Average strength of the effective interaction in multistep direct reactions

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The effective *NN*-interaction strength for the first *NN* collision in multistep reactions, averaged along the trajectory of the incident nucleon with respect to both the nuclear density and the first *NN*-collision probability, is obtained. Good agreement between the calculated values of this quantity and its phenomenological values is found. The account for the finite-range corrections to the involved local density approximation has a small effect on the average strength of the *NN* interaction. It is concluded that the nuclear-density dependence of the effective *NN* interaction may account for the phenomenological V_0 values which have been underestimated at low energies when the trend of the optical-model potential only has been followed. [S0556-2813(97)00309-9]

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Various semiclassical models and quantum-statistical theories (e.g., [1]) describe the energy equilibration in nuclear reactions by a sequence of nucleon-nucleon (NN)interactions leading to particle-hole excitations. A real finiterange Yukawa potential with a range parameter $r_0 = 1$ fm has been generally used within the multistep-direct (MSD) and multistep-compound (MSC) reaction theory of Feshbach, Kerman, and Koonin (FKK) [2]. The respective two-body interaction strength V_0 has been the only free parameter of the FKK calculations adjusted to reproduce the experimental data. However, there exist quite large discrepancies in the systematics of the phenomenological V_0 values although a consistent model-parameter set as well as several other effects has been considered [3,4]. A particular meaning in this respect has the eventual scaling of V_0 in order to compensate for some effects which have been neglected and should be added to the theory [5,6]. Here we should mention that the microscopic calculations of V_0 [7] lead to unsatisfactory results for the real proton-nucleus scattering potentials. Meanwhile, it has been found that the so-called M3Y interaction may perhaps not be as good as assumed [6].

The monotonic decrease of V_0 with the projectile energy is consistent with a similar trend of the real part of the nucleon optical-model potential (OMP), in agreement with the simple folding model [8]. Cowley et al. [9] assumed that V_0 has the same energy variation as the real optical potential, normalized to the value obtained by Austin [8] from a survey of the inelastic proton scattering to discrete final states in the energy range 20-50 MeV. On the other hand, the different behavior of the actual V_0 systematics below 30 MeV has recently been inquired [10]. Thus, a strength \overline{V}_0 averaged along the trajectory of the incident nucleon with respect to both the nuclear density and the first NN-collision probability has been obtained in the local-density approximation (LDA). The fulfilled conditions under which one may consider the NN collision in the nucleus as localized and adopt actually the basic assumptions of the semiclassical distorted wave (SCDW) model [11] have been discussed in Ref. [10]. First results from the applications of the \overline{V}_0 calculated values to analyses of preequilibrium-reaction cross sections have been shown in [12]. The aims of the present work are (i) to complete the study of the average *NN*-interaction strength using various realistic optical potentials, (ii) to include finiterange corrections [13] to the LDA considerations, (iii) to study the surface effects on \overline{V}_0 calculated within both the LDA and the improved local-density approximation (ILDA), and (iv) to give a comparison of the predicted and phenomenological values of V_0 .

In a finite nucleus (FN) the simple folding model approximation for the real part of the optical potential, in terms of the nuclear density and effective *NN* interaction [13],

$$V^{\rm FN}(r) = \int d\mathbf{r}' \ \rho(\mathbf{r}') v_0(|\mathbf{r} - \mathbf{r}'|), \qquad (1)$$

becomes in the frame of the local density approximation (e.g., [14]),

$$V^{\text{LDA}}(r) = \rho(r) \int d\mathbf{r}' \ v_0(r'). \tag{2}$$

In the case of the 1-fm-range Yukawa *NN* interaction this relation leads to the following radial and energy dependence of the effective strength:

$$V_0(\rho) = \frac{1}{4\pi} \frac{V^{\text{LDA}}(r)}{\rho(r)}.$$
 (3)

On the other hand, the Brueckner-Hartree-Fock nuclear matter calculations performed by Jeukenne *et al.* [13] showed that the contribution of the OMP isoscalar component in Eq. (3) can be parametrized so that we may have [10]

$$V_0(\rho) = \frac{1}{4\pi} F(E_i) \left[1 - d\rho^{2/3}(r) \right], \tag{4}$$

where d=2.03 fm² and $F(E_i)=(903-7.67E_i+0.022E_i^2)$ MeV fm³ in the energy range [10,140 MeV]. This form of

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FIG. 1. Radial dependence of the first *NN*-collision probability for 20 MeV incident neutrons and protons on 93 Nb. The OMP parameter sets of Lagrange-Lejeune [19] (LL), Walter-Guss [20] (WG), and Becchetti-Greenlees [21] (BG) have been used. The arrow indicates the value of the half-density radius [17] of the nuclear-matter density distribution for 93 Nb.

the local density-dependent strength of the 1-fm-range Yukawa interaction is in agreement with Myers' expression [15], used first in FKK-MSD analyses by Bonetti and Colombo [16]. The parametrization of Negele [17] has been used to describe the realistic nuclear matter distribution.

The method to calculate the average strength of the effective NN interaction along the trajectory of the incident nucleon in a complex distorting optical potential [10,12] originates from the geometry-dependent hybrid (GDH) model [18] for preequilibrium emission (PE) processes. Thus, *l*-dependent average Fermi energies given by the LDA were introduced. Additionally we have derived average quantities of interest for the reaction description by introducing the radial-dependent probability P(r) of the first twobody collision between the projectile and one of the target nucleons. The form and behavior of the probability P(r) at low incident energies has pointed out [10] a surface character of the respective PE. This result is illustrated in Fig. 1 for 20 MeV nucleons on ⁹³Nb by means of three realistic OMP's which are chosen mainly due to the possibility to provide both neutron and proton optical potentials with asymmetry terms being different only by the opposite signs. The first is the phenomenological local OMP parameter set obtained by Lagrange and Lejeune [19] for ⁹³Nb, in close agreement with the microscopic OMP of Jeukenne *et al.* [13]. The second is the global OMP of Walter and Guss [20], proposed as part of the standard parameter set which can be used for systematic FKK analyses [3]. The last one is the well-known global OMP of Becchetti and Greenless [21].

Under these circumstances, the effective NN-interaction strength averaged along the trajectory of the incident nucleon



FIG. 2. Comparison of the effective *NN*-interaction strengths as functions of the incident energy, obtained from FKK analyses of nucleon-induced reactions on ⁹³Nb using a 1-fm-range Yukawa form factor, with the calculated (a),(d) only energy-dependent V_0 values given by the volume integral per nucleon of the real optical potentials (used in Fig. 1), and the average values \overline{V}_0 of the local strengths provided by the same potentials while the first *NN*-collision probability is either (b),(e) not taken into account, i.e., P(r)=1, or (c),(f) it is considered as given by the respective OMP parameters. The phenomenological values are from Watanabe *et al.* [3] (solid triangles and squares), Demetriou *et al.* [23] (open triangles), Chadwick *et al.* [24,25] (open squares, solid circles), and Cowley *et al.* [9] (open circles).

with respect to both the nuclear density and the first *NN*-collision probability becomes

$$\overline{V}_0 = \frac{\int d\mathbf{r} \ \rho(\mathbf{r}) P(r) V_0(\rho)}{\int d\mathbf{r} \ \rho(\mathbf{r}) P(r)} = \frac{\int_0^{R_s} dr \ r^2 \rho(r) P(r) V_0(\rho)}{\int_0^{R_s} dr \ r^2 \rho(r) P(r)}.$$
(5)

The upper limit of the integral is chosen to be equal to the radius $R_s = r_D A^{1/3} + 6a_D$ at which the surface part of the imaginary potential is 1% of its central depth. The present formalism makes it possible to integrate over the whole nuclear volume without any additional assumption about the localization of the first *NN* collision [22].

The calculated average values \overline{V}_0 corresponding to the target nucleus ⁹³Nb are compared in Fig. 2 with phenomenological V_0 values obtained from (n,n') and (p,p') reaction analyses. Actually, some phenomenological values [3] correspond to the FKK analysis of (p,p') processes on ⁹³Nb, ⁹⁸Mo, and ¹⁰⁶Pa. However, since neither the phenomenological V_0 values nor the calculated average values \overline{V}_0 show a significant dependence on the target atomic mass, we have involved in the present study only the calculated \overline{V}_0 for the target ⁹³Nb. The comparison carried out in the energy range of the Walter-Guss OMP is especially meaningful for low energies where the two-step contribution to the MSD process is negligible. We begin with the values of the 1-fm-range Yukawa-interaction strength given by the volume integral per nucleon of the respective real optical potentials, which are shown in Figs. 2(a) and 2(d). Next, the average values \overline{V}_0 of the local strengths (3) provided by the same potentials are shown in two cases. First, there are shown in Figs. 2(b)



FIG. 3. Same as Fig. 2, except the local strengths are provided by the parametrization based on the Brueckner-Hartree-Fock nuclear matter calculations within (a),(c) LDA and with the first *NN*-collision probability P(r) given by the respective OMP parameters, and (b),(d) including finite range corrections to the LDA, and by the use of P(r) corresponding to the optical potential of Lagrange-Lejeune [19].

and 2(e) the values which are given by Eq. (5) irrespective of the position where the first *NN* collision takes place, i.e., for P(r)=1. Then, the curves in the Figs. 2(c) and 2(f) are obtained by means of the probability function P(r) calculated using the corresponding potential.

The average strengths obtained by using the parametrization (4) based on the Brueckner-Hartree-Fock nuclear matter calculations are shown in Figs. 3(a) and 3(c). They correspond to the probabilities P(r) given by each of the OMP's discussed [19–21]. Similar to the results in Fig. 2, an improvement can be seen of the agreement between the calculated and the phenomenological V_0 values at lower energies when not only the nuclear-density dependence of the *NN*-interaction strength is taken into account but also the surface localization of the first *NN* collision (Fig. 1). This remark may be related to the well-known increase of the effective *NN* interaction when the nuclear density reduces [15].

One comment may concern the underestimation of the phenomenological V_0 values for the (p,p') reactions (at the lowest energies) by the average values obtained by using the parametrization based on nuclear matter calculations [13]. It may correspond to the similar behavior of the expression $F(E_i)$ fitting the isoscalar real potential (Fig. 1 of [13]) at small nuclear density. The mentioned shortcoming originates from the properties of the effective *NN* interaction at low densities for which the nuclear matter results are not reliable [7].

The lack of reliable methods by means of which the nuclear matter results can be mapped onto finite-nuclei theory has led to the use of the LDA in many applications. The successful GDH model [18] is based on the LDA in order to account for the PE surface effects despite the LDA not being accurate in the surface region [13]. In order to take

into account more accurately the effect of the nonuniform medium on the effective NN interaction, Jeukenne *et al.* introduced a range of the interaction in a semiphenomenological way using a Gaussian form factor [13]

$$V_0(\rho, |\mathbf{r} - \mathbf{r}'|) = (t\sqrt{\pi})^{-3} \exp(-|\mathbf{r} - \mathbf{r}'|^2 / t^2) \ V_0(\rho), \quad (6)$$

where the range parameter t = 1.2 fm, and $V_0(\rho)$ is the LDA form defined by either Eq. (3) or Eq. (4). Within this ILDA with an account for finite-range effects, the nominator of Eq. (5) becomes

$$\int_{0}^{R_{s}} dr \ r^{2} \rho(r) P(r) \int_{0}^{R_{s}} d\mathbf{r}' (t \sqrt{\pi})^{-3} \\ \times \exp(-|\mathbf{r} - \mathbf{r}'|^{2} / t^{2}) V_{0}(\rho(r')) \\ = \frac{2}{t \sqrt{\pi}} \int_{0}^{R_{s}} dr \ r \ \rho(r) P(r) \int_{0}^{R_{s}} dr' \ r' \\ \times \exp[-(r^{2} + r'^{2}) / t^{2}] \sinh(2rr' / t^{2}) V_{0}(\rho(r')).$$
(7)

The results for the ILDA average values of the local effective *NN*-interaction strength, Figs. 3(b) and 3(d), show an improvement of the agreement with the phenomenological V_0 values at low energies by around 3% in the case when the parametrization of the Brueckner-Hartree-Fock nuclear matter calculations are used. In the other cases the V_0 values are almost unchanged. The small changes of \overline{V}_0 when the suggested ILDA is used in comparison with the LDA predictions can prove therefore the correctness of the average quantities which are introduced and used in our method.

We have investigated surface effects on the average strength of the effective NN interaction when its nucleardensity dependence is taken into account besides the energy dependence. Both types of local effective NN-interaction strengths given by the real part of the OMP or based on the Brueckner-Hartree-Fock nuclear matter calculations have been considered. Three of the most widely used optical potentials have been involved in the present analysis in order to establish the respective sensitivity of the results. Good agreement between the average strength of the effective NN interaction and the phenomenological values has been obtained. It is pointed out that the account for the finite-range corrections to the LDA (in the approach used) has a small effect. We have concluded that the nuclear-density dependence of the effective NN interaction may account for the low-energy phenomenological V_0 values which have been underestimated in other studies. The suggested method can provide the strength values requested in order to obtain the agreement between the experimental data and the FKK multistep reaction theory without using free model parameters.

Actually, the specific experimental data could be equally well reproduced in terms of different approaches by adjustment of parameters always involved even in the "parameterfree" models [26]. This comes from various assumptions on both the *special* PE parameters and *external* ones which describe general nuclear quantities [27]. The effects of the latter class on calculated cross sections can be minimized by using the fit of different types of independent experimental data in order to establish or validate a consistent set of external parameters as the OMP parameter sets, partial state density (PSD) parameters, and PSD spin distributions [12,22,28,29]. However, even following this way, different assumptions considered for the external parameters (e.g., single-particle state density) lead to different V_0 values solely adjusted in order to reproduce the experimental data [28]. It is the final aim of this work to obtain on a different

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basis a definite V_0 value which should be used for eventual further studies of the general nuclear quantities involved as external parameters.

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