Low-energy extensions of the eikonal approximation to heavy-ion scattering

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We discuss different schemes devised to extend the eikonal approximation to the regime of low bombarding energies (below 50 MeV per nucleon) in heavy-ion collisions. From one side we consider the first- and second-order corrections derived from Wallace's expansion. As an alternative approach we examine the procedure of accounting for the distortion of the eikonal straight-line trajectory by shifting the impact parameter to the corresponding classical turning point. The two methods are tested for different combinations of colliding systems and bombarding energies, by comparing the angular distributions they provide with the exact solution of the scattering problem. We find that the best results are obtained with the shifted trajectories, the Wallace expansion showing a slow convergence at low energies, in particular for heavy systems characterized by a strong Coulomb field. [S0556-2813(97)05808-1]

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I. INTRODUCTION

The eikonal approximation is widely used for the description of high-energy potential scattering in different fields of physics [1–8]. Due to its appealing simplicity, different prescriptions have been advanced in order to extend the validity of the model to lower bombarding energies. A number of these approaches are based on the eikonal expansion developed by Wallace [9–14], in which the phase shift is expressed as a series involving powers and derivatives of the potential. This expansion offers a consistent mathematical framework in which systematic corrections to the eikonal approximation can be derived in closed form. The actual evaluation of the different terms in the series, however, turns out to be rather involved for high-order terms. In addition, unphysical features such as flux production in absorptive complex potentials may appear when the expansion is truncated to few terms [13].

Other approaches formally maintain the eikonal form of the phase shift and try to account for deviations from the straight-line trajectory by shifting the impact parameter to effective values, typically to the turning point of the corresponding classical trajectory [15–19]. These methods have a more pragmatic character and can be easily extended to coupled-channel scattering processes. They have also been applied to the microscopic description of nucleus-nucleus collisions based on the Glauber model [20–25].

These rather different ways of correcting the eikonal approximation are compared in this paper. We consider elastic heavy-ion collisions, which are characterized by strong Coulomb fields and strong absorption, and we take profit of the variety of projectile-target combinations to modulate the relative importance of these effects. Although we describe the nucleus-nucleus interaction in terms of an optical potential, our results are also relevant to the microscopic Glauber approach due to the common features of the two models.

The paper is organized as follows. In Sec. II we discuss the Wallace expansion and apply it to the elastic scattering of heavy ions. In Sec. III we introduce trajectory corrections through the use of effective impact parameters, and use them to study the same heavy-ion systems investigated in Sec. II. Discussions and conclusions are given in Sec. IV.

II. EIKONAL EXPANSION

In the impact parameter representation, the scattering amplitude by a spherically symmetric potential $V(r)$ is written as

$$f(\theta)=ik\int_0^\infty db\,b\,J_0(qb)[1-e^{i\chi(b)}], \tag{1}$$

where k is the center-of-mass momentum and $q=2k\sin(\theta/2)$ is the momentum transfer. A systematic expansion of the phase $\chi(b)$ in powers of $1/k$ has been proposed some time ago by Wallace [9]. He obtained a sequence of approximations

$$\chi(b)=\chi_0(b)+\chi_1(b)/k+\chi_2(b)/k^2+\cdots, \tag{2}$$

in which the first term is the usual eikonal formula

$$\chi_0(b)=-\frac{1}{\hbar v}\int_{-\infty}^{\infty}dz\,V(r), \tag{3}$$

v being the incident velocity and $r=\sqrt{b^2+z^2}$. The first- and second-order corrections to the eikonal term are, respectively,

$$\chi_1(b)=-\frac{1}{2(\hbar v)^2}\left(1+b\frac{\partial}{\partial b}\right)\int_{-\infty}^{\infty}dz\,V^2(r), \tag{4}$$

and

$$\chi_2(b)=-\frac{1}{6(\hbar v)^3}\left(3+5b\frac{\partial}{\partial b}+b^2\frac{\partial^2}{\partial b^2}\right)\int_{-\infty}^{\infty}dz\,V^3(r)+\epsilon_2(b), \tag{5}$$

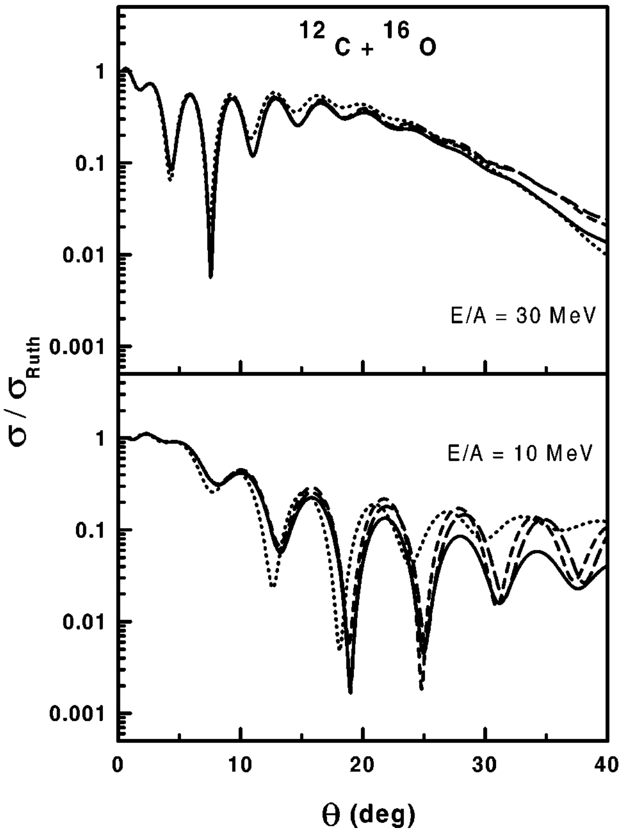

FIG. 1. Elastic cross section (as a ratio to the Rutherford) for the system $^{12}C+^{16}O$ at bombarding energies E/A equal to 30 MeV (upper frame) and 10 MeV (lower frame). The exact results are represented by solid lines, while the dotted lines represent the eikonal approximation. The short and long dashes show the Wallace expansion taken to first and second order, respectively. The optical potential parameters (cf. text) are taken from [26].

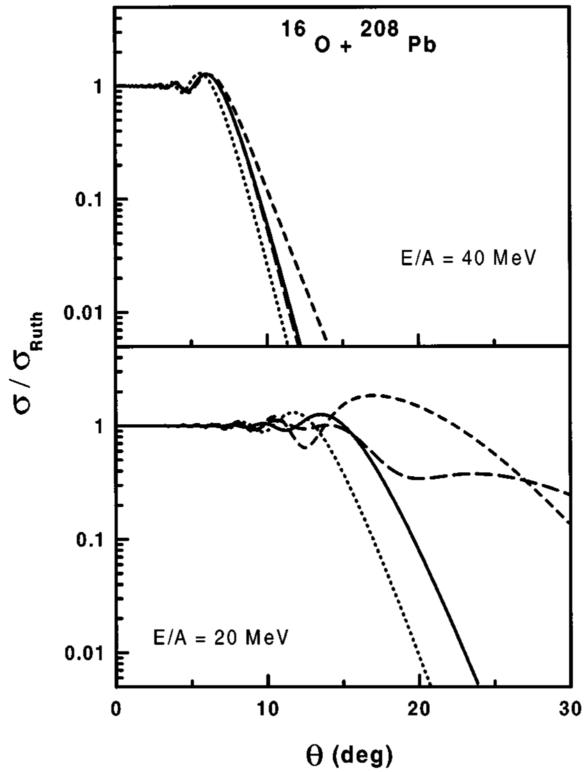

FIG. 2. Elastic cross section (as a ratio to the Rutherford) for the system $^{16}O+^{208}Pb$ at bombarding energies E/A equal to 40 MeV (upper frame) and 20 MeV (lower frame). The optical potential parameters (cf. text) are taken from [27]. For the meaning of the different curves, cf. caption to Fig. 1.

where

$$\epsilon_2(b)=-\frac{1}{24}b[\chi_0'(b)]^3+\frac{i}{8}\chi_0'(b)[b\chi_0''(b)+\chi_0'(b)]. \tag{6}$$

The Wallace expansion has been applied successfully to proton-nucleus scattering at relatively low energies (down to 200 MeV) [11,12]. It has also been used to study collisions involving light nuclei ($\alpha+^{58}$Ni,^{6}Li$+^{12}$C) at energies as low as 20 MeV per nucleon [12,13]. A version of this expansion was recently used to improve the accuracy of a few-body Glauber calculation of ^{11}Be$+^{12}$C scattering at 25 and 50 MeV/nucleon [14].

In Fig. 1 we show the result of applying the Wallace expansion, truncated at second order, to the elastic scattering of $^{12}C+^{16}O$. We use a standard Woods-Saxon shape for the complex nuclear potential, with parameters $V_r=-63.7$ MeV, $R_r=5.1$ fm, and $a_r=0.63$ fm for the real part, and $V_i=-27.2$ MeV, $R_i=5.1$ fm, $a_i=0.69$ fm for the imaginary part [26]. In order to better evidence the effect of the bombarding energy, we keep fixed the parameters of the optical potential, disregarding any dynamical energy dependence. The calculations were made at two relatively low energies, 30 and 10 MeV/nucleon. The dotted lines represent the eikonal approximation, and the short and long dashes show the Wallace expansion taken to first and second order, respectively. The exact angular distributions, obtained through numerical solution of the partial wave Schrödinger equation, are given by the solid lines. We see that the non-eikonal corrections improve appreciably the eikonal approximation, even though discrepancies remain at large angles and at the lowest energy. The convergence of the expansion is also seen to be slow, as the second order correction does not add much to the first-order result.

The same expansion gives rather different results for a heavier projectile-target system. In Fig. 2 we present calculations of $^{16}O+^{208}Pb$ scattering at energies 40 and 20 MeV/nucleon. We use a Woods-Saxon nuclear potential with parameters $V_r=-50$ MeV, $V_i=-42.2$ MeV, $R_r=R_i=9.15$ fm, $a_r=a_i=0.755$ fm [27]. The curves have the same meaning as in Fig. 1. We note that the first-order correction to the eikonal term, though large, does not lead to a better agreement with the exact result. This is at variance with what we found above for the lighter system $^{12}C+^{16}O$, where the first-order correction already gave reasonable results. The second-order Wallace approximation reproduces quite well the exact calculation at 40 MeV/nucleon, but is seen to fail badly at the lower energy.

The difficulties met by the Wallace expansion for heavy systems are further illustrated by looking at the transmission factor $T(b)=1-|e^{i\chi(b)}|^2$, which can be compared to the exact partial wave result $T_l=1-|e^{2i\delta_l}|^2$ using the semiclassical relation between angular momentum and impact parameter, $l+1/2=kb$. In Fig. 3 the transmission coefficients are plot-

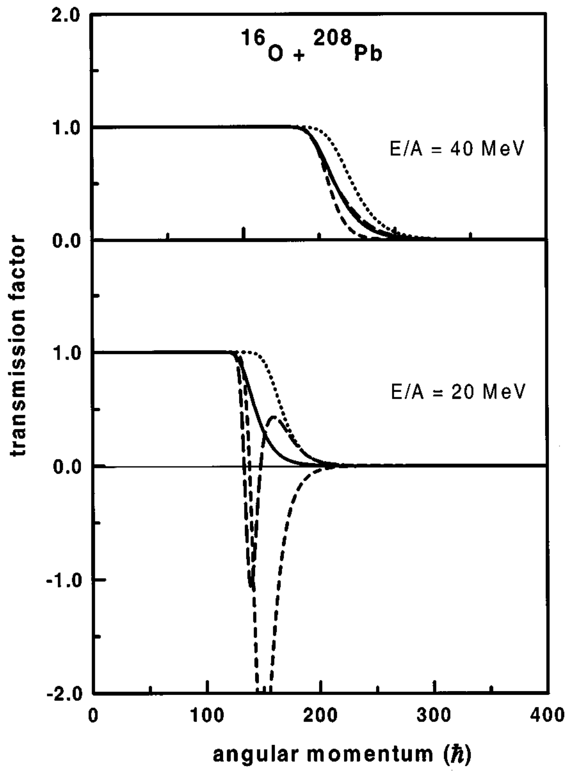

FIG. 3. Transmission factors associated with the elastic scattering of $^{16}O+^{208}Pb$ at the two bombarding energies displayed in Fig. 2. For the meaning of the different curves, see caption to Fig. 1.

ted against l for the same nuclei and energies as in Fig. 2. One sees that at the lowest energy the noneikonal corrections yield negative transmission probabilities (flux is created in the elastic channel). It is also clear from Fig. 3 that this violation of unitarity comes mostly from the first-order correction.

In order to understand why these flux conservation problems arise in the case of heavy systems, let us consider in more detail the first-order noneikonal correction. To this order the scattering phase can be put into the form

$$\chi(b) \simeq \chi_0(b)+\chi_1(b)/k=-\frac{1}{\hbar v}\int_{-\infty}^{\infty} dz V_{\mathrm{eff}}(r), \tag{7}$$

where the effective potential $V_{\mathrm{eff}}(r)$ is [9]

$$V_{\mathrm{eff}}(r)=V(r)+\frac{1}{4E}\left(2+r\frac{\partial}{\partial r}\right)V^2(r), \tag{8}$$

and E is the center-of-mass energy. Writing the ion-ion potential as $V(r)=Z_1Z_2e^2/r+V_n(r)$, and further dividing the nuclear potential in its real and imaginary parts, $V_n(r)=U_n(r)+iW_n(r)$, we obtain for the imaginary part of $V_{\mathrm{eff}}(r)$

$$\mathrm{Im}\{V_{\mathrm{eff}}\}=W_n+\frac{Z_1Z_2e^2}{2E}W_n' + \frac{1}{2E}[U_nW_n+rU_n'W_n+rU_nW_n'], \tag{9}$$

where the prime denotes derivative with respect to the radius r. For large enough distances the nuclear potential falls exponentially, $V_n(r)\sim\exp(-r/\alpha)$, with $\alpha\simeq 0.7$ fm. In this region the last term in Eq. (9) is negligible because of its short range ($\alpha/2$), and we are left with

$$\mathrm{Im}\{V_{\mathrm{eff}}(r)\}\simeq(1-a_c/\alpha)W_n(r), \tag{10}$$

where we have introduced the Coulomb distance parameter $a_c=Z_1Z_2e^2/2E$. One notes from Eq. (10) that for $a_c>\alpha$ the tail of the effective potential has a positive imaginary part, producing flux instead of absorbing it. This is likely to occur for heavy systems at low energies, and we see an example in Fig. 3. At the lowest energy shown in this figure, 20 MeV/nucleon, the value of the Coulomb distance is $a_c=1.6$ fm, much larger than the diffuseness $\alpha=a_i=0.755$ fm of the imaginary nuclear potential. Correspondingly, violations of the unitarity bounds are huge. At the highest energy, 40 MeV/nucleon, we have $a_c=0.8$ MeV, and such violations are very small. In the case of the light system $^{12}C+^{16}O$, we find that the critical condition $a_c=a_i$ only occurs lowering the energy to 7 MeV/nucleon.

III. TURNING POINT CORRECTIONS

The eikonal approximation is based on the idea that at high energies the colliding particles follow straight-line trajectories. In particular, this means that the impact parameter is also the distance of closest approach between projectile and target. The strong Coulomb field present in heavy-ion collisions distorts significantly these straight-line trajectories even at relatively high energies, breaking down the eikonal approximation. Due to the short range nature of the nuclear potential, the most important feature of such a distortion is the difference that appears between the Coulomb turning point and the impact parameter. A simple way to take this into account has been proposed by Fäldt and Pilkuhn for pion-nucleus scattering [16] and applied to heavy-ion collisions in Ref. [17]. Dividing the eikonal phase into its Coulomb and nuclear parts

$$\chi_0(b)=\chi_c(b)+\chi_n(b), \tag{11}$$

an approximate correction for the Coulomb distortion of the trajectory is achieved by the substitution

$$\chi_n(b)\rightarrow\chi_n[d_c(b)], \tag{12}$$

where $d_c(b)$ is the Coulomb distance of closest approach

$$d_c(b)=a_c+\sqrt{a_c^2+b^2}, \tag{13}$$

and a_c is the Coulomb parameter introduced in the previous section. In the same semiclassical spirit, in order to assure the conservation of the angular momentum, one can also change the asymptotic velocity that is used to calculate the nuclear eikonal phase by the tangential velocity at the turning point,

$$v_c(b)=\frac{b}{d_c(b)}v. \tag{14}$$

The Coulomb turning point correction then takes the form

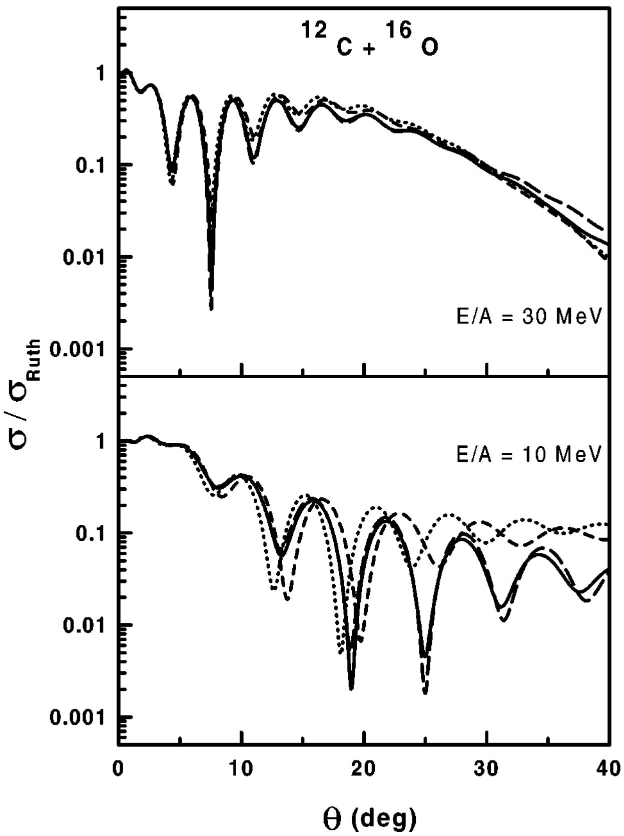

FIG. 4. Elastic cross section (as a ratio to the Rutherford) for the system $^{12}C+^{16}O$ at bombarding energies E/A equal to 30 MeV (upper frame) and 10 MeV (lower frame). The exact results are represented by solid lines, while the dotted lines represent the eikonal approximation. The short and long dashes show the results obtained by including the correction due to the Coulomb field and the nuclear+Coulomb fields, respectively. The optical potential is the same as in Fig. 1.

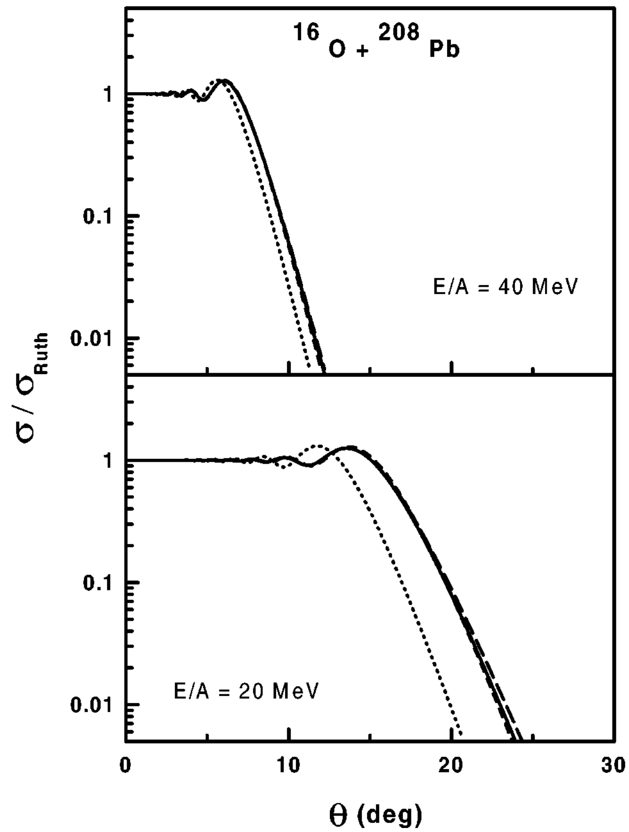

FIG. 5. Elastic cross section (as a ratio to the Rutherford) for the system $^{16}O+^{208}Pb$ at bombarding energies E/A equal to 40 MeV (upper frame) and 20 MeV (lower frame). The optical potential is the same as in Fig. 2. For the meaning of the different curves, see caption to Fig. 4.

$$\chi_n(b)\rightarrow\frac{d_c(b)}{b}\chi_n[d_c(b)]. \tag{15}$$

Furthermore, we could introduce an additional correction for the trajectory distortion caused by the nuclear field [18,23–25,8] (note that, in general, the corresponding Coulomb+nuclear turning point $d_n(b)$ will be complex [28]). The turning point correction is then written, in analogy to Eq. (15), as

$$\chi_n(b)\rightarrow\frac{d_n(b)}{b}\chi_n[d_n(b)]. \tag{16}$$

We have used these corrections to study the low-energy scattering of the same projectile-target systems discussed in the previous section. The results for $^{12}C+^{16}O$ collisions are shown in Fig. 4. The dotted and solid lines represent the eikonal and exact results, respectively. The short-dashed lines were obtained with the Coulomb correction, Eq. (15), and the long-dashed ones correspond to the Coulomb-nuclear correction of Eq. (16). We see that the Coulomb correction by itself does not improve the eikonal approximation. However, by taking into account the concurrent effects of Coulomb and nuclear interactions, we obtain a reasonably good agreement with the exact calculation.

The application of turning point corrections to the heavier system $^{16}O+^{208}Pb$ is shown in Fig. 5. The Coulomb correction alone improves markedly the eikonal result, contrary to what happened with the comparatively light system of Fig. 4. The nuclear correction is very small, again in contrast to the case of lighter systems.

IV. DISCUSSION

A comparison of Figs. 1 and 4 shows that for the $^{12}C+^{16}O$ system the Wallace expansion and the impact parameter shifts both give good results, of similar quality. For the heavier $^{16}O+^{208}Pb$ system the situation is different: at low energies the turning point methods are clearly much better than the Wallace approach (see Figs. 2 and 5). We should note that the optical potential we have used for the $^{16}O+^{208}Pb$ system has a strong absorptive part which dies out only at large distances. This characteristic is partially responsible for the success of an eikonal-like approach at such low energies, as the deviation from pure Rutherford scattering is dominated by a narrow window of impact parameters around the grazing value for which the eikonal condition $|V|/E\ll 1$ is valid. Trajectories associated with smaller impact parameters, which penetrate deeply and feel a stronger nuclear field, are completely damped by the absorption.

On the other hand, this strong absorption is also responsible for the problems met by the Wallace approach in the case of heavy systems; we see from Eq. (10) that the unitarity violation is proportional to the strength of the imaginary potential in the surface region. This raises the question of whether our results hold for more ''surface-transparent'' heavy-ion potentials, which give rise to rainbow scattering [28] (our optical potential for $^{16}O+{}^{208}Pb$ leads to an angular distribution that is closer to a Fresnel diffraction pattern). To check for this we reduced by a few fermis the radius of the imaginary part of the $^{16}O+{}^{208}Pb$ potential, so that the scattering became essentially rainbow dominated. Even in this weak absorption case the Wallace corrections still produced unacceptable results, while the turning point approach remained quite successful. We found that the latter method breaks down only for very small absorption radii, when the interference between the near-side and the more penetrating far-side trajectories becomes apparent in the angular distribution.

To summarize, the use of the distance of closest approach as an effective impact parameter in the eikonal formula provides a simple and efficient way for studying heavy-ion scattering at relatively low energies. Low-order truncation of the Wallace expansion seems to be a good alternative to this method only for light systems.

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[1] R. J. Glauber, *Lectures on Theoretical Physics* (Interscience, New York, 1959), Vol. I.

[2] R. G. Newton, *Scattering of Waves and Particles* (Springer, New York, 1982).

[3] C. J. Joachain, *Quantum Collision Theory* (North-Holland, Amsterdam, 1976).

[4] T. W. Donnelly, J. Dubach, and J. D. Walecka, Nucl. Phys. **A232**, 355 (1974).

[5] H. Feshbach and J. Hüfner, Ann. Phys. (N.Y.) **56**, 268 (1970).

[6] F. W. Byron, Jr. and C. J. Joachain, Phys. Rep., Phys. Lett. **34C**, 233 (1977).

[7] F. A. Cucinotta, G. S. Khandelwal, L. W. Townsend, and J. W. Wilson, Phys. Lett. B **223**, 127 (1989).

[8] S. M. Lenzi, A. Vitturi, and F. Zardi, Z. Phys. A **352**, 303 (1995).

[9] R. J. Wallace, Ann. Phys. (N.Y.) **78**, 190 (1973).

[10] R. J. Wallace, Phys. Rev. D **8**, 1846 (1973).

[11] D. Waxman, C. Wilkin, J. F. Germond, and R. J. Lombard, Phys. Rev. C **24**, 578 (1981).

[12] G. Fäldt, A. Ingemarsson, and J. Mahalanabis, Phys. Rev. C **46**, 1974 (1992).

[13] F. Carstoiu and R. J. Lombard, Phys. Rev. C **48**, 830 (1993).

[14] J. S. Al-Khalili, J. A. Tostevin, and J. M. Brooke, Phys. Rev. C **55**, R1018 (1997).

[15] R. da Silveira, J. Galin, and C. Ngô, Nucl. Phys. **A159**, 481 (1970).

[16] G. Fäldt and H. Pilkuhn, Phys. Lett. **46B**, 337 (1973).

[17] A. Vitturi and F. Zardi, Phys. Rev. C **36**, 1404 (1987).

[18] M. H. Cha, Phys. Rev. C **46**, 1026 (1992).

[19] S. K. Gupta and P. Shukla, Phys. Rev. C **52**, 3212 (1995).

[20] S. M. Lenzi, A. Vitturi, and F. Zardi, Phys. Rev. C **38**, 2086 (1988).

[21] S. M. Lenzi, A. Vitturi, and F. Zardi, Phys. Rev. C **40**, 2114 (1989).

[22] S. M. Lenzi, F. Zardi, and A. Vitturi, Phys. Rev. C **42**, 2079 (1990).

[23] S. K. Charagi and S. K. Gupta, Phys. Rev. C **41**, 1610 (1990).

[24] S. K. Charagi and S. K. Gupta, Phys. Rev. C **46**, 1982 (1992).

[25] S. K. Charagi, Phys. Rev. C **48**, 452 (1993).

[26] M. Lozano and G. Madurga, Nucl. Phys. **A334**, 349 (1980).

[27] M. C. Mermaz *et al.*, Z. Phys. A **326**, 353 (1987).

[28] R. A. Broglia and A. Winther, *Heavy Ion Reactions* (Addison-Wesley, New York, 1990).