Two-particle rapidity correlations from the Bose-Einstein effect in central ^{28}Si+Au collisions at 14.6A GeV/c and intermittency

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In previous work, the E802 Collaboration at the BNL-AGS used negative binomial distribution (NBD) fits to charged particle multiplicity distributions from central collisions of ^{16}O+Cu at 14.6A GeV/c to derive the two-particle short-range rapidity correlation length and strength. These turned out to be much shorter and weaker than the values for hadron collisions, which led to a simple and elegant explanation of intermittency. In the present work, a direct measurement of the two-particle correlation of identified pions in the E802/E859 magnetic spectrometer is performed in the interval $1.5\leqslant y\leqslant 2.0$ for central ^{28}Si+Au collisions, both in terms of $Q_{\rm inv}=\sqrt{|\vec{q}|^2-q_0^2}$, where $q=p_2-p_1=(\vec{q},q_0)$, and also in terms of $|\eta_2-\eta_1|$ and $|y_2-y_1|$, where p, η, and y are the four-momentum, pseudorapidity, and rapidity of the pions. It is demonstrated that the two-pion correlation in rapidity (and pseudorapidity) is entirely due to the Bose-Einstein interference. The directly measured correlation length in both η and y is $\xi=0.19\pm0.03$ for two π^-, with strength $R(0,0)\sim 1\%$, in agreement with the previous E802 indirect measurements derived from the NBD analysis of intermittency. [S0556-2813(97)05109-1]

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I. INTRODUCTION

The intermittency formalism as a method to study non-Poisson fluctuations of charged particle multiplicity distributions in small pseudorapidity intervals $\delta\eta\leqslant 1$ has intrigued particle and relativistic heavy ion physicists for over a decade [1]. In previous work, the E802 Collaboration [2] at the BNL-AGS analyzed the evolution of charged particle multiplicity distributions from central collisions of ^{16}O+Cu at 14.6A GeV/c as a function of the width of the pseudorapidity interval $\delta\eta$, in the range $1.2\leqslant\eta\leqslant 2.2$, both by the method of normalized factorial moments and by direct measurements of the shape of the distributions. The charged multiplicity distributions were well represented by negative binomial distributions (NBD's) and simply characterized by the NBD parameter $k(\delta\eta)$ which represents the first departure of a distribution from a Poisson fluctuation:

$$\frac{1}{k(\delta\eta)}=\frac{\sigma^2}{\mu^2}-\frac{1}{\mu}=F_2(\delta\eta)-1=K_2(\delta\eta), \tag{1}$$

where $\mu\equiv\langle n(\delta\eta)\rangle$ is the mean multiplicity on the interval,

*Deceased.

$\sigma \equiv \sqrt{\langle n^2\rangle - \langle n\rangle^2}$ is the standard deviation, $F_2(\delta\eta)$ is the second order normalized factorial moment on the interval,

$$F_2 = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2} = \frac{\langle n^2\rangle - \langle n\rangle}{\langle n\rangle^2} = \frac{\langle n\rangle^2 + \sigma^2 - \langle n\rangle}{\langle n\rangle^2}, \quad (2)$$

and $K_2(\delta\eta)$ is a normalized factorial cumulant [3]. The factorial moments of the multiplicity on an interval $\delta\eta$ are simply related to the integrals of the q-particle inclusive rapidity densities $\rho_q(y_1, \ldots, y_q)$:

$$\int^{\delta\eta} dy_1 \quad \rho_1(y_1) = \langle n\rangle, \quad (3)$$

$$\int^{\delta\eta} dy_1 dy_2 \quad \rho_2(y_1, y_2) = \langle n(n-1)\rangle = \langle n\rangle^2 F_2, \quad (4)$$

$$\int^{\delta\eta} dy_1 \cdots dy_q \quad \rho_q(y_1, \ldots, y_q) = \langle n(n-1)\cdots(n-q+1)\rangle$$
$$= \langle n\rangle^q F_q. \quad (5)$$

These integrals (or moments) are sensitive to any short-range rapidity correlation in particle production, since if there were no correlation, then

$$\rho_q(y_1, \ldots, y_q) = \rho_1(y_1)\rho_1(y_2)\cdots\rho_1(y_q), \quad (6)$$

in which case all the F_q reduce to unity, a Poisson distribution. Mueller [3] introduced a series of functions to describe correlations in multiparticle emission. For instance, the normalized two-particle short-range rapidity correlation function $R_2(y_1, y_2)$ is defined as

$$R_2(y_1,y_2) \equiv \frac{C_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} \equiv \frac{\rho_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1$$
$$= R(0,0)e^{-|y_1-y_2|/\xi}, \quad (7)$$

where $\rho_1(y)$ and $\rho_2(y_1,y_2)$ are the inclusive densities for a single particle (at rapidity y) or two particles (at rapidities y_1 and y_2), $C_2(y_1,y_2) = \rho_2(y_1,y_2) - \rho_1(y_1)\rho_1(y_2)$ is the Mueller correlation function for two particles (which is zero for the case of no correlation), and ξ is the two-particle short-range rapidity correlation length [3,4] for an exponential parametrization. These equations may be combined to yield the relationship [5,6]

$$\frac{1}{k(\delta\eta)} = K_2(\delta\eta) = F_2(\delta\eta) - 1$$
$$= \frac{\int^{\delta\eta} dy_1 dy_2 \rho_1(y_1)\rho_1(y_2) R_2(y_1,y_2)}{\int^{\delta\eta} dy_1 dy_2 \rho_1(y_1)\rho_1(y_2)}. \quad (8)$$

Thus, the evolution of the NBD parameter (or equivalently of the normalized factorial moments) with $\delta\eta$ gives a measurement of the two-particle short-range rapidity correlation.

The values of the two-particle correlation length and strength [7] determined for central ^{16}O+Cu collisions, $\xi = 0.18 \pm 0.05$ and $\overline{R(0,0)} = 0.031 \pm 0.005$, were much shorter and weaker than the values for hadron collisions. This result yielded a simple and elegant explanation of intermittency—the "large" bin-by-bin fluctuations in individual event rapidity distributions from Si+AgBr interactions in cosmic rays [8,9] are a consequence of the apparent statistical independence of the multiplicity in rapidity bins of size $\delta\eta \sim 0.2$ due to the surprisingly short two-particle rapidity correlation length.

In fact, the weakened, but finite, short-range rapidity correlations in the collisions of relativistic heavy ions had been predicted in the context of intermittency moment analyses [10–13]. In nucleus-nucleus collisions, the conventional hadron short-range correlations should be washed out by the random superposition of many sources of correlated particles [11,14,15], so that eventually only the quantum-statistical Bose-Einstein (BE) correlations of identical particles remain [10,11,16]—for example, the conventional two-particle correlations apply only to two pions from the same nucleon-nucleon collision out of many collisions in a complicated central heavy ion reaction, whereas any two identical pions are affected by the Bose-Einstein correlation [17]. As the Bose-Einstein effect represents a very-short-range correlation in the difference of the four-momenta ($q = p_2 - p_1$) of the two identical particles, it should come to dominate the conventional short-range correlation at very small intervals and should produce dramatic differences between the case of identical or nonidentical particles—even in hadron collisions—especially for intermittency analyses which study small volumes in multidimensional phase space. In fact, the relationship between intermittency and BE correlations has been convincingly demonstrated in other experiments [18–21] using nonidentified charged particles. If BE correlations were the entire effect, then direct measurements of BE correlations in terms of the pseudorapidity and rapidity differences of the two particles, $\eta_2 - \eta_1 = \Delta\eta$ and $y_2 - y_1 = \Delta y$, instead of the usual variables [22] $Q_{\rm inv}$, $|\vec{q}\,|$, q_0—where $Q_{\rm inv} = \sqrt{|\vec{q}\,|^2 - q_0^2}$, $q = p_2 - p_1 = (\vec{q}, q_0)$, and $p = (\vec{P}, E)$—should reproduce the short-range rapidity correlation parameters derived by E802 from the evolution of $k(\delta\eta)$, when adjusted for the charged particle composition. This paper presents such direct measurements of correlations in ^{28}Si+Au collisions.

II. MEASUREMENTS OF BE CORRELATIONS IN ^{28}Si+ Au COLLISIONS

The BE correlation analysis is performed for pairs of negatively charged pions detected in the E802/E859 spectrometer from 14.6A GeV/c ^{28}Si+Au$\rightarrow 2\pi^- + X$ central collisions. The centrality is defined by a target multiplicity array (TMA) which measures the nonidentified charged particle multiplicity over the polar angular interval from $6° \leq 140°$ with nearly full azimuthal coverage of 300°. For the present arrangement, which had slightly smaller azimuthal coverage than previous publications [23], central collisions are defined by the upper 10% of the distribution, which corresponds to 100 or more detected charged particles.

A sample of events with two-particles detected in the small aperture (25 msr) spectrometer was selected using a two-particle second-level trigger [24]. Tracks with a measured $1/\beta$ (where β is the velocity) within 3σ of the value for a pion with the same reconstructed momentum, for momenta below 1.82 GeV/c (the π-K 3σ crossing), are identified as pions. For the present measurement, the magnetic field in the spectrometer was set to optimize the acceptance of negative particles and the spectrometer aperture spanned polar angles from 14° to 28°, accepting pions with $150 \lesssim p_T \lesssim 700$ MeV/c and $1.5 < y < 2.0$, where the nucleon-nucleon center-of-mass rapidity is $y_{NN} = 1.7$. The azimuthal coverage ranged from 0.5 rad at the smallest polar angle to 0.25 rad at the largest, with an average value of $\delta\phi = 0.4$ rad.

The total number of π^- pairs used for this analysis was 229 210.

III. TWO-PARTICLE BOSE-EINSTEIN CORRELATION FUNCTION

Bose-Einstein intensity interferometry [25,22] exploits the fact that identical bosons emitted by a chaotic source of spatial extent R exhibit a correlation in relative momentum $q = p_2 - p_1$ which vanishes for $|q| \gtrsim \hbar/R$. The correlation function for BE interferometry measurements is closely related to the normalized two-particle correlation function $R_2(p_1, p_2)$:

$$C_2^{\mathrm{BE}}(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)} = 1 + R_2(p_1, p_2). \tag{9}$$

For $p_2 - p_1$ outside the region of BE correlation, it is assumed that there is negligible other correlation so that $\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2)$ and therefore $C_2^{\mathrm{BE}} \to 1$ and $R_2 \to 0$. For small values of the argument $p_2 - p_1$, where there is full BE correlation: $C_2^{\mathrm{BE}} \to 2$, $R_2 \to 1$. The BE correlation is traditionally represented by a Gaussian in pair relative momenta quantities, such as Q_{inv},

$$C_2^{\mathrm{BE}}(Q_{\mathrm{inv}}) = 1 + \lambda e^{-Q_{\mathrm{inv}}^2 R_{\mathrm{inv}}^2}, \tag{10}$$

where the empirical parameter $\lambda \leqslant 1$ is introduced to account for the fact that not all detected pions come from a single chaotic source [22].

IV. MEASURED TWO-PARTICLE CORRELATION FUNCTION

In order to form a correlation function corresponding to Eq. (9) from the measured sample of negative pion pairs (the *actual* distribution of two-pion events), a *background* sample must be found which exhibits all correlations induced by phase space, dynamics, experimental acceptance, etc., *except* those resulting from BE correlations [22,26]. The method chosen is that of event mixing, which was originally suggested by Kopylov [27] and subsequently used by most BE measurements in hadron or heavy ion collisions [28,29,22]. The prescription for creating the event-mixed *background* from the *actual* sample is very simple. Two events from the *actual* sample are selected at random, with a check that the same event is not matched with itself. Then one pion from each event is chosen at random to form a background pair. In order to not be limited by the statistics of the mixed events, approximately 5 times as many background pairs are formed for the present analysis [24,30], $\sim 1 \times 10^6$ (999 792, to be exact).

The measured correlation functions $\mathcal{C}_2^{\mathrm{BE}}(v)$ are defined to be the ratios of the *actual* distribution of negative pion pairs to the event mixed *background*, plotted as a function of a variable v,

$$\mathcal{C}_2^{\mathrm{BE}}(v) = \frac{A(v)}{B(v)}, \tag{11}$$

where $v = Q_{\mathrm{inv}}$, Δy, or $\Delta\eta$. We correct for the inefficient measurement of two tracks with small opening angles in the *actual* distribution. The correlation functions (numerator and denominator) are restricted to opening angles where this correction is less than or equal to 2 [24]. Precisely the same set of pion pairs, and corrections, are used for all three of the correlation functions, which are in effect just projections in the different variables. No corrections besides that for close-track inefficiency are applied to the data in this analysis—*i.e., no Coulomb corrections are applied.*

The principal advantage of reconstructing the two-particle *background* sample from the *actual* sample of two pions is that this procedure automatically solves the problem that the class of two-particle events may be different from the class of one-particle events [26,30]. For heavy ion collisions, dynamical considerations and conservation of energy are not an issue since the central events have no structure (e.g., jets) and only 2 out of the more than 100 particle are used. One problem that event mixing does not solve is that the integral of the *actual* sample of correlated pairs [Eq. (4)] is not equal to the integral of the background distribution of pairs of uncorrelated single particles [the square of Eq. (3)]—even in the ideal case—since $F_2 > 1$ for most distributions.[1] However, for the present analysis, the mixed event sample is purposely taken to be much larger than the *actual* sample, and so the correct normalization is obtained most simply by evaluating it as a parameter in the fit to the correlation function.

The measured BE correlation function [31] in Q_{inv} is traditionally fit to the Gaussian form

$$\mathcal{C}_2^{\mathrm{BE}}(Q_{\mathrm{inv}}) = \mathcal{N}[1 + \lambda_Q\, e^{-Q_{\mathrm{inv}}^2 R_{\mathrm{inv}}^2}] \tag{12}$$

or, equivalently,

$$\mathcal{C}_2^{BE}(Q_{\mathrm{inv}}) = \mathcal{N}[1 + \lambda_Q e^{-Q_{\mathrm{inv}}^2/(2\sigma_Q^2)}]. \tag{13}$$

The fit parameter $\mathcal{N}$ is the normalization constant, which just depends on the number of mixed events chosen for the *background* distribution compared to the *actual* sample. The only assumption required is that there is no other correlation outside the region of the BE correlation, so that $C_2^{\mathrm{BE}} \to 1$, $R_2 \to 0$. In practice, if the data exhibit a clear correlation peak and a significant region of constant $\mathcal{C}_2^{\mathrm{BE}}$, then the normalization constant can be well determined by this procedure [e.g., see Fig. 1(a)]. There remains the possiblity that the region of constant $\mathcal{C}_2^{\mathrm{BE}}$ corresponds, e.g., to a conventional two-particle correlation, $R_2 = \epsilon$, with much larger correlation length than the range of measurement. In this case, the above

[1]This may explain why some authors [26,30] divide the correlation function [Eq. (9)] by F_2. However, in such cases $C_2^{\mathrm{BE}} \to 1/F_2$ (rather than unity) in the region outside the BE correlation.

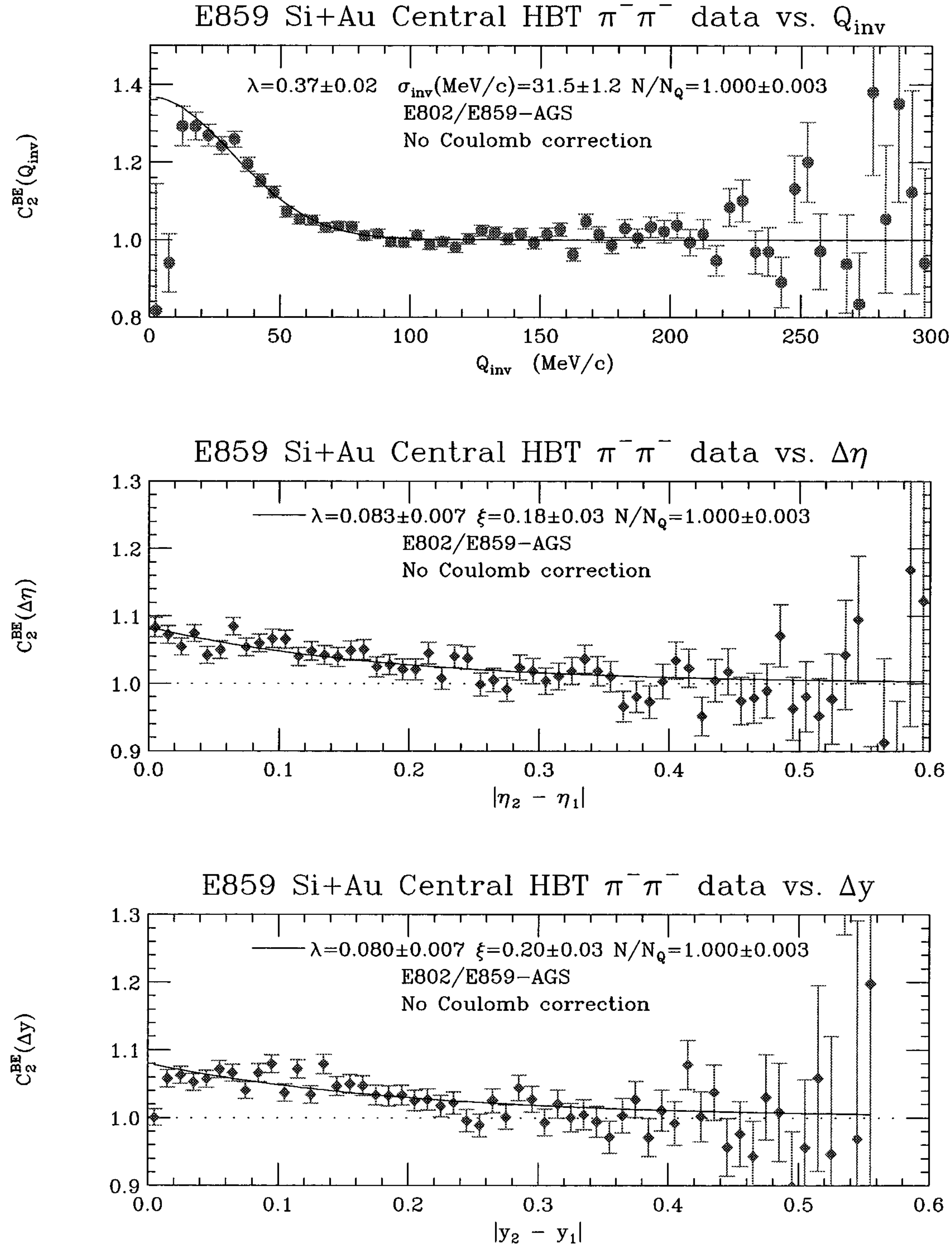

FIG. 1. The Bose-Einstein correlation function $C_2^{\rm BE}$ as a function of the three variables $Q_{\rm inv}$, $|\eta_2-\eta_1|$, and $|y_2-y_1|$ for pairs of identified π^- in central ^{28}Si+Au collisions. The lines are the fits discussed in the text. The plotted data have been divided by the fitted value of $\mathcal{N}_Q=0.222\,68$.

normalization procedure will lead to a systematic mismeasurement of λ_Q by a factor of $1/(1+\epsilon)$. It is evident, for the present analysis, that the same normalization constant (and possible systematic error on λ) applies to any projection of the correlation function, $\mathcal{C}_2^{\rm BE}(v)=A(v)/B(v)$, since precisely the same *actual* and *background* events are used for all projections.

The correlation functions in the rapidity (or pseudorapidity) are parametrized as exponential, as in Eq. (7) (although a Gaussian works just as well for the present data):

$$\mathcal{C}_2^{\rm BE}(y_1,y_2)=\mathcal{N}[1+\lambda_y\, e^{-|y_1-y_2|/\xi_y}], \tag{14}$$

$$\mathcal{C}_2^{BE}(\eta_1,\eta_2)=\mathcal{N}[1+\lambda_\eta e^{-|\eta_1-\eta_2|/\xi_\eta}]. \tag{15}$$

These are just projections of the BE correlation function onto the longitudinal direction, integrated over the limited azimuthal aperture of the spectrometer, $\delta\phi=0.40$ rad, and other variables such as $\Delta E=E_2-E_1$, the energy difference of the two pions. Thus $\lambda_y,\lambda_\eta\neq R(0,0)$, since $R(0,0)$ [Eq. (7)] represents the strength of the two-particle rapidity correlation integrated over the full azimuth, and in general λ_y, $\lambda_\eta\leqslant\lambda_Q$—a full discussion is given below in Sec. VII.

V. RELATION TO MULTIPLICITY DISTRIBUTIONS AND MOMENTS

For intermittency moment analyses and NBD fits to multiplicity distributions, the data are measured in a pseudorapidity interval of full width (denoted $\delta\eta$) in order to determine the multiplicity distribution and the two-particle normalized factorial moment (cumulant), $F_2(K_2)$, which is nothing other than the integral of the two-particle correlation function on the interval. Recalling Eq. (8),

$$K_2(\delta\eta)=F_2(\delta\eta)-1=\frac{\int^{\delta\eta}dy_1dy_2\rho_1(y_1)\rho_1(y_2)R_2(y_1,y_2)}{\int^{\delta\eta}dy_1dy_2\rho_1(y_1)\rho_1(y_2)}.$$

The integrand in the numerator,

$$\rho_1(y_1)\rho_1(y_2)R_2(y_1,y_2)=\rho_2(y_1,y_2)-\rho_1(y_1)\rho_1(y_2),$$

is just the Mueller correlation function $C_2(y_1,y_2)$ and the denominator is simply $\langle n(\delta\eta)\rangle^2$, the square of the mean multiplicity on the interval. As noted above, for the case of a NBD, $K_2(\delta\eta)=1/k(\delta\eta)$. If the inclusive single-particle density $\rho_1(y)=dn/dy$ is assumed constant on the interval, then the integral can be performed analytically (specifically on the interval $0\leqslant y_1\leqslant\delta\eta$, $0\leqslant y_2\leqslant\delta\eta$) to obtain the normalized factorial moment $F_2(\delta\eta)$ or normalized factorial cumulant $K_2(\delta\eta)$ in terms of the parameters of Eq. (7) [5,6,10,11,32]:

$$K_2(\delta\eta)=R(0,0)G(\delta\eta/\xi), \tag{16}$$

where the function $G(x)$ is defined as

$$G(x)=2\frac{(x-1+e^{-x})}{x^2}. \tag{17}$$

The correlation length ξ (actually ξ_η) and strength for central ^{16}O+Cu collisions were derived by E802 [2] from a fit of the measured $K_2(\delta\eta)=1/k(\delta\eta)$ to this integral function.

It is worth dwelling for an instant on some general properties of Eqs. (8), (16), and (17). It is clear in this formulation [Eq. (8)] that the normalized factorial cumulant $K_2(\delta\eta)$ is simply the mean value of the normalized two-particle correlation function $R_2(y_1,y_2)$ on the interval $\delta\eta$, with weighting by the square of the inclusive single-particle density:

$$K_2(\delta\eta)=\langle R_2(y_1,y_2)\rangle|_{\delta\eta}. \tag{18}$$

Similarly, the normalized factorial moment $F_2(\delta\eta)=1+K_2(\delta\eta)$ is just $\langle C_2^{\mathrm{BE}}(y_1,y_2)\rangle|_{\delta\eta}$. For constant single-particle density, the evolution of $K_2(\delta\eta)$ as a function of $\delta\eta$ depends on the scaled variable $x=\delta\eta/\xi$, and from the limits of $G(x)$ [$G(x)=1$, for $x\ll1$; $G(x)=2/x$ for $x\gg1$] is quite easy to understand:

$$K_2(\delta\eta)=\langle R_2(y_1,y_2)\rangle|_{\delta\eta}=R(0,0)\quad\text{for}\quad\delta\eta\ll\xi, \tag{19}$$

$$K_2(\delta\eta)=\langle R_2(y_1,y_2)\rangle|_{\delta\eta}=2\frac{R(0,0)}{\delta\eta/\xi}\quad\text{for}\quad\delta\eta\gg\xi\ . \tag{20}$$

For intervals much smaller than the correlation length, the average two-particle correlation strength is just the maximum value, while for intervals much larger than the correlation length, the average two-particle correlation strength becomes inversely proportional to the interval (for uniform density) since the correlation only exists for particles within approximately one correlation length of each other.

VI. RESULTS

The present measurement of the $\pi^-\pi^-$ correlation function $C_2^{\mathrm{BE}}(p_1,p_2)$ is shown in Fig. 1 as a function of the variable Q_{inv}, together with the same data plotted as a function of the pseudorapidity and rapidity differences of the two pions, $|\eta_2-\eta_1|$ and $|y_2-y_1|$. (In all three projections the plotted data have been divided by the best fit value of the normalization $\mathcal{N}_Q=0.222\,68$ whose error then appears in the fitted value of $\mathcal{N}/\mathcal{N}_Q$.) An evident correlation effect is visible for $Q_{\mathrm{inv}}\leqslant80$ MeV/c, along with a clear region of constant C_2^{BE} for $Q_{\mathrm{inv}}\geqslant100$ MeV/c, where there is no correlation so that the normalization constant $\mathcal{N}_Q$ can be precisely determined. In the $|\eta_2-\eta_1|$ and $|y_2-y_1|$ projections, the data show an $\sim$8% drop over the range from 0 to 0.5; however, it is not clear whether the correlation function has become constant or would continue to decrease for values greater than 0.5. The solid lines are obtained by a fit which constrains the normalization in all three projections to the same value $\mathcal{N}$ which is well determined in Q_{inv}. Fits obtained without this normalization constraint are inadequate to make any statistically significant conclusion for the determination of the correlation length ξ.

As emphasized above, the key point for the present analysis is that the parameter $\mathcal{N}$ just represents the relative number of events in the *actual* and *background* distributions which is identical for the projections in the three different variables Q_{inv}, $\Delta\eta$, and Δy. Thus, this constraint is applied in a common fit of $\mathcal{C}_2^{\mathrm{BE}}$ for all three projections [Eqs. (13), (14), and (15)], which allows statistically significant values λ_η, ξ_η, λ_y, and ξ_y to be obtained. The results of the fit are given in Table I for both exponential and Gaussian parametrizations of the (pseudo)rapidity correlation. To be consistent with the custom in short-range rapidity correlation analyses, no Coulomb correction is applied to the data. This correction is significant for the two lowest points in Q_{inv}, and so they (and the lowest point in Δy) are not used in the fit—thus the absence of a Coulomb correction has no effect on the determination of the key normalization parameter. The fitted exponential correlation lengths $\xi_\eta=0.18\pm0.03$ and $\xi_y=0.20\pm0.03$ are equal within errors, as they should be since the pions are relativistic. Both values (for ^{28}Si+Au

TABLE I. Fit parameters for the correlation functions with exponential form in Δy, $\Delta\eta$, and Gaussian in Q_{inv} all constrained to the same normalization constant, $\mathcal{N}$ (uppermost entry of table). The middle section of the table gives the parameters when Gaussians forms are used for all three projections. The lowest entry in the table shows the small systematic change in $\mathcal{N}$ when only the Q_{inv} data are used in the fit. The errors are statistical only. The systematic error in ξ is determined by the statistical error in $\mathcal{N}$, see text.

Projection	Form	λ	ξ or σ	$\mathcal{N}/\mathcal{N}_Q$	χ^2/NDF
$\lvert y_2-y_1\rvert$	exponential	0.080 ± 0.007	0.20 ± 0.03		
$\lvert\eta_2-\eta_1\rvert$	exponential	0.083 ± 0.007	0.18 ± 0.03		
Q_{inv} (MeV/c)	Gaussian	0.367 ± 0.020	31.5 ± 1.2	1.000 ± 0.003	198.8/169
$\lvert y_2-y_1\rvert$	Gaussian	0.065 ± 0.005	0.155 ± 0.015		
$\lvert\eta_2-\eta_1\rvert$	Gaussian	0.067 ± 0.005	0.149 ± 0.015		
Q_{inv} (MeV/c)	Gaussian	0.367 ± 0.021	31.3 ± 1.1	1.0013 ± 0.0026	181.7/169
Q_{inv} (MeV/c)	Gaussian	0.367 ± 0.021	30.9 ± 1.1	1.0034 ± 0.0031	81.5/57

central collisions) agree impressively well with the previous indirect measurement [2] (for ^{16}O+Cu central collisions). The main systematic error for the present work is the uncertainty of the normalization $\mathcal{N}$, which leads to a common (one standard deviation) systematic error in ξ_η and ξ_ϕ of ±0.01.

In order to determine whether the measured rapidity correlation is entirely due to the Bose-Einstein effect, two tests were performed. The $\pi^+\pi^-$ correlation, which has no BE interference [24,25], shows a constant value of C_2^{BE} in the $|y_2-y_1|$ projection [see Fig. 2(a)]—this clearly establishes the correlation as being an identical particle effect. Second, for the identical $\pi^-\pi^-$, the region of $Q_{\mathrm{inv}}\leqslant100$ MeV/c (where the correlation effect is exhibited) was eliminated and the data were again plotted in terms of $|\eta_2-\eta_1|$ and $|y_2-y_1|$ (Fig. 2): The data at larger values of $|\eta_2-\eta_1|$ or $|y_2-y_1|$ are identical, but the correlation at low values has vanished as illustrated by direct comparison with the uncut data points. This demonstrates that the two-pion short-range rapidity correlation in ^{28}Si+Au collisions is entirely due to the Bose-Einstein interference.

VII. DISCUSSION AND RELATION TO OTHER RESULTS

In order to relate the strength parameter λ_η [Eq. (15)] measured within the spectrometer ϕ aperture to the usual two-particle short-range strength $R(0,0)$ defined for the full azimuth [Eq. (7)], the dependence of the two-particle normalized correlation function on $\Delta\phi$ must be known. The simplest model of the source to explain all the E802/E859 two-pion BE correlation data [24] is that of a spherically symmetric Gaussian with finite lifetime:

$$R_2(|\vec{q}|,q_0)=\lambda_Q e^{-|\vec{q}|^2R^2-q_0^2\tau^2}. \tag{21}$$

This expression can be evaluated to lowest order [33] in ΔE, $\Delta\eta$, and $\Delta\phi$:

$$R_2(p_1+\Delta p,p_1)=\lambda_Q\exp-\{[(\Delta E/\beta)^2+(p_T\Delta\eta)^2+(p_T\Delta\phi)^2]R^2+[(\Delta E)^2]\tau^2\}, \tag{22}$$

which implies the relationship $\sigma_{\Delta\phi}=\sigma_{\Delta\eta}=1/(\sqrt{2}p_TR)$ and $\sigma_{\Delta E}=1/\sqrt{2(R^2/\beta^2+\tau^2)}$, where $\beta=P/E$ is the velocity of either pion [33]. From the latest E802/E859 measurement [24] using the same data as the present analysis, $R=2.80\pm0.11$ fm, with $\langle p_T\rangle=300$ MeV/c. This yields the prediction $\sigma_{\Delta\phi}=\sigma_{\Delta\eta}=0.166\pm0.007$, which is in excellent agreement with the Gaussian fit in the present analysis, $\sigma_{\Delta\eta}=0.149\pm0.015$. Consequently, we assume that $\sigma_{\Delta\phi}=\sigma_{\Delta\eta}=0.149\pm0.015$ to extrapolate from the fit within the spectrometer aperture $\delta\phi=0.40$ rad to the full azimuth.

For simplicity in averaging, Eq. (22) is converted to an exponential form [cf. Eq. (7)] by taking $\xi_\phi=0.20\simeq\xi_\eta$—the empirical ratio of ξ_η to $\sigma_{\Delta\eta}$ from the present fit (see Table I) is $1.3(\sim\sqrt{2})$—and, similarly, by assuming $\xi_E\simeq\sqrt{2}\sigma_{\Delta E}=1/\sqrt{R^2/\beta^2+\tau^2}$:

$$R_2(p_1+\Delta p,p_1)=\lambda_Q e^{-|E_1-E_2|/\xi_E}e^{-|\phi_1-\phi_2|/\xi_\phi}e^{-|\eta_1-\eta_2|/\xi_\eta}. \tag{23}$$

The average of this correlation function over the three-dimensional interval $0\leqslant\eta_1,\eta_2\leqslant\delta\eta$, $0\leqslant\phi_1,\phi_2\leqslant\delta\phi$, $0\leqslant E_1,E_2\leqslant\delta E$ is then very similar in form to Eq. (16):

$$\begin{aligned}K_2(\delta\eta,\delta\phi,\delta E)&=\langle R_2(p_1+\Delta p,p_1)\rangle|_{\delta\eta,\delta\phi,\delta E}\\&=\lambda_QH(\delta E/\xi_E)G(\delta\phi/\xi_\phi)G(\delta\eta/\xi_\eta),\end{aligned} \tag{24}$$

where $G(x)$ is the same function as Eq. (17) and $H(x)$ is another function [34,23]. Comparison of Eq. (24) to Eqs. (7)–(16) makes it clear that the determination of λ_η involves an average over δE as well as $\delta\phi$:

$$R(0,0)=\lambda_QH(\delta E/\xi_E)G(2\pi/\xi_\phi), \tag{25}$$

$$\lambda_\eta=\lambda_QH(\delta E/\xi_E)\ G(\delta\phi/\xi_\phi). \tag{26}$$

The ratio of Eqs. (25) and (26) can be used to obtain $R(0,0)=0.9\%$ from the measured value of λ_η (Table I), without knowledge of $H(\delta E/\xi_E)$. This result is for identical pions, and must be reduced by a factor of ~2 for comparison to a mixture of pions with equal numbers of π^+ and π^-. This yields an overall $R(0,0)\simeq0.45\%$, which is comparable to, but a factor of ~2 lower than previous direct measurements [35,36] of $\sim1\%$ for nonidentified charged particles. The same effect is seen in extrapolating the present $R(0,0)$ to correspond to the 200° aperture used in the E802 NBD

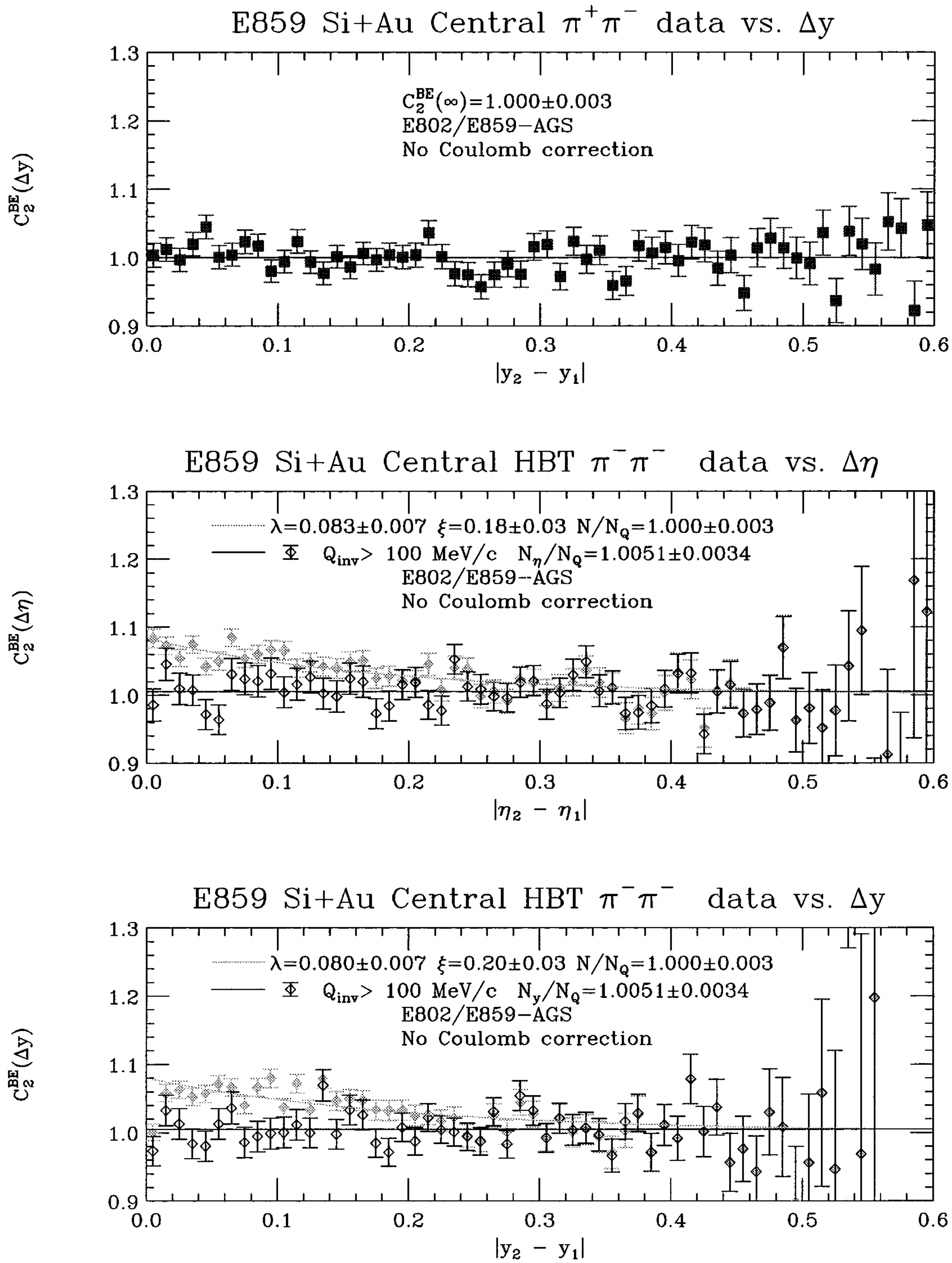

FIG. 2. The Bose-Einstein correlation function C_2^{BE} as a function of $|y_2-y_1|$ and $|\eta_2-\eta_1|$ for $\pi^-\pi^-$ or $\pi^+\pi^-$ pairs in central ^{28}Si+Au collisions. (a) $\pi^+\pi^-$ data for $|y_2-y_1|$. (b) and (c) $\pi^-\pi^-$ data with the requirement $Q_{\mathrm{inv}}\geq 100$ MeV/c in comparison to the uncut data. The horizontal lines shown are the best fits to a constant. The best fits for the strength of a correlation with $\xi=0.2$ (fixed) are $\lambda=0.013\pm0.007$ for (a), $\lambda=0.010\pm0.007$ for (b) and (c).

analysis [2,7,37]: Using $R(0,0)=0.45\%$ yields $\overline{R(0,0)}\simeq 0.8\%$ in comparison with the measured $3.1\%\pm0.5\%$ for nonidentified charged particles. The agreement is quite good, considering the crude extrapolation approximations [38]. Furthermore, the value of $H(\delta E/\xi_E)$ is difficult to estimate in general. For the present analysis, $H(\delta E/\xi_E)\simeq 0.4$ can be estimated from Eq. (26) using the measured values (Table I) of λ_η and λ_Q [39].

On a final note, the E802 analysis was for ^{16}O+Cu, while the present analysis is for ^{28}Si+Au (and a different rapidity range), again leading to an uncertainty. However, to the extent that $\xi_\eta=\xi_\phi\simeq 1/(\langle p_T\rangle R)$ is a reasonable approximation, the correlation lengths should be in the ratio of $A^{1/3}$. Thus, the present measurement for ^{28}Si, $\xi_\eta=0.18\pm0.03$, should be shifted to 0.22 ± 0.03—an effect comparable to one standard deviation—for comparison with $\xi=0.18\pm0.05$ from the

^{16}O+Cu NBD analysis. Hence, the effect of the extrapolation is minimal for the correlation length measurement, and the data agree very well.

VIII. CONCLUSIONS

The two-pion correlation in rapidity has been determined using a sample of pion pairs in the E802/E859 spectrometer from 14.6A GeV/c ^{28}Si+Au$\rightarrow 2\pi^- + X$, 10% central (TMA) collisions, for which a Bose-Einstein correlation analysis is available. Within the spectrometer aperture, no rapidity correlation is observed for $\pi^+\pi^-$ pairs, while $\pi^-\pi^-$ pairs exhibit a rapidity correlation of maximum strength 8.0% $\pm$0.7%, with exponential correlation length $\xi_y = 0.20 \pm 0.03$. The $\pi^-\pi^-$ rapidity correlation vanishes (1.0%$\pm$0.7%) when the region of the Bose-Einstein correlation (in Q_{inv}) is eliminated, thus demonstrating that the pion-pair short-range rapidity correlation in ^{28}Si+Au central collisions is entirely due to the Bose-Einstein effect. The present direct measurements are in very good agreement with the previous indirect measurement [2] of the two-particle short-range rapidity correlation length and strength from the evolution with $\delta\eta$ of the NBD fit parameter $k(\delta\eta)$ in central ^{16}O+Cu collisions. Taken together, these measurements provide quantitative confirmation of the interrelationship of non-Poisson multiplicity fluctuations, the negative binomial distribution, short-range rapidity correlations, intermittency, and the Bose-Einstein effect, which has had much previous theoretical and experimental support.

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fit. The original Mueller two-particle correlation function $C_2(p_1,p_2)=\rho_1(p_1)\rho_1(p_2)[C_2^{\mathrm{BE}}(p_1,p_2)-1]$ is different from the two-particle correlation function used in BE correlations.

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[38] Thus no errors are quoted for extrapolated quantities.

[39] The value of $\lambda_Q=0.37\pm 0.02$ for the present analysis with no Coulomb correction is reasonable in comparison to the fully corrected value [24] of 0.57 ± 0.02 which is, however, considerably smaller than the value of 1.0 expected for BE correlations.