Freeze-out time in ultrarelativistic heavy ion collisions from Coulomb effects in transverse pion spectra

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The influence of the nuclear Coulomb field on transverse spectra of π^+ and π^- measured in Pb+Pb reactions at 158*A* GeV has been investigated. Pion trajectories are calculated in the field of an expanding fireball. The observed enhancement of the π^-/π^+ ratio at small momenta depends on the temperature and transverse expansion velocity of the source, the rapidity distribution of the net positive charge, and mainly the time of the freeze-out. [S0556-2813(97)00109-X]

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In heavy ion collisions at ultrarelativistic energies, nuclear systems with a total net positive charge of around $Z=160$ can now be created and studied. It has recently been observed [1] in central Pb+Pb collisions at CERN-SPS energies (158*A* GeV) that the ratio of negative pions to positive pions is significantly increased at low transverse masses ($m_\perp \lesssim m_\pi + 50$ MeV) for central rapidities. A possible cause for this effect is the Coulomb interaction between the produced pions and the positive charge from the reaction partners. Such effects are already known in reactions at intermediate energies [2].

If the two incoming ions pass through each other without stopping, the combined electromagnetic field of the two moving ions has only a minimal effect on the motion of the pions that are produced around midrapidity. On the average, a charged pion experiences at time τ_f and transverse distance R a momentum change of $\sim Ze^2R/(\gamma_{\rm c.m.}\tau_f)^2$, where Z is the combined charge of the target and projectile nuclei and $\gamma_{\rm c.m.}$ is the Lorentz factor. This effect is inversely proportional to the bombarding energy. On the other hand, if the colliding ions are strongly decelerated (large nuclear stopping), the total charge stays together for a sufficient amount of time to accelerate or decelerate the produced charged pions significantly. Under such conditions distortions of the pion spectra can occur, leading to nonuniform pion yield ratios.

In this paper we report on a study within a dynamical model of the enhancement of the π^-/π^+ ratio as a function of the properties of the participant fireball and investigate the sensitivity to dynamical features such as the total participant charge, the transverse expansion velocity, and the temporal and spatial extent of the source.

In relativistic heavy ion collisions at SPS energies most of the pions are produced rather late in the collision history. They are formed either directly from strings or in the decay of resonances. In a dense medium they undergo several collisions so that the pionic ensemble comes close to thermal equilibrium, as can be deduced from properties of measured transverse spectra. As a result of the long range nature of the Coulomb force, the details of the production mechanism are not very important for our purpose. We thus consider the system at a proper time τ_f sufficiently large that the violent stage of the collision has ceased, i.e., close to freeze-out. Already at this time a certain charge separation has occurred during the equilibration phase due to the inhomogeneous Coulomb field. However, the main Coulomb acceleration effect occurs after freeze-out during the free motion in the field of the source. At this time the pions already have a spectrum very similar to that measured in experiment, since only the Coulomb field leads to a further significant modification of the spectrum shape, particularly for slowly moving particles.

We assume that the pions can be described by a thermal distribution (characterized by a temperature T) that is superimposed on a collective outward flow of the matter characterized by a four-velocity field u^μ:

$$u^\mu=(\gamma_\perp \cosh y', \gamma_\perp \sinh y', \mathbf{u}_\perp), \gamma_\perp=\sqrt{1+\mathbf{u}_\perp^2},$$

$$\mathbf{u}_\perp(r)=\frac{\overline{\beta}}{\sqrt{1-\overline{\beta}^2}}\frac{\mathbf{r}_\perp}{R}. \quad (1)$$

This field describes slices which move with rapidity y' in the longitudinal z direction (i.e., along the beam axis). We assume that the reaction zone has an axially symmetric cylindrical shape and that the transverse four-velocity $\mathbf{u}_\perp$ scales linearly with the radial distance $\mathbf{r}_\perp$ from the axis. The transverse velocity $\overline{\beta}$ describes the motion of a characteristic radius R.

Assuming that the rapidity slices follow a Gaussian distribution centered at the center-of-mass rapidity $y_{\rm c.m.}$ the pion source distribution at freeze-out reads

$$\frac{d^6N}{dy\,d\mathbf{p}_\perp^2\,dz\,d\mathbf{r}_\perp^2}\sim\int dy'\exp\left[-\frac{(y'-y_{\rm c.m.})^2}{2\Delta y^2}\right]\times\delta(z-\tau_f\sinh y')F\left(\frac{r_\perp}{R}\right)\times\frac{u_\mu p^\mu}{\exp[u_\mu(p^\mu-eA^\mu)/T]-1}, \quad (2)$$

where p^μ, e, and A^μ denote the pion momentum, charge, and electromagnetic four-potential of the source, respectively. The distribution is a function of the rapidity y, the transverse momentum $\mathbf{p}_\perp$, and the longitudinal and transverse extensions z and $\mathbf{r}_\perp$. In the spirit of the Bjorken picture the longitudinal distance z is related to the rapidity y' of the cells via an effective freeze-out time τ_f although in reality there will be a distribution of freeze-out times. The width Δy is taken from experiment. For the transverse density profile $F(r_\perp/R)$ we find that two different profiles of the transverse pion distribution—a sharp cutoff at radius $F(r_\perp) \sim \Theta(1-r_\perp/R)$ and a Gaussian distribution $\exp(-2r_\perp^2/R^2)$, both with the same mean square radius—give similar pion ratios within a few percent. As a result of the Coulomb potential A^μ, the shapes of the distributions are different for positively and negatively charged pions.

The subsequent motion of the pions in the expanding electromagnetic field created by the net charge is described by the four-velocity w^μ which satisfies the equation of motion in the electromagnetic potential A^μ:

$$m\frac{d}{d\tau}w^\mu = e w_\nu(A^{\nu,\mu}-A^{\mu,\nu}), \tag{3}$$

where τ denotes the proper time and m is the pion mass. The final spectra are obtained by sampling over a set of different initial conditions using Eq. (2).

The potential A^μ is obtained from the current distribution of the (positive) net charge. Most of this charge is carried by the baryons which are not fully stopped [3,4] and, thus, may have a wider rapidity distribution than the pions which are preferentially produced at midrapidity. In the case of symmetric reactions we parametrize the charge distribution as

$$\begin{aligned} f_{\rm ch}(y) = {} & Z_{\rm cent}\frac{1}{\sqrt{8\pi}\Delta y_{\rm ch}}\left[\exp\left(-\frac{(y-y_1)^2}{2\Delta y_{\rm ch}^2}\right)\right. \\ & \left. +\exp\left(-\frac{(y+y_1)^2}{2\Delta y_{\rm ch}^2}\right)\right] \\ & + z_t\delta(y-y_t) + z_p\delta(y-y_p), \end{aligned} \tag{4}$$

describing the participant net charge $Z_{\rm cent}$ in the fireball as two distributions of width $\Delta y_{\rm ch}$ centered at rapidities $\pm y_1$ in the center-of-mass system. For noncentral collisions the rest charges $z_{t,p}$ of the target and projectile continue to move with their original rapidities y_t and y_p.

The participant zone further expands due to the collective transverse flow (1) and the random thermal motion. We model the time evolution of the charge distribution by a Monte Carlo sample of charged test particles. These test particles move on straight trajectories with initial conditions given by Eq. (2) but now with the longitudinal distribution (4) instead of the single Gaussian distribution. The potential A^μ is then calculated by summing up the retarded potentials of the test particles. As a result of the large retardation effects, an essential part of the electromagnetic potential is generated by the charge distribution prior to freeze-out. We describe this situation between $0<\tau<\tau_0$ by the hydrodynamic flow (2) of the matter. During this period the transverse flow is assumed to increase linearly with time. The

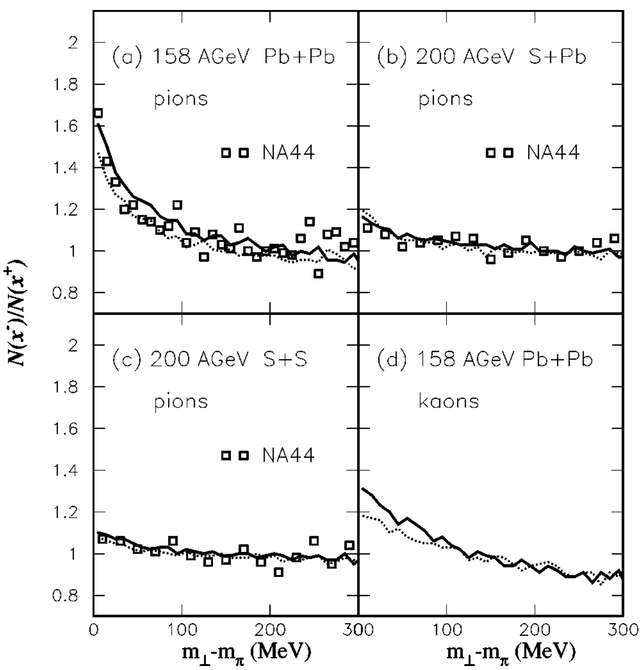

FIG. 1. Experimental π^-/π^+ ratios [1] for the reactions (a) Pb+Pb, (b) S+Pb, and (c) S+S as a function of transverse mass compared to calculations (solid lines) using the dynamical Coulomb model with a best-fit freeze-out time of 7 fm/c. The dotted lines show the results for a detector located exactly at midrapidity. Panel (d) shows the predicted arbitrarily normalized K^-/K^+ ratio for the Pb+Pb reaction.

potential depends somewhat on the particle composition of the charged zone since the thermal velocity of the particles depends on their masses. We have included a 10% excess of negative pions over positive pions compensated by the positive kaon net charge [5].

In Figs. 1(a)–1(c) we compare the result of the calculations to recent data from the NA44 experiment for Pb+Pb collisions at $E=158A$ GeV and for S+Pb and S+S at 200A GeV [1], all for central collisions and rapidities $3.3<y<4.0$. The calculations take the actual NA44 detector acceptance [1] into account. In addition, we also plot calculations corresponding to a detector located exactly at midrapidity. Since absolute π^-/π^+ ratios are not available, we normalize our calculated ratios to unity in the region 200 MeV $<m_t-m_\pi<$400 MeV, as is done for the experimental results.

Lead data were taken with a trigger selecting 15% of the total interaction cross section. In a sharp cutoff model this implies an average charge of $Z_{\rm cent}=122$ in the fireball. In the calculations we fix the charge of the participant zone to this value. We transform the transverse area into a circle of $R_{\rm geom}=5.8$ fm. At freeze-out we take a radius of $R=R(\tau_f)=R_{\rm geom}+\overline{\beta}\tau_f/2$ assuming the linear increase of the transverse velocity. The widths of the pion rapidity distributions were chosen to be $\Delta y=1.3$ corresponding to measurements in S+S and S+Pb [6] and with measured transverse energy spectra in Pb+Pb collisions [7]. We use a temperature of $T=120$ MeV and $\overline{\beta}=$ 0.62 corresponding to a mean transverse expansion velocity $\langle\beta\rangle=0.42$. These parameters are well compatible with the values obtained from a systematic analysis of measured transverse π, K, and p spectra

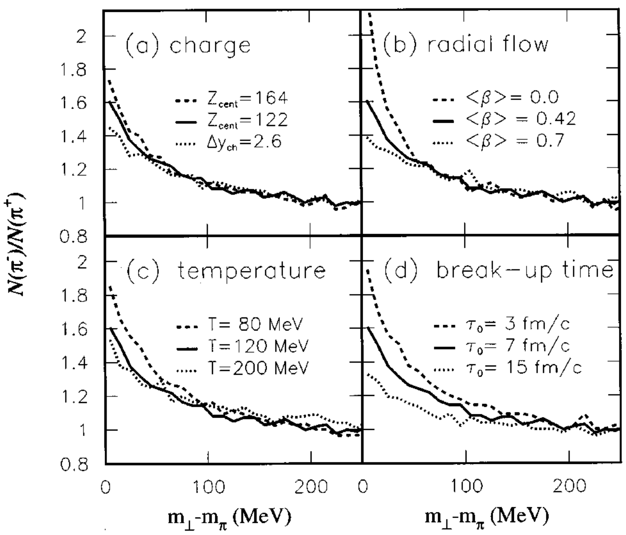

FIG. 2. Comparison of the sensitivity of the calculations to variation of the main parameters. Solid lines represent the best fit calculations for the Pb+Pb reaction shown in Fig. 1. Variation of (a) total participant charge $Z_{\rm cent}$ and width of rapidity distribution, (b) average transverse expansion velocity $\langle\beta\rangle$, (c) temperature at freeze-out, and (d) freeze-out time τ_f.

from Pb+Pb data [8], where a definite anticorrelation between extracted temperature and transverse velocity was established. Recent dN/dy vs y data from the NA44 [3] and NA49 [4] experiments provide us with the rapidity distribution of protons which can be characterized by the parameters $\Delta y_{\rm ch}=0.84$ and $y_1=1.1$, implying a rapidity loss of 1.7.

Using these values a good description of the experimental data [see Fig. 1(a)] was obtained using a freeze-out time of 7 fm/c. Figures 1(b) and 1(c) show a comparison for lighter systems. Here we use a higher temperature of 130 MeV and a smaller velocity $\langle\beta\rangle=0.29$ [6,9]. A fit to the S+S data [6] indicates a smaller rapidity loss of 1.3, leading to $\Delta y_{\rm ch}=1.25$, $y_1=1.7$. For the asymmetric S+Pb reaction a good description of the data is obtained [see Fig. 1(b)] by assuming that 30 protons from the Pb target and all protons from the projectile participate in the fireball decelerated to their respective rapidities y_1 in Eq. (4).

The temperatures, transverse flow, and charge distributions used in Fig. 1 were obtained from best fits to experimental $p_\perp$ and rapidity distributions. The effect of varying the various parameters is exhibited in Fig. 2. The magnitude of the enhancement clearly scales with the total participant charge and diminishes if the charge is distributed over a wider rapidity range [Fig. 2(a)]. If the system expands fast, the slower pions are overtaken by the expanding potential and experience a smaller net charge. This can be caused by increasing collective flow [Fig. 2(b)] or by increasing thermal motion [Fig. 2(c)] and results in a decrease of Coulomb effects. A stronger effect is observed when the freeze-out time τ_f increases [see Fig. 2(d)], reflecting a more diluted charge distribution. The profile of the rapidity distribution of the net charge also plays a role. Further, we remark that the pion ratio decreases slightly from 1.60 to 1.55 if the net charges of kaons and pions are not taken into account.

Hanbury-Brown–Twiss (HBT) analysis provides two independent comparisons of the freeze-out time. In a longitudinally expanding system with Bjorken scaling the freeze-out time is related to the longitudinal HBT radius by $\tau_f=R_l\sqrt{m_\perp/T}$. From the NA44 data $R_l\simeq5.5$ fm [10] we estimate $\tau_f\simeq8$ fm/c. From transverse HBT radii we obtain $R=2R_s\simeq10$ fm. Comparing to $R=R_{\rm geom}+\overline{\beta\tau_f}/2$ we deduce $\tau_f=8$ fm/c; this value should, however, be corrected for transverse flow which affects the HBT radii. We conclude that both freeze-out times extracted from the longitudinal and transverse HBT radii are compatible with the one we find from π^-/π^+. We also remark that calculations using the relativistic quantum molecular model (RQMD) predict substantially larger values of $\tau_f=15$ fm/c [11].

A simple estimate of the Coulomb effect can be made from the retarded electric field resulting from a net charge distribution, $dN^{\rm ch}/dy$, which is approximately constant in rapidity. Such a longitudinally streaming charge distribution generates in the transverse direction an electric field $\mathbf{E}\simeq2e(dN^{\rm ch}/dy)\mathbf{r}_\perp/(tR^2)$. Here, t is the time after collision and R is the transverse size of the charge distribution. On average this field leads to a momentum change of the $\pi^\pm$ by

$$\frac{\delta\mathbf{p}_\perp}{\mathbf{p}_\perp}\simeq\pm e^2\frac{dN^{\rm ch}}{dy}\frac{2f}{mR}\equiv\pm\Delta, \tag{5}$$

for small transverse momenta. The factor f is of order the inverse of the average proton velocity but also depends on the freeze-out time and the transverse expansion [12]. Assuming a thermal transverse momentum distribution at freeze-out, $dN/dp_\perp^2\sim\exp(-m_\perp/T)$, as has been used for the spectra [8], we obtain, for small transverse momenta after correcting for the Coulomb effect of Eq. (5),

$$\frac{N(\pi^-)}{N(\pi^+)}=\frac{dN^-/dp_\perp^2}{dN^+/dp_\perp^2}\simeq1+4\Delta\left(1-\frac{m_\perp-m}{T}+\cdots\right). \tag{6}$$

This approximate result shows that the linear decrease of the ratio takes place on a scale of $m_\perp-m\simeq T$. The enhancement for Pb+Pb collisions at midrapidities can be estimated using $dN^{\rm ch}/dy\simeq35$ [4], $R\simeq10$ fm, and $f\simeq2$ to be $4\Delta\simeq0.6$. This approximate value and Eq. (6) are in qualitative agreement with the pion ratio in Fig. 1.

We have also calculated the kaon ratio K^-/K^+ in Pb+Pb collisions using the same parameter set as used for the pions [Fig. 1(d)]. As the kaon is heavier we expect the corresponding K^-/K^+ ratio to be smaller by a factor of $m_K/m_\pi\sim3$ accordingly to Eq. (5).

In summary, we have found that the observed π^-/π^+ ratios can be qualitatively explained as resulting from the Coulomb acceleration of pions by the positive participant charge in a rapidly expanding system. We have demonstrated that a measurement of this charge ratio in large systems supplements the standard HBT analyses, providing an independent constraint on the freeze-out time scale, the flow, and the transverse size. The charge ratio is mostly influenced by the freeze-out time. The quantitative agreement with experimental Pb+Pb data requires a relatively short freeze-out time of about 7 fm/c assuming a mean transverse flow velocity of $\langle\beta\rangle=0.42$ and a temperature of 120 MeV. Such values are compatible with analyses of transverse pion, kaon, and proton spectra. Larger freeze-out times require less amount of flow. The freeze-out time obtained is compatible with freeze-out times and source sizes extracted in HBT analyses.

Finally, we note that the magnitude of the Coulomb effects in pion spectra is sensitive to the degree of stopping and the resulting distribution of the positive charge. At RHIC and LHC energies the stopping and in particular the net charge at midrapidity are expected to be smaller, and consequently we predict [see Eq. (5) and Fig. 2(a)] correspondingly smaller Coulomb effects.

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