a nuclear anapole moment—an electromagnetic multipole violating the fundamental symmetries of parity and charge conjugation invariance. The anapole moment was introduced by Zel'dovich [2] just after the discovery of parity violation. He pointed out that a particle should have a parity-violating electromagnetic form factor, in addition to the usual electric and magnetic form factors. The first realistic example, the anapole moment of the nucleus, was considered in Ref. [3] and calculated in Ref. [4]. In these works it was also demonstrated that atomic and molecular experiments could detect anapole moments. Subsequently, a number of experiments were performed in Paris, Boulder, Oxford, and Seattle [5] and some limits on the magnitude of the anapole moment were established. However, the first unambiguous detection of the nuclear anapole moment (14% accuracy) has just been completed [1].

The existence of the anapole moment is due to parity nonconserving nuclear forces which create spin and magnetic moment helical structures inside the nucleus. (A detailed discussion of the spin helix produced by the weak interaction is contained in Ref. [6]). The wave function of the unpaired nucleon can be presented as (see, e.g., [4])

$$\psi=e^{i\theta\boldsymbol{\sigma}\cdot\mathbf{r}}\psi_0, \tag{3}$$

i.e., the spin $\mathbf{s}=\frac{1}{2}\boldsymbol{\sigma}$ is rotated around the vector $\mathbf{r}$. Here the angle of rotation $2\theta r$ is proportional to the strength of the weak interaction [$\theta=-(G/\sqrt{2})g\rho$, see Eq. (17)] and ψ_0 is the unperturbed wave function. The correction to the electromagnetic currents due to this spin rotation has a toroidal structure. The toroidal electromagnetic current density $\mathbf{j}$ produces a magnetic field inside the torus like that inside a classical toroidal coil. In the limit of a pointlike nucleus the vector potential corresponding to this magnetic field can be presented as [3,4]

$$\mathbf{A}=\mathbf{a}\delta(r),$$

$$\mathbf{a}=-\pi\int\mathbf{j}(\mathbf{r})r^2d^3r=\frac{1}{e}\frac{G}{\sqrt{2}}\frac{K\mathbf{I}}{I(I+1)}\kappa_a, \tag{4}$$

where $\mathbf{a}$ is an anapole moment vector directed along the nuclear spin $\mathbf{I}$, $K=(I+\frac{1}{2})(-1)^{I+1/2-l}$ (l is the orbital angular momentum of the external nucleon), and e is the electric charge of the proton. We separated the Fermi constant of the weak interaction (G) and introduced the dimensionless constant κ_a. The operator of the anapole moment, $\hat{\mathbf{a}}$ ($\mathbf{a}=\langle\psi|\hat{\mathbf{a}}|\psi\rangle$) is given by the following formula [7]:

$$\hat{\mathbf{a}}=\frac{\pi e}{m}\left[\mu(\mathbf{r}\times\boldsymbol{\sigma})-\frac{q}{2}(\mathbf{p}r^2+r^2\mathbf{p})\right], \tag{5}$$

where m is the mass of a nucleon, $\mathbf{r}$ and $\mathbf{p}$ are the position and momentum operators of the nucleon, μ is the nucleon magnetic moment in nuclear magnetons, and $q=0$ (1) for a neutron (proton). The dominant contribution to the nuclear anapole is given by the spin current [the first term in Eq. (5)]. The contribution of the second term (the convection or orbital current contribution) is very small. Moreover, to a large extent it is canceled out by the contribution of the contact current (see Refs. [3,4,8]). The only other sizable contribution is due to the spin-orbit current considered in Ref. [8] and is about −20% of the dominant spin contribution.

The interaction between atomic electrons and the magnetic field of the nuclear anapole produces a nuclear spin dependent PNC effect in atoms, which was first calculated in Ref. [9] and has been measured in Ref. [1]. The PNC amplitudes for different hyperfine transitions were found to be different. This difference is produced by the magnetic interaction of the atomic electron and the anapole vector potential $\mathbf{A}$:

$$V_a=e\boldsymbol{\alpha}\cdot\mathbf{A}=e\boldsymbol{\alpha}\cdot\mathbf{a}\delta(r)=\frac{G}{\sqrt{2}}\frac{K\mathbf{I}\cdot\boldsymbol{\alpha}}{I(I+1)}\kappa_a\delta(r). \tag{6}$$

Note that there are other mechanisms that produce (small) atomic effects similar to the anapole moment. This means that the atomic electron's interaction with the nucleus should

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actually be described by Eq. (6) with κ_a replaced by a new constant, κ (more on this below).

Accurate atomic calculations of the PNC effect produced by the interaction (6) have been done in Refs. [10–13]. The result of the many-body calculation in Ref. [11] is very close to the semiempirical calculation in Ref. [10]. The result of the Hartree-Fock calculation [13] differs by about 10% since it does not include many-body corrections. To reduce the theoretical error we calculate the ratio of the nuclear spin dependent PNC amplitude to the main spin independent PNC amplitude. Using the most complete many-body calculation of the nuclear spin dependent PNC amplitude [11], the calculation of the main PNC amplitude [14] (which was done using the same method and computer codes) and the experimental data for different hyperfine transitions from Ref. [1] we obtain the following equations:

$$E(1+0.05814\kappa)=1.6349(80)\ \ \mathrm{mV/cm}\quad (\text{for the } 6S_{F=4}\rightarrow 7S_{F=3}\ \text{transition}),$$

$$E(1-0.05148\kappa)=1.5576(77)\ \ \mathrm{mV/cm}\quad (\text{for the } 6S_{F=3}\rightarrow 7S_{F=4}\ \text{transition}). \tag{7}$$

The solution to these equations is

$$E=1.5939(56)\ \ \mathrm{mV/cm}, \tag{8}$$

$$\kappa=0.442(63). \tag{9}$$

The calculated ratio of the nuclear spin dependent PNC amplitudes in Eq. (7) to the main PNC amplitude (E) is known very accurately, i.e., there is practically no theoretical error in the extracted value of κ. This value of κ contains three contributions:

$$\kappa=\kappa_a-\frac{K-1/2}{K}\kappa_2+\frac{I+1}{K}\kappa_Q, \tag{10}$$

where $K=4$ and $I=\frac{7}{2}$ for ^{133}Cs, κ_a is the anapole moment contribution (4), $\kappa_2=1.25(2\sin^2\theta_W-\frac{1}{2})\approx-0.05$ is the contribution of that part of the weak electron-nucleus interaction that depends on the nuclear spin (see, e.g., [6,9]), and κ_Q is the contribution of the combined action of the nuclear spin independent electron-nucleus weak interaction and the hyperfine interaction [15] (see also Refs. [16,17]):

$$\kappa_Q=-\frac{1}{3}Q_W\frac{\alpha\mu_N}{mR_N}=2.5\times10^{-4}A^{2/3}\mu_N=0.017. \tag{11}$$

Here Q_W is the weak charge of the nucleus, $\alpha=e^2=1/137$, $R_N=r_0A^{1/3}$ is the nuclear radius, and μ_N is the magnetic moment of the nucleus in nuclear magnetons (for ^{133}Cs $\mu_N=2.58$). The value of κ_Q obtained in the more complete calculation in Ref. [17] is about 1.5 times larger (as it contains some average radius of the nucleon distribution, $\overline{R}$ instead of R_N, as in the above equation), i.e., $\kappa_Q\approx0.025$. From the above results it follows that $\kappa_a=0.370(63)$.

The Hamiltonian of the electron-nucleon interaction (6) is presented for a pointlike nucleus. However, a real nucleus has a finite size. Therefore, the ‘‘anapole moment’’ measured in the experiment [1] is in fact different than the anapole moment defined in Eq. (4). The ‘‘anapole moment’’ that was measured in the experiment can be defined as [18]

$$\tilde{\mathbf{a}}=-\pi\int \mathbf{j}(\mathbf{r})r^2[1-Z^2\alpha^2u(r)]d^3r\approx(1-0.3Z^2\alpha^2)\mathbf{a}, \tag{12}$$

where $u(r)\approx\frac{1}{4}(r/R_N)^2-\frac{1}{30}(r/R_N)^4$. For Cs $Z^2\alpha^2=0.16$. The interaction due to this ‘‘anapole moment’’ is just Eq. (6) with $\mathbf{a}$ replaced by $\tilde{\mathbf{a}}$, i.e., the ‘‘anapole moment’’ is placed at the center of the nucleus. However, in the previous atomic calculations [10–13] a different ‘‘regularization’’ prescription was used: $\delta(r)$ was replaced by a finite range function, $\tilde{\delta}(r)$ that has the shape of the nuclear density. The electron part of the anapole moment interaction (6) mainly mixes $s_{1/2}$ and $p_{1/2}$ electron orbitals. Using the electron wave functions inside the nucleus presented in Ref. [6] we have

$$\langle\psi_s|e\boldsymbol{\alpha}\cdot\tilde{\mathbf{a}}\delta(r)|\psi_p\rangle=\frac{\langle\psi_s|e\boldsymbol{\alpha}\cdot\tilde{\mathbf{a}}\tilde{\delta}(r)|\psi_p\rangle}{1-0.4Z^2\alpha^2}$$
$$=\frac{1-0.3Z^2\alpha^2}{1-0.4Z^2\alpha^2}\langle\psi_s|e\boldsymbol{\alpha}\cdot\mathbf{a}\tilde{\delta}(r)|\psi_p\rangle. \tag{13}$$

This means that to accurately take into account the finite nuclear size the results of the atomic calculations of the anapole moment contribution [10–13] should be multiplied by $(1-0.3Z^2\alpha^2)/(1-0.4Z^2\alpha^2)\approx1+0.1Z^2\alpha^2=1.016$. Therefore the true value of κ_a will be 1.6% smaller than 0.37:

$$\kappa_a=0.364(62). \tag{14}$$

The value 0.36 has also been obtained in [19].

In Ref. [4] analytical and numerical calculations of κ_a have been done. The approximate analytical formula was obtained by using the wave function (3) to calculate the mean value of the anapole moment operator (5). The result is

$$\kappa_a=\frac{9}{10}\frac{\alpha\mu}{mr_0}A^{2/3}g_p=0.08g_p, \tag{15}$$

where μ is the magnetic moment of the external nucleon in nuclear magnetons and $r_0=1.2$ fm. The more accurate numerical calculations [4,8] in a Saxon-Woods potential with a spin-orbit correction give the following for ^{133}Cs:

$$\kappa_a=0.06g_p. \tag{16}$$

Here g is the dimensionless strength constant in the weak nucleon-nucleus potential:

$$\hat{W}=\frac{G}{\sqrt{2}}\frac{g}{2m}[\boldsymbol{\sigma}\cdot\mathbf{p}\rho(r)+\rho(r)\boldsymbol{\sigma}\cdot\mathbf{p}], \tag{17}$$

where $\rho(r)$ is the number density of core nucleons ($g=g_p$ for a proton).

The proton-nucleus and neutron-nucleus constants can be expressed in terms of the meson-nucleon parity nonconserving interaction constants [4,20] (we use the notation of Ref. [21]):

$$g_p=2.0\times10^5 W_\rho\left[176\frac{W_\pi}{W_\rho}f_\pi-19.5h_\rho^0-4.7h_\rho^1+1.3h_\rho^2-11.3(h_\omega^0+h_\omega^1)-1.7h_\rho^{1'}\right],$$

$$g_n=2.0\times10^5 W_\rho\left[-118\frac{W_\pi}{W_\rho}f_\pi-18.9h_\rho^0+8.4h_\rho^1-1.3h_\rho^2-12.8(h_\omega^0-h_\omega^1)+1.1h_\rho^{1'}\right]. \tag{18}$$

The parameters W_ρ and W_π are present in the above equation to take into account the nucleon-nucleon repulsion at small distances and the finite range of the true interaction potential. As in Ref. [4], we use the calculations of PNC for neutron and proton scattering on ^{4}He [22], and take $W_\rho=0.4$ and $W_\pi=0.16$. Using the ''best'' values of the f and h constants listed in Ref. [21] (from here on we will refer to these as the DDH ''best'' values) one obtains $g_p=4.5$, $g_n=0.2$, and $\kappa_a=0.27$. Note that this is a single-particle shell-model value of the anapole moment constant. Shell-model calculations usually have an accuracy of about 30%. Thus, the agreement between the experimental value (0.364 ± 0.062) and the theoretical value (0.27) is as good as could be expected. (Moreover, it was shown in Ref. [23] that the RPA corrections to the weak potential increase g_p by 30%, thus κ_a could be increased to very close to the central experimental number of 0.364.)

Comparing the measured value of κ_a (14) with the theoretical expression (16) gives

$$g_p=6\pm1(\text{expt}). \tag{19}$$

We do not present here the theoretical error from the nuclear calculation of κ_a (about 30%).

Now we can use the expression for g_p in terms of the meson-nucleon interaction constants to find f_π. It was stated in the recent review [24] that experiments give values of the ρ and ω weak constants very close to the DDH ''best'' values (these constants can be found from, e.g., p-p and p-α PNC experiments). The contribution of ρ and ω to g_p is $g_p(\rho,\omega)=2$. The main controversy is about the value of $f_\pi\equiv h_\pi^1$. Comparison between Eqs. (19) and (18) gives

$$f_\pi=(g_p-2)\times1.8\times10^{-7}=[7\pm2(\text{expt})]\times10^{-7}. \tag{20}$$

We stress once more that the theoretical error in the nuclear calculation of κ_a is ignored here. Then, using this value of f_π and the DDH ''best'' values of h_ρ and h_ω in Eq. (18) we obtain

$$g_n=-0.38\times10^7 f_\pi+1.9=-0.9\pm0.7(\text{expt}). \tag{21}$$

There are other nuclear calculations of κ_a [25,12,17,26]. Reference [25] contains a detailed calculation of the π-meson contribution to the anapole moment and Refs. [12,17] include some configuration mixing effects. The most complete calculation of the anapole moment has been done in Ref. [26]: they included all single-particle contributions (spin, spin-orbit, convection, and contact currents) and many-body corrections in the RPA approximation (e.g., the induced PNC interaction and the above-mentioned RPA renormalization of the weak potential, which were considered in Refs. [23,27]). For comparison, it is convenient to present the result of their calculation in a form that stresses the role of g_p:

$$\kappa_a=0.05(g_p+0.16g_n-0.07g_{pp}-0.01g_{np}), \tag{22}$$

where g_{pp} and g_{np} are the constants of the proton-proton and neutron-proton weak interactions; these are related to g_p and g_n by the formulas $g_p=(Z/A)g_{pp}+(N/A)g_{pn}$ and $g_n=(Z/A)g_{np}+(N/A)g_{nn}$ (see Ref. [28]). The authors of Ref. [26] estimated the theoretical error in Eq. (22) as smaller than 20%. For the DDH ''best'' values of the meson-nucleon weak interaction constants we have $g_n=0.2$, $g_{pp}=1.5$, and $g_{np}=-2.2$ [28] and so we obtain

$$\kappa_a=0.05(g_p-0.05). \tag{23}$$

Thus, to an accuracy of $\sim1\%$ κ_a is still proportional to g_p. Comparing this with the experimental value of κ_a in Eq. (14) we obtain

$$g_p=7.3\pm1.2(\text{expt})\pm1.5(\text{theor}). \tag{24}$$

Once again we can use the value of g_p to find a value of f_π. Comparing the expression for g_p (18) with its numerical value (24) we obtain

$$f_\pi\equiv h_\pi^1=[9.5\pm2.1(\text{expt})\pm3.5(\text{theor})]\times10^{-7}. \tag{25}$$

We increased the theoretical error here from 2.7 to 3.5 to take into account the uncertainty in the relation between g_p and f_π (18). As before, we use this value of f_π and the DDH ''best'' values of h_ρ and h_ω in Eq. (18) and we obtain

$$g_n=-1.7\pm0.8(\text{expt})\pm1.3(\text{theor}). \tag{26}$$

We have presented two sets of estimates of g_p, f_π, and g_n to give an indication of the possible spread of the results due to theoretical uncertainty. These two sets of results agree with each other to within their errors. In the abstract we presented values based on the more complete many-body calculations.

Now we will compare our estimates of f_π, Eqs. (20) and (25), with other estimates in the literature. There is no contradiction between these values of f_π and the QCD calculations, which give $f_\pi\equiv h_\pi^1=5$–6×10^{-7} [29,30]. The DDH ''best'' value of f_π is $f_\pi = 4.6 \times 10^{-7}$. However, there are also smaller estimates of f_π in the literature, going down to the value $|f_\pi| < 1.3 \times 10^{-7}$ derived from a ^{18}F PNC measurement (see, e.g., the review [31]).

Note that there could also be a more exotic interpretation of the results of the κ measurement: κ_2 may not be described by the standard electroweak theory and so may have a larger magnitude, thus implying a smaller value of κ_a [see Eq. (10)], and hence f_π. However, such an explanation would be very improbable since the results of measurements of atomic weak charges and PNC in deep inelastic electron-nucleon scattering agree with the standard model. To clear this question it would be interesting to measure the anapole moment of the ^{207}Pb nucleus, which contains an external neutron. The constant g_n contains f_π with a negative sign in this case [see Eq. (18)].

Just before the submission of this paper it was brought to our attention that an analysis of nucleon weak interactions, based on the experiment [1], has also been done in the recent work [32].

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