Polarized nuclear matter using extended Seyler-Blanchard potentials

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In the present work the equation of state (EOS) is derived using three types of potentials for polarized nuclear matter. The potentials used are the extended Seyler-Blanchard (SB), modified Seyler-Blanchard (MSB) and the generalized Seyler-Blanchard (GSB) potentials. It is found that the equation of state derived using SB potential is a stiff EOS whereas the equations of state derived using MSB and GSB potentials are soft ones. The phase diagram for nuclear matter is also studied. The shapes are similar for the three potentials used but the critical temperatures are slightly different. [S0556-2813(97)07108-2]

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I. INTRODUCTION

The equation of state is used to study the properties of nuclear matter. Static properties of nuclear matter, e.g., binding energy, asymmetric energy, incompressibility, etc., can be determined successfully by the EOS. Also, at finite temperatures, the thermal properties of nuclear matter can be studied, e.g., free energy, entropy, effective mass, chemical potential, and all possible phases in which the matter may exist. Liquid-gas mixture exists at very low temperatures in the crusts of neutron star and at temperatures of 5 to 10 MeV in supernovas [1]. Such matter is also observed in high energy heavy ion collisions. The emission of intermediate mass fragments during the decay of a highly excited nuclear system was interpreted as being due to statistical droplet formation in liquidlike or gaslike phases of the system [2–5].

In early experiments [6], from the energy distribution of the fragments, the critical temperature of the hot fragmenting nuclear matter is deduced to be about 12 MeV. Below this critical temperature nuclear matter coexists in the liquid and in the gas phase; it becomes a single uniform phase above it. A liquid-gas phase transition was considered [7] from the EOS of hot nuclear matter using SB potential. Later works [8] considered the Skyrme interactions which offered simplicity of the calculations; the critical temperature was found to be about 15–20 MeV. Most of the calculations considered the symmetric nuclear matter case whereas few authors studied the thermostatic properties of asymmetric nuclear matter. The polarized nuclear matter has been studied by Dabrowski and Haensel [9–11] at zero temperature and by Hassan *et al.* [12] at finite temperature. Rudra and De [13] used a density dependent modified Seyler-Blanchard (MSB) potential to study the equation of state of asymmetric nuclear matter. In addition to the density dependent term Myers and Swiatecki [14] added a reverse momentum-dependent $(1/P)$ term to study the nuclear matter properties. In a previous work [15] the thermostatic properties of polarized nuclear matter were calculated using an extended form of the Seyler-Blanchard potential.

In the present work we construct a general form for the SB potential which is suitable for polarized nuclear matter. This potential contains the old SB potential plus a density-dependent term and a reverse momentum-dependent term. Some of the previous calculations [15] are presented here after correcting them by applying the proper physical condition $P=0$ at $\rho/\rho_0=1$ which was not satisfied in their calculation.

In the next section we present the theory and in Sec. III we present the results and discussion.

II. THEORY

A. Nuclear matter at zero temperature

Nuclear matter is an infinite system of nucleons with a definite ratio of neutrons to protons numbers. We refer the reader to Ref. [15] where the detailed method of calculation is explained. Polarized nuclear matter is composed of numbers $N\uparrow$ $(N\downarrow)$ of spin up (spin down) neutrons and $P\uparrow$ $(P\downarrow)$ of spin up (spin down) protons, with corresponding densities $\rho_{n\uparrow}$, $\rho_{n\downarrow}$, $\rho_{p\uparrow}$, and $\rho_{p\downarrow}$, respectively; thus

$$A=N\uparrow+N\downarrow+P\uparrow+P\downarrow$$

is the total number of particles and the total density ρ is given by

$$\rho=\rho_n+\rho_p=\rho_{n\uparrow}+\rho_{n\downarrow}+\rho_{p\uparrow}+\rho_{p\downarrow}. \tag{1}$$

For polarized nuclear matter, we define the following parameters': the neutron excess parameter

$$X=(\rho_n-\rho_p)/\rho, \tag{2}$$

the neutron spin-up excess parameter

$$\alpha_n=(\rho_{n\uparrow}-\rho_{n\downarrow})/\rho, \tag{3}$$

the proton spin-up excess parameter,

$$\alpha_p=(\rho_{p\uparrow}-\rho_{p\downarrow})/\rho, \tag{4}$$

$$Y=\alpha_n+\alpha_p, \tag{5}$$

and

$$Z=\alpha_n-\alpha_p. \tag{6}$$

In the present work we used the following form of the Seyler-Blanchard potential [13,14,16] which is generalized in our case to polarized nuclear matter (see also Ref. [15])

TABLE I. Parameters of the SB potential.

Input data [15]	Output
$k_f = 1.312\ \mathrm{fm}^{-1}$	$b = 2.0683\ \mathrm{fm}^{-1}$
$\rho_0 = 0.1533\ \mathrm{fm}^{-3}$	$\alpha = \beta = 0$
$E_V = -16.4$ MeV	$C_{LL} = 229.41$ MeV
$E_X = 33.4$ MeV	$C_{Lu} = 536.91$ MeV
$E_Y = 31.5$ MeV	$C_{uL} = 654.42$ MeV
$E_Z = 36.5$ MeV	$C_{uu} = 345.18$MeV
$a = 0.557$ fm	$K = 310$ MeV

$$V(r,k) = -C_{\xi,v}\,\frac{e^{-r/a}}{r/a}\left[1-\frac{k^2}{b^2}-\alpha(\rho_1+\rho_2)^n+\beta\,\frac{1}{k}\right], \tag{7}$$

where r is the separation distance between two nucleons and k is the relative momentum. $C_{\xi,v}$ is the strength of the interaction where ξ and v refer to the likeness L and unlikeness u for spin and isotopic spin, respectively. a, b, α, β, and n are parameters of the interaction. $\rho_1(r_1)$ and $\rho_2(r_2)$ are the densities at the two sites of the interacting nucleons [16]. Using the above generalized interaction the total energy per particle of the polarized nuclear matter is given by

$$E = E_V + X^2E_X + Y^2E_Y + Z^2E_Z, \tag{8}$$

where

$$E_V = \frac{3\hbar^2C^{2/3}}{10m}\,\rho^{2/3} + \left(-\frac{a^3C}{3\pi}\,\rho + \alpha\,\frac{a^3 2^n C}{3\pi}\,\rho^{n+1} + \frac{2a^3C^{5/3}}{5\pi b^2}\,\rho^{5/3} - \beta\,\frac{2a^3C^{2/3}}{5\pi}\,\rho^{2/3}\right)C_1 \tag{9}$$

and

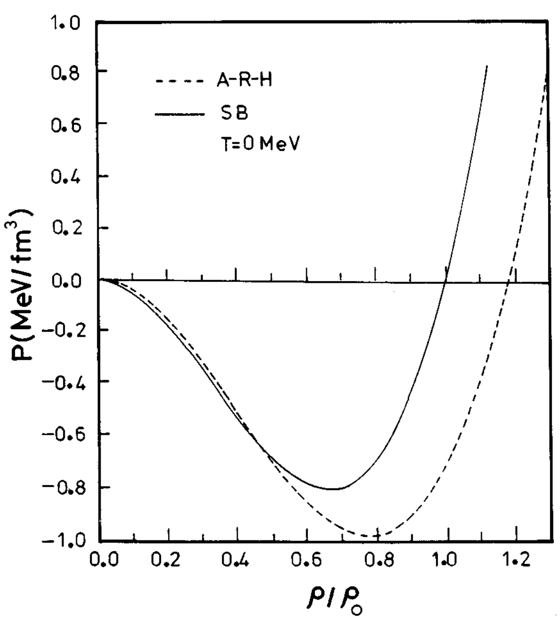

FIG. 1. The pressure dependence on density for the SB potential in comparison with ARH data [15].

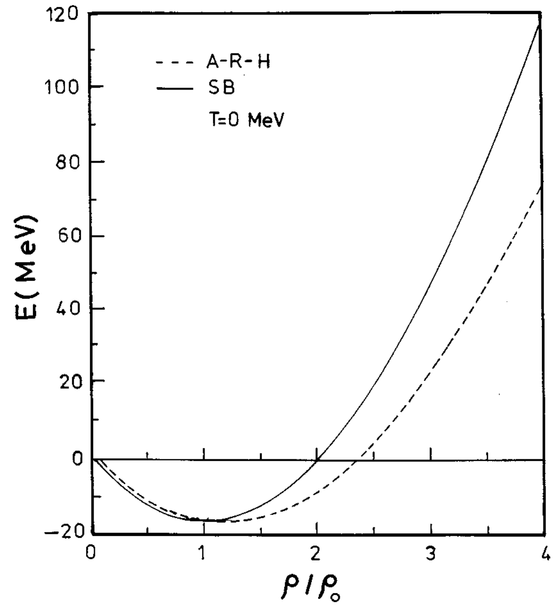

FIG. 2. The energy per particle for the SB potential in comparison with Ref. [15].

$$E_i = \frac{\hbar^2C^{2/3}}{6m}\,\rho^{2/3} - \frac{a^3C}{3\pi}\,\rho C_{i1} + \alpha\,\frac{a^3 2^n C}{3\pi}\,\rho^{n+1}C_{i1} + \frac{4a^3C^{5/3}}{9\pi b^2}\,\rho^{5/3}C_{i2} - \beta\,\frac{2a^3C^{2/3}}{9\pi}\,\rho^{2/3}C_{i3} \quad (i = X,\ Y,\ \text{and}\ Z), \tag{10}$$

where m is the nucleon mass and

$$C = 3\pi^2/2,$$

$$C_1 = C_{LL} + C_{Lu} + C_{uL} + C_{uu},$$

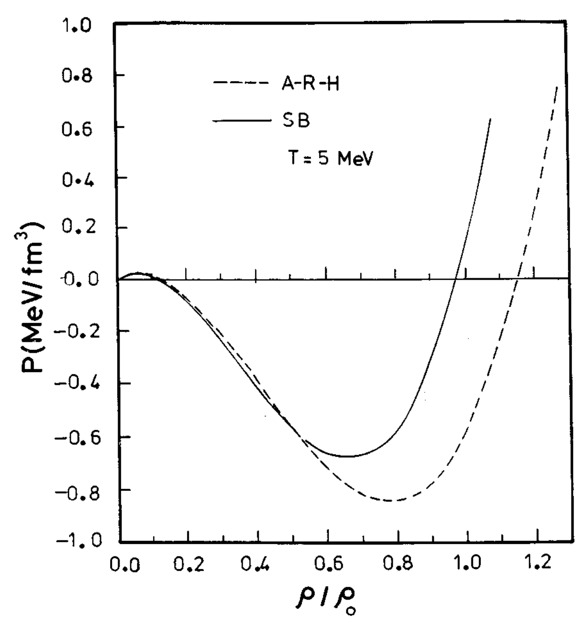

FIG. 3. The pressure at $T = 5$ MeV of the SB potential in comparison with Ref. [15].

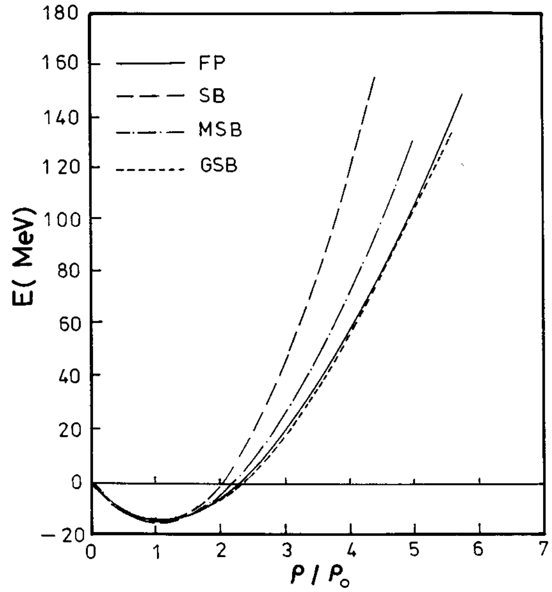

FIG. 4. A comparison of the energy per particle for SB, MSB, and GSB of the present work with the FP potential [25].

$$C_{X1}=C_{LL}+C_{Lu}-C_{uL}-C_{uu},$$

$$C_{Y1}=C_{LL}-C_{Lu}+C_{uL}-C_{uu},$$

$$C_{Z1}=C_{LL}-C_{Lu}-C_{uL}+C_{uu},$$

$$C_{X2}=2C_{LL}+2C_{Lu}-C_{uL}-C_{uu},$$

$$C_{Y2}=2C_{LL}-C_{Lu}+2C_{uL}-C_{uu},$$

$$C_{Z2}=2C_{LL}-C_{Lu}-C_{uL}+2C_{uu},$$

$$C_{X3}=C_{LL}+C_{Lu}-2C_{uL}-2C_{uu},$$

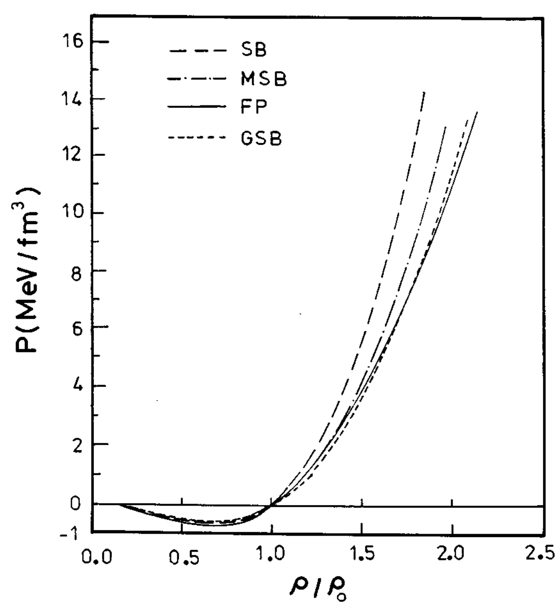

FIG. 5. A comparison of the pressure for SB, MSB, and GSB of the present work with the FP potential [25].

TABLE II. Parameters of the MSB potential.

Input data	Ref.	Output
$k_f=1.32$ fm^{-1}	[21]	$b=9.678$ fm^{-1}
$\rho_0=0.1555$ fm^{-3}		$\alpha=0.956,\ n=1/3,\ \beta=0$
$E_V=-16.1$ MeV	[21]	$C_{LL}=-422.9$ MeV
$E_X=33.9$ MeV	[22]	$C_{Lu}=940.9$ MeV
$E_Y=31.3$ MeV	[23]	$C_{uL}=1013.5$ MeV
$E_Z=36.5$ MeV	[24]	$C_{uu}=861.6$ MeV
$a=0.565$ fm		$K=241.3$ MeV

$$C_{Y3}=C_{LL}-2C_{Lu}+C_{uL}-2C_{uu},$$

and

$$C_{Z3}=C_{LL}-2C_{Lu}-2C_{uL}+C_{uu}.$$

The pressure of nuclear matter is defined as

$$p=\rho^2\,\frac{\partial E}{\partial \rho}, \tag{11}$$

the incompressibility as

$$K=9\rho^2\,\frac{\partial^2 E}{\partial \rho^2}, \tag{12}$$

and the velocity of sound is given by

$$V_s=\left(\frac{\partial P}{\partial \rho}\right)^{1/2}. \tag{13}$$

Terms higher than quadratic in X, Y, and Z are neglected in Eq. (8). The parameters a, b, α, β, n, C_{LL}, C_{Lu}, C_{uL}, and C_{uu} are adjusted to fit the values of k_f, E_V, E_X, E_Y, E_Z, and K for polarized nuclear matter.

B. Nuclear matter at finite temperature

It is well known from classical thermodynamics [17] that the thermodynamic properties of matter are determined completely if the free energy F, is known in terms of the density ρ and temperature T, where

$$F=E-TS, \tag{14}$$

E being the total energy and S is the entropy. Following Ref. [15] and using the T^4 approximation [18] we obtain the entropy S, the free energy F, and the pressure P as follows:

TABLE III. Parameters of the GSB potential.

Input data	Reference	Output
$k_f=1.32$ fm^{-1}	[21]	$b=9.892$ fm^{-1}
$\rho_0=0.1555$ fm^{-3}		$\alpha=1.001,\ n=1/3,\ \beta=0.215$ fm^{-1}
$E_V=-16.1$ MeV	[21]	$C_{LL}=-207.4$ MeV
$E_X=33.9$ MeV	[22]	$C_{Lu}=648.3$ MeV
$E_Y=31.3$ MeV	[23]	$C_{uL}=704$ MeV
$E_Z=36.5$ MeV	[24]	$C_{uu}=587.5$ MeV
$a=0.565$ fm		$K=210$ MeV

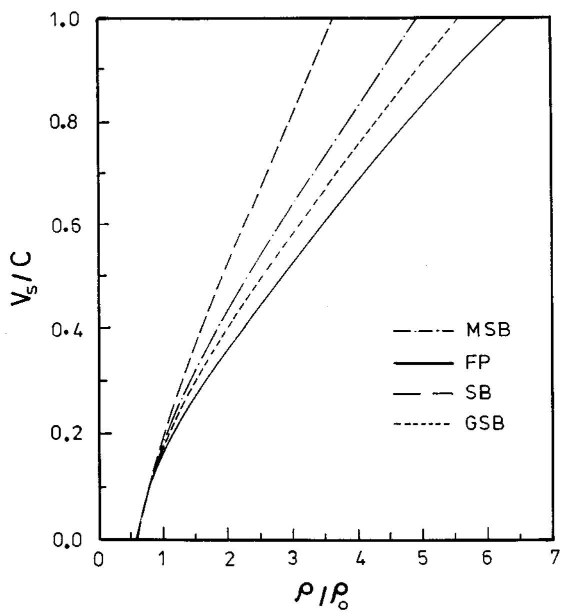

FIG. 6. A comparison of the sound velocity for the SB, MSB, and GSB potentials of the present work with the FP potential [25].

$$S=\frac{T}{6}\left(\frac{3\pi^3}{2}\right)^{1/3}\left(\frac{2m^*}{\hbar^2}\right)\rho^{-2/3}-\frac{7T^3}{1080}\left(\frac{2m^*}{\hbar^2}\right)^3\rho^{-2}, \tag{15}$$

$$F=E(T=0)-\frac{T^2}{6}\left(\frac{3\pi^2}{2}\right)^{1/3}\left(\frac{2m^*}{\hbar^2}\right)\rho^{-2/3}+\frac{7T^4}{12960}\left(\frac{2m^*}{\hbar^2}\right)^3\rho^{-2}, \tag{16}$$

and

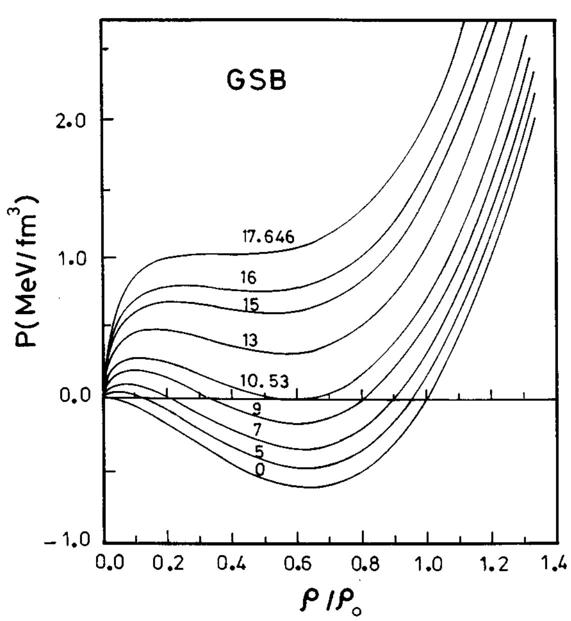

FIG. 7. The pressure-density isotherms for polarized nuclear matter using the GSB potential at different temperatures.

TABLE IV. The critical temperatures.

	Present work					
Reference	SB	MSB	GSB	[7]	[16]	[25]
T_c (MeV)	18.53	16.9	17.55	17.35	15–18	17.5

$$P=P(T=0)+\frac{T^2}{9}\left(\frac{3\pi^2}{2}\right)^{1/3}\left(\frac{2m^*}{\hbar^2}\right)\rho^{1/3}-\frac{7T^4}{6480}\left(\frac{2m^*}{\hbar^2}\right)^3\rho^{-1}, \tag{17}$$

where m^* is the effective mass

$$m^*=m\left[1+\frac{mC_1}{2\hbar^2}\left(\frac{2a^2}{3\pi b^2}k_f^3+\frac{a^3\beta}{3\pi}\right)\right]^{-1}, \tag{18}$$

k_f being the Fermi momentum.

III. RESULTS AND DISCUSSION

In the present work we used three types of potentials: namely the standard Seyler-Blanchard (SB; $\alpha=\beta=0$), the modified SB (MSB; $\alpha\neq0$, $\beta=0$), and the generalized SB (GSB; $\alpha\neq\beta\neq0$) potentials which are extended to study the properties of polarized nuclear matter. In a previous work [15] with SB it is noticeable that the equilibrium condition ($P=0$ at $\rho/\rho_0=1$) is not satisfied at zero temperature. This is clear from Fig. (2.1) of that reference, where the pressure curves until $T=10$ MeV, cross the ρ/ρ_0 axis at $\rho/\rho_0>1$. Here we present their calculation taking into account the equilibrium condition. A new set of parameters (Table I) of the SB potential are obtained using the same input data given in Ref. [15].

From Table I we notice that the incompressibility $K\approx310$ MeV, which is similar to other previous works [7,19] with the SB potential, is still higher than that given by Blaizot [20] ($K=210\pm30$ MeV). This shows that any equation of state derived using the SB potential will be a stiff one.

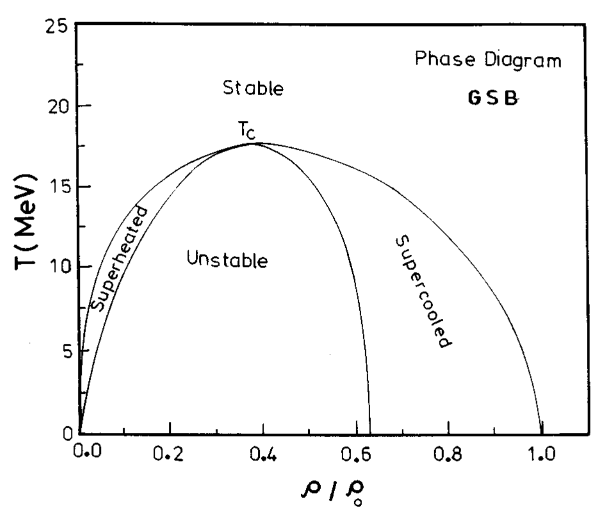

FIG. 8. The phase diagram for polarized nuclear matter using the GSB potential in the ρ-T plane.

TABLE V. The effective mass ratio.

	Present work					
Reference	SB	MSB	GSB	[7]	[25]	[28]
m^*/m	0.38	0.9	0.9	0.38	0.7	0.92

Figure 1 gives the pressure-density relation at $T=0$ in comparison with that of Ref. [15]. It is clear that the saturation condition is not satisfied in their calculation. The correct energy per particle is shown in Fig. 2 again in comparison with those of Ref. [15]. Figure 3 shows the correct behavior of the pressure at $T=5$ MeV in comparison with that of Ref. [15]. Tables II and III give the same set of parameters as in Table I but for the MSB and GSB potentials. In Fig. 4 we present a comparison between our work for the energy per particle using SB, MSB, and GSB with that of Friedman and Pandharipande (FP) [25]. Our results with GSB are very close to the FP calculation. Also, in Fig. 5 we present a comparison between the EOS's for the SB, MSB, and GSB of our work with that of FP. The GSB results give a softer EOS than that of MSB. The values of the velocity of sound using SB, MSB, GSB, and FP potentials are presented in Fig. 6. Here again we notice that GSB values are closer to the FP calculation.

The thermostatic properties of polarized nuclear matter were studied for the potentials used here, using the T^4 approximation. Because they are similar in shape we only present the calculation for the GSB potential in Fig. 7 up to the critical temperature $T_c=17.646$ MeV. We note that the limiting temperature is $T_{\lim}=10.53$ MeV. In Table IV we present our values of T_c in comparison with previous works. The agreement is good.

A phase diagram of our results is given in Fig. 8 for the GSB potential. The diagrams are similar to those of the SB and MSB potentials in shape. The outside region, marked

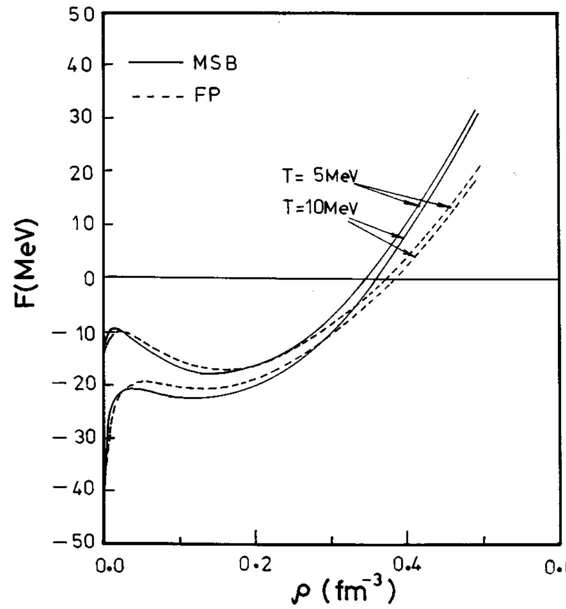

FIG. 10. The free energy of polarized NM using the MSB potential at $T=5$ MeV and $T=10$ MeV in comparison with those of FP.

''stable,'' consists of uniform nuclear matter. The other two regions are the metastable regions. The system can be put in a metastable state, supercooled or superheated, which might not decay in a typical time interval characteristic of nuclear collisions. The region of metastability ends when $(\partial P/\partial\rho)|_T=0$ whereas the system with $(\partial P/\partial\rho)<0$ is absolutely unstable. The phase diagram is similar to those obtained by previous authors [21,26,27].

In Table V we present the effective mass ratio for SB, MSB, and GSB potentials in comparison with different calculations. It is noted that $m^*/m=0.38$ for the SB potential is

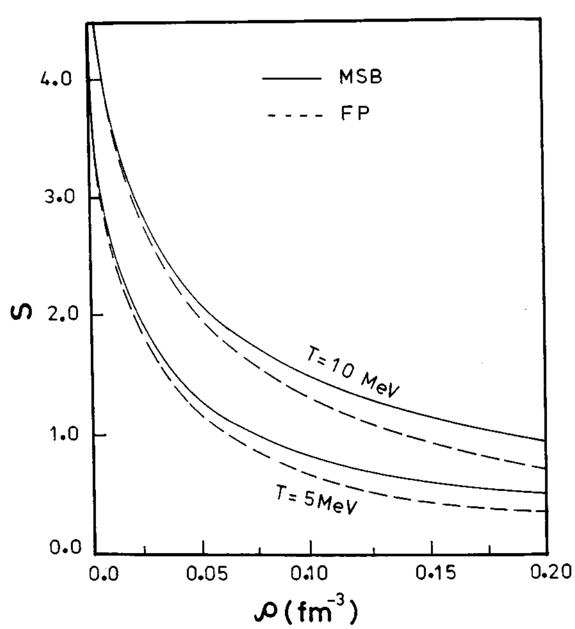

FIG. 9. The entropy of polarized NM using the MSB potential at $T=5$ MeV and $T=10$ MeV in comparison with those of FP.

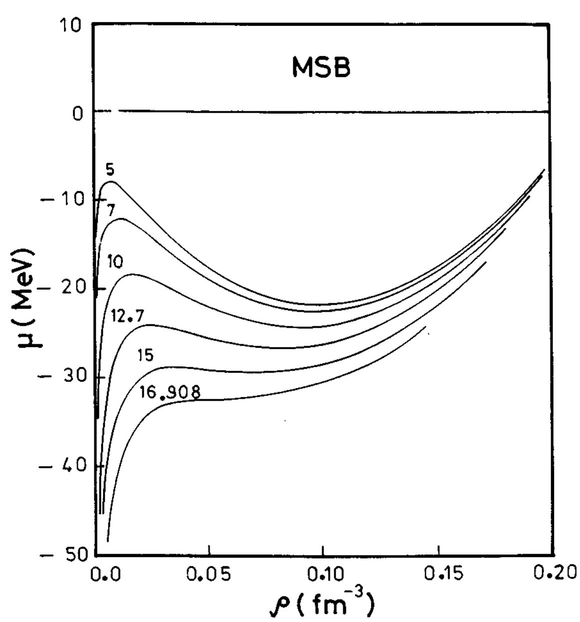

FIG. 11. The chemical potential using the MSB potential at different temperatures.

small. This is a poor characteristic of the SB potential. MSB and GSB potentials give better values for m^*/m. The results show that the SB potential gives a stiff EOS whereas the MSB and GSB give a soft EOS. This is clear from high K values ($K=310$ MeV) and the low effective mass ratio ($m^*/m=0.38$) using the SB potential, whereas $K\approx 240$ and 210 MeV and $m^*/m\approx 0.9$ for the MSB and GSB potentials.

The phase diagram for polarized nuclear matter shows that the choice of the N-N potential slightly affects the shape of the diagram. We obtained critical temperatures, in the range 17.0–18.5 MeV for the potentials considered in this work. The T^4 approximation is reliable here to study the polarized nuclear matter properties for small temperatures. The thermodynamic quantities for polarized NM (e.g., free energy, entropy, and chemical potential) using the extended SB, MSB, and GSB potentials have almost the same behavior. Figures 9–11 show the behavior of such physical quantities using the MSB potential. The entropy per nucleon is given in Fig. 9 in comparison with FP calculations. Our results coincide with those of FP in the low density region, but at higher densities it is lower. It is noted that the entropy decreases as the density increases and the values become larger at higher temperatures. The effect of temperature on the energy-density relation is illustrated in Fig. 10. Good agreement is obtained with those values of FP. The chemical potential-density relation is shown in Fig. 11 at different temperatures. It is noticed that at the critical temperature $T_c=16.908$, the chemical potential curve has no minimum, which means that the nuclear matter is in stable (equilibrium) phase [26] above this temperature. An alternative method for extending the MSB potential for polarized nuclear matter will be presented elsewhere by one of the authors (H.M.M.M.).

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