Possible deviation of the sum of strengths for the double giant dipole resonance from the harmonic oscillator limit

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(Received 14 April 1997)

It is shown that the part of the nuclear Hamiltonian which contains the products of pp and hh pair operators (the scattering terms) and which has been usually neglected in the calculations of the giant dipole resonance turns out to be definitely important in the study of the double giant dipole resonance (DGDR). The complete energy-weighted sum of strengths (EWSS) for the DGDR and a possible deviation from its value in the harmonic oscillators' limit are derived for the first time with the full nuclear Hamiltonian taken into account. The numerical calculations within a schematic model show an example where this deviation turns to a strong enhancement of the EWSS for the DGDR at various particle numbers. [S0556-2813(97)00509-8]

PACS number(s): 21.60.Jz, 24.30.Cz

I. INTRODUCTION

The recent observation of the double giant dipole (DGDR) in relativistic heavy-ion reactions of a wide range of nuclei [1–6] has attracted considerable attention. The observed excitation energy of the DGDR is about twice as much compared to the energy of the giant dipole resonance (GDR). Its width is larger than the GDR width by 1.6 times [1,2,5,6]. While these parameters of the DGDR are in a good agreement with the representation of the multiphonon giant resonance, which is formed by noninteracting harmonic oscillators (phonons), the most striking point is that its strength has been found to be strongly enhanced as compared to any theoretical estimations available so far [7–10]. Recently, making use of a sum rule approach, Kurasawa and Suzuki [11] have demonstrated that, if the mean energy of the DGDR is twice as much compared to the GDR energy, the DGDR strength is 2 times square of that of the GDR. Thus, there is a discrepancy between the strength of the DGDR determined by the sum rules and the experimental data.

In the previous work [12], taking into account the multiphonon structure of the wave function of the GDR and DGDR, we have shown that there is a possibility for the enhancement of the $E1$ decay as well as excitation of the DGDR in a two-step process. The aim of the present paper is to study the deviation of the DGDR sum rule from the value, defined in the harmonic oscillators' (independent phonons) limit, when the full multiphonon structure of the nuclear Hamiltonian is adequately included. It is our hope that this effort will serve as a further step in improving our present understanding of the quenching of the DGDR strength in theoretical calculations as compared to the experimental systematics.

The paper is organized as follows. In Sec. II the complete sum rule for the DGDR is derived, making use of a model Hamiltonian with a two-body separable residual interaction. The formalism is tested in a solvable model in Sec. III for several systems with different mass numbers. Conclusions are given in the last section, where the paper is summarized.

II. MODEL HAMILTONIAN AND SUM RULE FOR DGDR

In this section we will derive for the first time the complete energy-weighted sum of the strengths (EWSS) for the DGDR taking into account the full structure of the nuclear Hamiltonian.

The EWSS $S_1^{(2)}$ for DGDR is defined in a similar way as that for GDR according to Ref. [11] as

$$S_1^{(2)}=\tfrac{1}{2}\langle 0|[D^2,[H,D^2]]|0\rangle, \tag{1}$$

where D is the standard electric dipole operator of nuclei [13], H denotes the nuclear Hamitonian and the expectation value is taken over the ground state $|0\rangle$ of the system. A simple algebraic derivation of the right-hand side (RHS) of Eq. (1) leads to

$$S_1^{(2)}=\langle 0|D[D,[H,D]]D|0\rangle+\tfrac{1}{2}\langle 0|[D,[H,D]]D^2|0\rangle + \tfrac{1}{2}\langle 0|D^2[D,[H,D]]|0\rangle. \tag{2}$$

If we now apply an approximation, which replaces all the double commutators $[D,[H,D]]$ on the RHS of Eq. (2) with their expectation values over the ground state $|0\rangle$, where

$$S_1^{(1)}\equiv\tfrac{1}{2}\langle 0|[D,[H,D]]|0\rangle, \tag{3}$$

which is nothing but the EWSS for the GDR, we obtain easily the model-independent relation between the sums for DGDR and GDR strengths in Ref. [11], namely,

$$(S_1^{(2)})_{\rm har}=4S_1^{(1)}S_0^{(1)}, \tag{4}$$

with

$$S_0^{(1)}=\langle 0|D^2|0\rangle \tag{5}$$

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being the non-energy-weighted strength for the GDR. The approximated decoupling scheme used in deriving Eq. (4), which is well known in the theory of the electron-phonon interaction in the lattice [14], is fairly course. In particular, it is insufficient to take into account the correlations between multiphonon configurations, which form the DGDR. As will be seen later, this approximation means that the DGDR is constructed from noninteracting phonons (or independent harmonic oscillators).

A complete relation for the EWSS $S_1^{(2)}$, including the contribution which arises from the multiphonon structure of the DGDR, can be derived considering a general nuclear Hamiltonian with two-body interaction. For simplicity, without reducing the generality of the results, we consider a two-body interaction in the separable form

$$H=\frac{1}{2}\sum_s \epsilon_s(B_{ss}^\dagger+B_{ss})-\frac{1}{4}k\sum_{kk'll'} f_{kl}f_{k'l'}(B_{kl}^\dagger+B_{kl})\times(B_{k'l'}^\dagger+B_{k'l'}). \tag{6}$$

The subscripts k,k',l,l' in Eq. (6) run over all particle (p) and hole (h) configurations. Hence the pair operators $B_{kl}^\dagger\equiv a_k^\dagger a_l$ represent the ph, pp, and hh pairs accordingly.

The phonon operator, which generates the collective ph excitation, is introduced in a standard way as

$$Q_\alpha^\dagger=\sum_{ph}(X_{ph}^\alpha B_{ph}^\dagger-Y_{ph}^\alpha B_{ph}). \tag{7}$$

Equation (7) and its adjoint can be inverted to give

$$B_{ph}^\dagger=\sum_\alpha (X_{ph}^\alpha Q_\alpha^\dagger+Y_{ph}^\alpha Q_\alpha). \tag{8}$$

Using Eq. (8), any one-body operator, including the dipole one D, can be expressed in terms of the phonon operators $Q_\alpha^\dagger$ and Q_α as

$$D=\sum_\alpha \mathcal{M}_\alpha(Q_\alpha^\dagger+Q_\alpha), \tag{9}$$

$$\mathcal{M}_\alpha=\sum_{ph}F_{ph}(X_{ph}^\alpha+Y_{ph}^\alpha),$$

where F_{ph} is the matrix element for the electric transition from the ground state to the state with multipolarity λ and parity π. For the $E1$ transitions one has $\lambda^\pi=1^-$. Equation (8) also allows us to transform all the ph pair operators $B_{ph}^\dagger$ and B_{ph} in the two-body interaction part of the Hamiltonian in Eq. (6) into the phonon ones $Q_\alpha^\dagger$ and Q_α. The Hamiltonian can be then expressed in terms of three parts H_Q, H_{QB}, and H_{BB}, which we are going to analyze below.

The part H_Q consists of only phonon operators and can be diagonalized within the random-phase approximation (RPA). The RPA solution yields the phonon energy ω_α, and the RPA amplitudes X_{ph}^α and Y_{ph}^α of the ph excitation, generated by the phonon operator $Q_\alpha^\dagger$ in Eq. (7), when it acts on the ground state $|0\rangle$ of an even-even nucleus, which serves as the phonon vacuum:

$$Q_\alpha|0\rangle=0. \tag{10}$$

Therefore the part H_Q can be represented as

$$H_Q=\sum_\alpha \omega_\alpha Q_\alpha^\dagger Q_\alpha, \tag{11}$$

which is a sum of independent harmonic oscillators with frequencies ω_α.

The part H_{QB} is a sum of terms, which are proportional to $(Q_\alpha^\dagger+Q_\alpha)(B_{ss'}^\dagger+B_{ss'})+\text{H.c.}$, where $\{ss'\}$ denotes the indices of a particle-particle or a hole-hole pair: $\{ss'\}=\{pp'\}$ and $\{hh'\}$. Using the exact commutation relations

$$[Q_\alpha,B_{pp'}^\dagger]=\sum_{\beta h_1}[\mathcal{U}_{ph_1p'h_1}^{\alpha\beta}Q_\beta+\mathcal{V}_{ph_1p'h_1}^{\alpha\beta}Q_\beta^\dagger], \tag{12}$$

$$[Q_\alpha,B_{hh'}^\dagger]=-\sum_{\beta p_1}[\mathcal{U}_{p_1h'p_1h}^{\alpha\beta}Q_\beta+\mathcal{V}_{p_1h'p_1h}^{\alpha\beta}Q_\beta^\dagger], \tag{13}$$

with

$$\mathcal{U}_{php'h'}^{\alpha\beta}=X_{ph}^\alpha X_{p'h'}^\beta+Y_{ph}^\beta Y_{p'h'}^\alpha,$$

$$\mathcal{V}_{php'h'}^{\alpha\beta}=-(X_{ph}^\alpha Y_{p'h'}^\beta+X_{ph}^\beta Y_{p'h'}^\alpha), \tag{14}$$

and their adjoints, it is easy to see that

$$\langle 0|[D,[H_{QB},D]]|0\rangle=0, \tag{15}$$

$$\langle 0|[D^2,[H_{QB},D^2]]|0\rangle=0. \tag{16}$$

Equations (15) and (16) can also be obtained using the expansion of $B_{ss'}^\dagger$ in terms of products of even numbers of ph pair operators $B_{ph}^\dagger$ and B_{ph}. The lowest-order terms of this expansion can be found in Ref. [15]. Therefore, the LHS of Eqs. (15) and (16) is always the expectation value of odd numbers of phonon operators over the phonon vacuum and, hence, always vanishes.

The last part H_{BB} can rewritten as

$$H_{BB}=\sum_{pp'hh'}\overline{V}_{pp',hh'}B_{pp'}^\dagger B_{hh'}^\dagger+\sum_{p_1p_1'p_2p_2'}V_{p_1p_1',p_2p_2'}B_{p_1p_1'}^\dagger B_{p_2p_2'}^\dagger+\sum_{h_1h_1'h_2h_2'}V_{h_1h_1',h_2h_2'}B_{h_1h_1'}^\dagger B_{h_2h_2'}^\dagger, \tag{17}$$

where

$$V_{p_1p_1',p_2p_2'}=\tfrac{1}{4}f_{p_1p_1'}f_{p_2p_2'},\quad V_{h_1h_1',h_2h_2'}=\tfrac{1}{4}f_{h_1h_1'}f_{h_2h_2'},$$

$$V_{pp',hh'}=\tfrac{1}{4}f_{pp'}f_{hh'},\quad \overline{V}_{kl,k'l'}=V_{kl,k'l'}+V_{k'l',kl}. \tag{18}$$

Applying the exact commutation relations in Eqs. (12) and (13), we calculate the expectation value of the double commutator $\langle 0|[D,[H_{BB},D]]|0\rangle$ as

$$\langle 0|[D,[H_{BB},D]]|0\rangle=\langle 0|[D,[H_{BB}^{(1)},D]]|0\rangle+\langle 0|[D,[H_{BB}^{(2)},D]]|0\rangle. \quad (19)$$

The first term on the RHS of Eq. (19) is equal to

$$\langle 0|[D,[H_{BB}^{(1)},D]]|0\rangle=2\sum_{\alpha\alpha'}\mathcal{M}_\alpha\mathcal{M}_{\alpha'}\sum_{pp'hh'\beta}\Bigg[\mathcal{F}^{\alpha\beta}_{php'h}\sum_{p_1p_1'}V_{pp',p_1p_1'}\mathcal{F}^{\alpha'\beta}_{p_1'h'p_1h'}+\mathcal{F}^{\alpha\beta}_{ph'ph}\sum_{h_1h_1'}V_{hh',h_1h_1'}\mathcal{F}^{\alpha'\beta}_{p'h_1p'h_1'}-\overline{V}_{pp',hh'}\sum_{p_1h_1}\mathcal{F}^{\alpha\beta}_{ph_1p'h_1}\mathcal{F}^{\alpha'\beta}_{p_1hp_1h'}\Bigg], \quad (20)$$

while the double commutator in the second term is

$$[D,[H_{BB}^{(2)},D]]=\sum_{\alpha\beta}\mathcal{M}_\alpha\mathcal{M}_\beta\Bigg\{\sum_{pp'hh'}\overline{V_{pp',hh'}}\Bigg[\sum_{h_1}(\mathcal{F}^{\alpha\beta}_{p'h_1ph_1}+\mathcal{F}^{\alpha\beta}_{ph_1p'h_1})B^\dagger_{hh'}-\sum_{p_1}(\mathcal{F}^{\alpha\beta}_{p_1hp_1h'}+\mathcal{F}^{\alpha\beta}_{ph_1p'h_1})B^\dagger_{pp'}\Bigg]$$
$$+\sum_{pp'p_1p_1'}V_{pp',p_1p_1'}\sum_{h_1}[(\mathcal{F}^{\alpha\beta}_{p'h_1ph_1}+\mathcal{F}^{\alpha\beta}_{ph_1p'h_1})B^\dagger_{p_1p_1'}+(\mathcal{F}^{\alpha\beta}_{p_1'h_1p_1h'}+\mathcal{F}^{\alpha\beta}_{p_1h_1p_1'h_1})B^\dagger_{pp'}]$$
$$-\sum_{hh'h_1h_1'}V_{hh',h_1h_1'}\sum_{p_1}[(\mathcal{F}^{\alpha\beta}_{p_1hp_1h'}+\mathcal{F}^{\alpha\beta}_{p_1h'p_1h})B^\dagger_{h_1h_1'}+(\mathcal{F}^{\alpha\beta}_{p_1h_1p_1h_1'}+\mathcal{F}^{\alpha\beta}_{p_1h_1'p_1h_1})B^\dagger_{hh'}]\Bigg\}. \quad (21)$$

In Eqs. (20) and (21) we introduce the shorthand notation

$$\mathcal{F}^{\alpha\beta}_{ph,p'h'}=\mathcal{U}^{\alpha\beta}_{php'h'}+\mathcal{V}^{\alpha\beta}_{p'h'ph}, \quad (22)$$

with $\mathcal{U}^{\alpha\beta}_{php'h'}$ and $\mathcal{V}^{\alpha\beta}_{p'h'ph}$ defined in Eqs. (14).

We now see that Eq. (20) can be combined with the expectation value of the double commutator $\langle 0|[D,[H_Q,D]]|0\rangle$ for the harmonic oscillators' part H_Q in Eq. (11) in calculating the EWSS as

$$\langle 0|[D,[H_Q+H_{BB}^{(1)},D]]|0\rangle=2\sum_{\alpha\alpha'}\mathcal{M}_\alpha\mathcal{M}_{\alpha'}\Bigg[\omega_\alpha\delta_{\alpha\alpha'}+\sum_{pp'p_1p_1'}V_{pp',p_1p_1'}\sum_{\beta hh'}\mathcal{F}^{\alpha\beta}_{php'h}\mathcal{F}^{\alpha'\beta}_{p_1'h'p_1h'}+\sum_{hh'h_1h_1'}V_{hh',h_1h_1'}\sum_{\beta pp'}\mathcal{F}^{\alpha\beta}_{ph'ph}\mathcal{F}^{\alpha'\beta}_{p'h_1p'h_1'}-\sum_{pp',hh'}\overline{V}_{pp',hh'}\sum_{\beta p_1h_1}\mathcal{F}^{\alpha\beta}_{ph_1p'h_1}\mathcal{F}^{\alpha'\beta}_{p_1hp_1h'}\Bigg]. \quad (23)$$

It is well known that the expectation value of the double commutator on the LHS of Eq. (23) for the electric dipole operator D is model independent and equal to twice of the Thomas-Reich-Kuhn (TRK) sum rule NZ/A [13]. Thus, the part $H_{BB}^{(1)}$ of the Hamiltonian is included to improve the RPA energy. This can be clearly seen in the diagonal approximation of Eq. (23) ($\alpha=\alpha'$),

$$\langle 0|[D,[H_Q+H_{BB}^{(1)},D]]|0\rangle=2\sum_\alpha\mathcal{M}_\alpha^2\tilde{\omega}_\alpha, \quad (24)$$

where the renormalized phonon energy $\tilde{\omega}_\alpha$ is

$$\tilde{\omega}_\alpha=\omega_\alpha+\Delta\omega_\alpha,$$

$$\Delta\omega_\alpha=\sum_{pp'hh'}\Bigg[\sum_{p_1p_1'}V_{pp',p_1p_1'}\mathcal{F}^{\alpha\beta}_{php'h}\mathcal{F}^{\alpha\beta}_{p_1'h'p_1h'}+\sum_{h_1h_1'}V_{hh',h_1h_1'}\mathcal{F}^{\alpha\beta}_{ph'ph}\mathcal{F}^{\alpha\beta}_{p'h_1p'h_1'}-\overline{V}_{pp',hh'}\sum_{p_1h_1}\mathcal{F}^{\alpha\beta}_{ph_1p'h_1}\mathcal{F}^{\alpha\beta}_{p_1hp_1h'}\Bigg]. \quad (25)$$

The estimation within the perturbation theory in Ref. [16] has shown that the correction $\Delta\omega_\alpha$ due to the part $H_{BB}^{(1)}$ is noticeable only for low-lying collective states in nuclei, which belong to the transitional region form spherical to deformed nuclei. In the region of GDR and DGDR this correction is expected to be small.

The most important part, which will be shown in the present paper to affect strongly the DGDR sum rule, comes from $H_{BB}^{(2)}$ in Eqs. (19) and (21). In the RPA, assuming the validity of the quasiboson approximation (QBA)

$$\langle 0|B^\dagger_{pp'}|0\rangle\simeq 0, \quad \langle 0|B^\dagger_{hh'}|0\rangle\simeq 0, \quad (26)$$

the expectation value of the double commutator in Eq. (21) over the ground state in Eq. (10) [the second term at the RHS of Eq. (19)] vanishes. This feature together with the small contribution of the part $H_{BB}^{(1)}$, mentioned above, is the reason why the part H_{BB} of the Hamiltonian was usually neglected in all the calculations of the GDR and other single-phonon giant resonances such as giant quadrupole resonances, etc. [18]. Doing so and taking into account Eq. (15), one would have, in this approximation,

$$\langle 0|[D,[D,H]]|0\rangle \simeq \langle 0|[D,[D,H_Q]]|0\rangle,$$

$$\langle 0|[D^2,[D^2,H]]|0\rangle \simeq \langle 0|[D^2,[D^2,H_Q]]|0\rangle, \qquad (27)$$

and the sum rule constraint in Eq. (4) would hold. This would mean that the effects given by the total nuclear Hamiltonian on the EWSS for the DGDR were the same as those given by its harmonic part. However, while in studying the GDR the part H_{BB} could be neglected also because it consists of terms of higher order in the number of phonon operators (four and higher) as compared to H_Q and H_{QB} in the boson expansion, it must be taken into account in the study of the DGDR and other multiphonon resonances. The reason is in studying the DGDR the contribution of three-phonon (and higher) terms and their interference with one- and two-phonon terms in the wave function of the DGDR state become decisively important [12]. These terms have the same order of phonon numbers as compared to the part H_{BB}, and hence the effect of the latter can be no more neglected in principle. The renormalized RPA, which includes the ground-state correlations beyond the RPA (see, e.g., Ref. [15]), allows an evaluation of the expectation values on the LHS of Eqs. (26) in the diagonal approximation, but not for the expectation value of the double commutator $\langle 0|[D^2,[H_{BB}^{(2)},D^2]]|0\rangle$. We shall show in the following that $\langle 0|[D^2,[H_{BB}^{(2)},D^2]]|0\rangle$ is not zero even within the QBA.

Taking into account the renormalization in Eqs. (23)–(25), together with Eqs. (4) and (16), we can write the complete EWSS for the DGDR in Eq. (1) in the form

$$S_1^{(2)} = (\widetilde{S}_1^{(2)})_{\rm har} + \Delta S_1^{(2)}, \qquad (28)$$

where the tilde means that in the sum $(S_1^{(2)})_{\rm har}$ in the harmonic limit [Eq. (4)] is evaluated with the renormalized phonon energy $\widetilde{\omega}_\alpha$ [Eqs. (23)–(25)]. The deviation $\Delta S_1^{(2)}$ from the value $(\widetilde{S}_1^{(2)})_{\rm har}$ is equal to

$$\Delta S_1^{(2)} = \langle 0|[D^2,[H_{BB}^{(2)},D^2]]|0\rangle = \langle 0|D[D,[H_{BB}^{(2)},D]]D|0\rangle + \langle 0|D^2[D,[H_{BB}^{(2)},D]]|0\rangle + \langle 0|[D,[H_{BB}^{(2)},D]]D^2|0\rangle. \qquad (29)$$

Using Eqs. (9) and (21), it follows that the last two terms on the RHS of Eq. (29) vanish in the QBA [Eq. (26)], while the first term can be calculated exactly, applying the commutation relations in Eqs. (12) and (13). The result yields

$$\Delta S_1^{(2)} = \sum_{\alpha\alpha'\beta\beta'} M_\alpha M_{\alpha'} M_\beta M_{\beta'} \Bigg\{ \sum_{p_1p_1'h_1h_1'} \Bigg[\sum_{pp'} \overline{V}_{pp',p_1p_1'} (\mathcal{F}^{\alpha\beta}_{p'h_1ph_1} + \mathcal{F}^{\alpha\beta}_{ph_1p'h_1}) \mathcal{U}^{\alpha'\beta'}_{p_1h_1'p_1'h_1'} + \sum_{hh'} \overline{V}_{hh',h_1h_1'} (\mathcal{F}^{\alpha\beta}_{p_1hp_1h'} + \mathcal{F}^{\alpha\beta}_{p_1h'p_1h}) \mathcal{U}^{\alpha'\beta'}_{p_1'h_1'p_1'h_1} \Bigg] - \sum_{pp'hh'} \sum_{p_1h_1} \overline{V}_{pp',hh'} [(\mathcal{F}^{\alpha\beta}_{p'h_1ph_1} + \mathcal{F}^{\alpha\beta}_{ph_1p'h_1}) \mathcal{U}^{\alpha'\beta'}_{p_1h'p_1'h} + (\mathcal{F}^{\alpha\beta}_{p_1hp_1h'} + \mathcal{F}^{\alpha\beta}_{p_1h'p_1h}) \mathcal{U}^{\alpha'\beta'}_{ph_1p'h_1}] \Bigg\}. \qquad (30)$$

Thus, we have shown that there is a finite deviation $\Delta S_1^{(2)}$ from the value in the harmonic limit $(\widetilde{S}_1^{(2)})_{\rm har}$ for the EWSS of the DGDR, and this deviation arises from the part $H_{BB}^{(2)}$, even though the latter does not contribute in the EWSS for the GDR. It is worth noticing that, although the three last terms on the RHS of Eqs. (23), (25), and (30) are noncoherent sums, Eq. (30) contains the interaction $\overline{V}$ instead of V. Numerical calculations in Ref. [17] showed that noncoherent sums can be small in several cases, depending on the effective interaction. In the next section we will see how varying the interaction parameter can affect the quantity in Eq. (30) in a simplified schematic model.

III. RESULTS IN SCHEMATIC MODEL

The advantage of using the sum rule approach to study the collective properties of complex nuclear systems is well known. As the sum rules do not depend on the detail structure of the single-particle levels and interactions, even simplified schematic models are able to reveal important features when applied to the sum rules. In this section we will estimate the deviation $\Delta S_1^{(2)}$ in Eq. (30) of the EWSS for the DGDR from its value in the harmonic limit within the framework of a schematic model, where the RPA has the analytical solution.

We notice that the RPA solution for the Hamiltonian in Eq. (6) is already rather simple as it comes from the dispersion relation [13,18]

$$1 - 2k \sum_{ph} \frac{f_{ph}^2(\epsilon_p - \epsilon_h)}{(\epsilon_p - \epsilon_h)^2 - \omega^2} = 0. \qquad (31)$$

Let us study it more closely in the degenerate case, putting all $(\epsilon_p - \epsilon_h) = \epsilon$ and all the matrix elements $f_{ph} = f$, $f_{pp'} = f_1$, and $f_{hh'} = f_2$ with a degeneracy $\Omega = N$, where N is

TABLE I. Parameters of the schematic model at several Ω.

Ω	TRK sum rule	ϵ (MeV)	F (MeV)	V_1-V (MeV)
40	5.0	10.0	0.11	5.0×10^{-3}
136	16.28	7.0	0.13	4.0×10^{-3}
208	24.84	6.0	0.14	1.0×10^{-3}

the particle number. The X and Y amplitudes of the phonon operator in Eq. (7) have a very simple form in this schematic model:

$$X=\frac{1}{2}\frac{(\epsilon+\omega)}{\sqrt{\Omega\epsilon\omega}},\quad Y=\frac{1}{2}\frac{(\epsilon-\omega)}{\sqrt{\Omega\epsilon\omega}}. \tag{32}$$

The phonon energy and its renormalized value in Eq. (25) are

$$\omega=\epsilon\sqrt{1+8\frac{\Omega V_{ph}}{\epsilon}}, \tag{33}$$

where $V_{ph}=-\frac{1}{4}kf^2$ and

$$\widetilde{\omega}=\omega+2\Omega^2(V_1-V)(X+Y)^2, \tag{34}$$

where $V_1=-\frac{1}{8}k(f_1^2+f_2^2)$ and $V=-\frac{1}{4}kf_1f_2$, respectively. The difference V_1-V is equal to $-\frac{1}{8}k(f_1-f_2)^2$. Therefore V_1-V is always positive provided $f_1\neq f_2$ since $k<0$ for the GDR. Here the completeness relations of the X and Y amplitudes give a Ω^2 factor instead of Ω^3 in this case, but this is not the case for the complete sum of the DGDR strengths below due to different indices in Eq. (30).

The EWSS for the DGDR in the harmonic limit defined in Eq. (4) becomes

$$(S_1^{(2)})_{\rm har}=4\omega\mathcal{M}^4, \tag{35}$$

with the EWSS, for the GDR,

$$S_1^{(1)}=\omega\mathcal{M}^2. \tag{36}$$

The complete EWSS for the DGDR in Eq. (28) becomes

$$S_1^{(2)}=4\mathcal{M}^4\{\omega+2\Omega^2(X+Y)^2(V_1-V)[1+\Omega(X^2+Y^2)]\}. \tag{37}$$

This sum is greater than its value in the harmonic limit $(S_1^{(2)})_{\rm har}$ if $V_1>V$, which is always the case when $f_1\neq f_2$. Thus we have found in this schematic model a possibility for the enhancement of the EWSS for the DGDR as compared to the harmonic limit.

We calculated the energies and EWSS for the GDR and DGDR in Eqs. (33)–(37) as a function of the interaction parameter $\Omega V/\epsilon\geqslant 0$ at various values of Ω. The parameters ϵ and the matrix element F of the dipole operator D in Eq. (9) are adjusted to fulfill the TRK sum rule. For simplicity we also put $V_{ph}=V$. The pp and hh interaction parameter V_1 is chosen so that the renormalized phonon energy $\widetilde{\omega}$ does not deviate appreciably from the RPA value ω to secure the small contribution of the part $H_{BB}^{(1)}$ [16]. The values of these parameters for $\Omega=$ 40, 136, and 208, which imitate ^{40}Ca, ^{136}Xe, and ^{208}Pb using the TRK sum rules, are listed in the Table I.

The energy ω in Eq. (33) (dotted curves) and its renormalized value $\widetilde{\omega}$ in Eq. (34) (solid curve) are displayed in Fig. 1 for $\Omega=$ 40 and 208 as a function of unitless interaction parameter $\Omega V/\epsilon$. The correction due to the term $H_{BB}^{(1)}$ [Eq. (24)] is small thanks to the parameters selected as in Table I. As a consequence the EWSS for the GDR calculated with the renormalized phonon energy $\widetilde{\omega}$ is slightly greater as compared to that calculated with the energy ω and the difference increases with increasing the interaction parameter (Fig. 2). The EWSS for the DGDR calculated with the renormalized phonon energy $\widetilde{\omega}$ and with the part $H_{BB}^{(2)}$ taken into account in Eq. (37) (top solid curves in Fig. 2), however, strongly differ from its value in the harmonic oscillators' limit (top dotted curves) especially as a stronger interaction parameter. At the value of $\Omega V/\epsilon\simeq 0.4$, where ω is about the GDR energy the complete EWSS $S_1^{(2)}$ is about 2.4 times larger than the value $(S_1^{(2)})_{\rm har}$ in the harmonic limit for $\Omega=$ 208 [Fig. 2(b)]. For $\Omega=$ 40 this ratio is about 1.6 [Fig. 2(a)]. From Eq. (37) it is seen that the deviation from the harmonic limit is a linear function of V_1-V. Therefore the larger the difference f_1-f_2 between pp and hh matrix elements, the stronger the enhancement will be for the EWSS of the

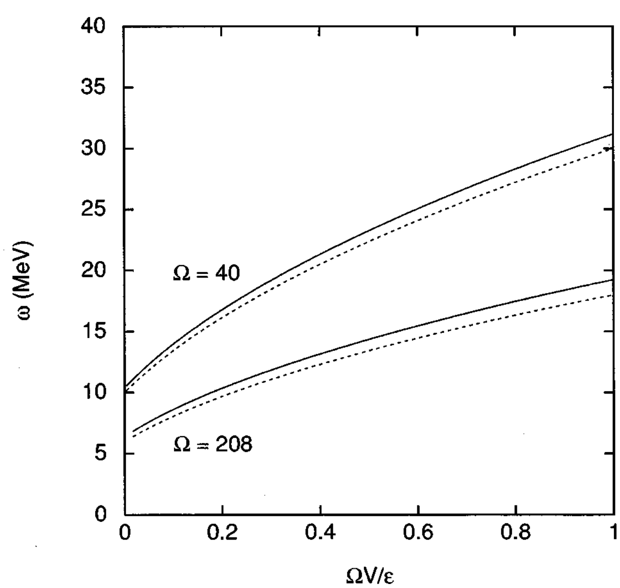

FIG. 1. Phonon energy ω (dotted line) and its renormalized value $\widetilde{\omega}$ due to the part $H_{BB}^{(1)}$ (solid line) as a function of the interaction parameter $\Omega V/\epsilon$.

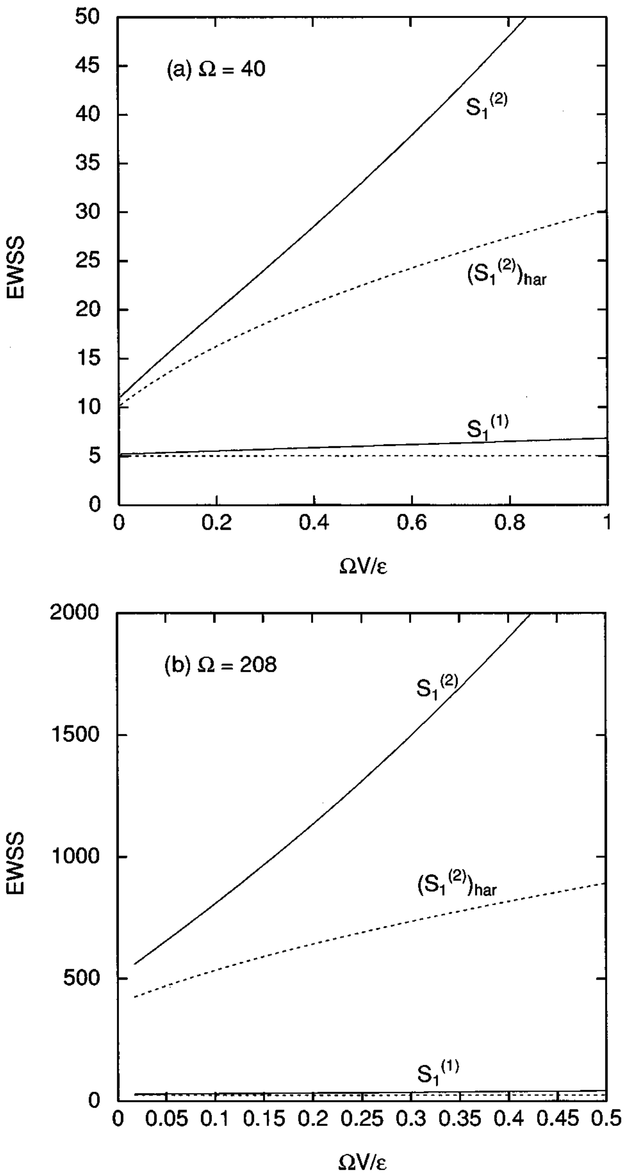

FIG. 2. EWSS $S_1^{(1)}$ for GDR and $S_1^{(2)}$ for DGDR at $\Omega = 40$ (a) and 208 (b). For the $S_1^{(1)}$ the notation is as in Fig. 1. For $S_1^{(2)}$ the dotted curves denote the values $(S_1^{(2)})_{\mathrm{har}}$ in the harmonic limit while the solid curves represent the complete $S_1^{(2)}$.

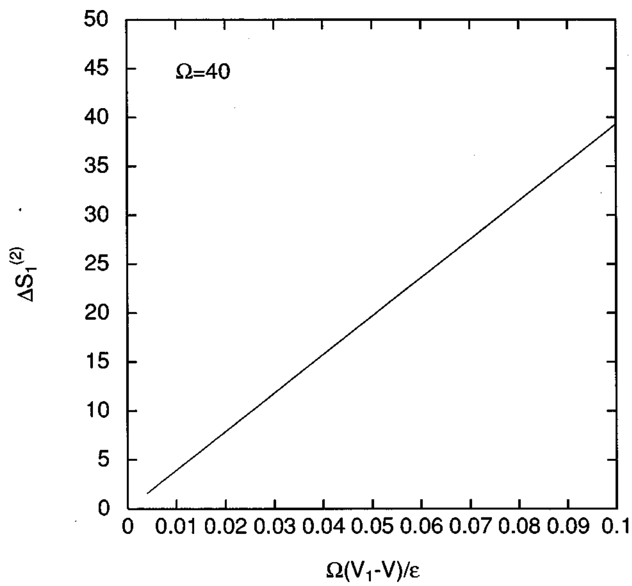

FIG. 3. Deviation $\Delta S_1^{(2)}$ from the harmonic limit ($\Omega = 40$).

DGDR as compared to its value in the harmonic limit. This feature is clearly seen in Fig. 3, which displays the deviation from the harmonic limit as a function of the difference $\Omega(V_1 - V)/\epsilon$ for $\Omega = 40$.

IV. CONCLUSIONS

In the present paper we have shown the importance of the part containing the products of the pp and hh pair operators (the so-called scattering terms) of the nuclear Hamiltonian in calculating the EWSS for the DGDR. This part is known to have a little influence on the phonon energy within the RPA except for only the lowest levels in transitional and deformed nuclei [16]. In the region of the GDR the contribution of this part is negligible. Therefore it has been neglected in all microscopic calculations of the GDR so far. It is shown in the present paper that it is no more true in the study of the DGDR. Even though the pp and hh pair operators cannot be expressed exactly in term of ph phonon operators, one can use the exact commutation relations of the former and the latter. The complete EWSS for the DGDR has been derived here for the first time and a possible deviation from the value of this EWSS in the harmonic limit has been found. The formalism has been tested within a framework of a degenerate two-level model, where the RPA has the simplest analytical solution. We found an example when this deviation is quite large and enhances strongly the EWSS for the DGDR at various particle numbers, while the EWSS for the GDR and the GDR energy change only slightly.

The present study, even though with the numerical calculations within a schematic model, shows that it is decisively important to include the full nuclear Hamiltonian in the study of the DGDR because some part of the Hamiltonian, while having a small influence on the GDR, may affect strongly the DGDR characteristics. It also shows that the DGDR is not a superposition of independent harmonic oscillators, but a complex of interacting multiphonon configurations. The latter makes the DGDR properties strongly deviated from the harmonic picture.

ACKNOWLEDGMENTS

The authors are grateful to H. Sagawa (University of Aizu), T. Suzuki (Nihon University), and T. Suzuki (University of Fukui) for fruitful discussions. N.D.D. acknowledges with thanks the financial support of Science and Technology Agency of Japan and the JISTEC, under which this work was done. He is also grateful to the Cyclotron Laboratory of RIKEN for the warm hospitality extended to him during his stay at RIKEN.

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